

DISCUSSION PAPER

November 2016

No. 78

Pollution from Cournot Duopoly  
and the Effect of Ambient Charges

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# Pollution from Cournot Duopoly Industry and the Effect of Ambient Charges

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## **Abstract**

From the perspective of oligopoly theory, this paper clarifies the effectiveness of ambient charges as a policy measure for reducing industrial non-point source pollution. Ganguli and Raju (2012) argue that ambient charges have perverse effects in the context of Bertrand competition. By contrast, this paper examines the effects of ambient charges in the context of Cournot competition, clarifying that they are an effective policy measure. The result presented here is consistent with previous results in the field of experimental economics.

Keywords: duopoly market; emissions; ambient charges; Cournot competition; non-point source pollution

JEL Classification: Q58, L13.

# **Pollution from Cournot Duopoly Industry and the Effect of Ambient Charges**

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## **1. Introduction**

This paper demonstrates the effectiveness of environmental policies that use ambient charges as a way to reduce pollutant emissions from Cournot duopoly industries.

Segerson (1988) suggests a scheme for controlling non-point source pollution (the existence of multiple sources of pollution). Her scheme envisages having emission cut-off levels for entire industries, not just for the individual firms in the industry. Any exceeding of the cut-off levels should result in all of the firms belonging to the relevant industry having to pay the same fines. Conversely, if the emissions stay below the cut-off level, then all of the firms belonging to that industry should be awarded the same subsidies.

Segerson's scheme addresses the de facto impossibility of a government measuring firm-specific emissions, by standardising fines (and subsidies) for all firms in one industry, regardless of

differences in actual emissions.

This paper follows Ganguli and Raju (2012) in referring to Segerson's (1988) scheme as ambient charges. The effectiveness of ambient charges as a policy measure is at the research stage. Ambient charges have never been practiced. There are studies on the efficacy of ambient charges in the field of experimental economics (e.g. Spraggon, 2002; Poe et al., 2004). However, there are few ambient charges studies in theoretical economics, and even fewer in oligopoly theory. The sole exception is Ganguli and Raju (2012), whose results merit attention.

They argue that the environmental effects brought about by ambient charges in the context of Bertrand competition are 'perverse'. Their results imply that if a government were to implement ambient charges to reduce industrial emissions, then this would actually result in an opposite effect—an increase in industrial emissions.

This paper demonstrates the effectiveness of ambient charges in the context of Cournot competition. It theoretically demonstrates that if a government were to implement ambient charges to reduce industrial emissions, then this would actually result in such a reduction. This result is in agreement with experimental economics results that relate to non-cooperative firm behaviour (e.g. Spraggon, 2002).

The paper explains the Cournot model in Section 2, and derives the Cournot equilibrium in Section 3. Section 4 concludes the paper with a summary.

## 2. The Cournot competition model

Suppose that there are two firms in the same industry, producing a homogenous product. The production quantity of firm  $i, i = 1, 2$  is represented as  $q_i$ . The market demand function for the product produced by these firms is defined as,

$$p = a - b(q_1 + q_2), \quad (1)$$

where  $p$  stands for the price on the product, and both  $a$  and  $b$  are positive constants. Further,  $a$  stands for the choke-off price, it is assumed to be a relatively large number. It is also assumed that the two firms' production technology is the same, and that the marginal costs of both firms are the positive constant  $c$ .

Firm  $i, i = 1, 2$  emits pollutants  $e_i q_i$  in connection with its production. It is not necessarily so that  $e_1$  and  $e_2$  have the same value. This means that the emission quantities may differ even when the

production quantities are the same. This can be said to reflect differences in levels of environmental technology (Ganguli and Raju, 2012) and, in industries with strong government (party) involvement (such as coal and oil industry in China), varying degrees of closeness to the bureaucracy.

It is possible for the government to measure an industry's total emission quantity (i.e.,  $e_1q_1 + e_2q_2$ ).

It is assumed that the environmental standard  $\bar{E}$  is provided exogenously.

Based on these assumptions, consider the following scheme. If  $e_1q_1 + e_2q_2 < \bar{E}$  (or  $e_1q_1 + e_2q_2 \geq \bar{E}$ ), then the government will award(levy) both firms the same subsidy(penalty), amounting to  $m$  times the difference between the total emission quantity and the environmental standard.

### **3. The Cournot equilibrium**

Both firms belonging to the same industry are mutually engaged in Cournot competition.

Based on the model described in Section 2, the profit function for firm  $i, i = 1, 2$  can be expressed as,

$$\begin{aligned}\pi_i &= pq_i - cq_i - m(e_1q_1 + e_2q_2 - \bar{E}) \\ &= (a - b(q_1 + q_2) - c - me_i)q_i - m(e_jq_j - \bar{E}), \quad i, j = 1, 2, j \neq i\end{aligned}\quad (2)$$

Equation (2) can be used to derive each firm's best response function, giving the following:

$$BR^i(q_j) = \frac{a - bq_j - c - me_i}{2b}, \quad i, j = 1, 2, j \neq i. \quad (3)$$

Solving the simultaneous equation that consists of firm 1 and 2's best response functions gives the

Cournot equilibrium. That is,

$$(q_1^*, q_2^*) = \left( \frac{a - c - m(2e_1 - e_2)}{3b}, \frac{a - c + m(e_1 - 2e_2)}{3b} \right). \quad (4)$$

When  $e_1$  and  $e_2$  are given, it follows that the industrial emission quantity realised under a Cournot

equilibrium can be calculated as,

$$e_1q_1^* + e_2q_2^* = \frac{(a - c)(e_1 + e_2) + 2m(e_1e_2 - e_1^2 - e_2^2)}{3b}. \quad (5)$$

This is a function of the policy parameter  $m$ , so denote  $e_1q_1^* + e_2q_2^*$  as a function  $E(m)$ , differentiating this function by  $m$ , gives the following:

$$E'(m) = \frac{2(e_1e_2 - e_1^2 - e_2^2)}{3b}. \quad (6)$$

With regard to equation (6), the sign of  $E'(m)$  corresponds to the sign of the bracketed part of the numerator (i.e.,  $e_1e_2 - (e_1^2 + e_2^2)$ ). Given  $e_1 > 0$  and  $e_2 > 0$ , the following proposition holds:

**Proposition 1.**  $E'(m) < 0$

(Proof)

Given  $e_1 > 0$  and  $e_2 > 0$ , the difference between the squares of  $\frac{e_1 + e_2}{2}$  and  $\sqrt{e_1e_2}$  is also positive. This is because,

$$\begin{aligned} & \left(\frac{e_1 + e_2}{2}\right)^2 - (\sqrt{e_1e_2})^2 \\ &= \frac{e_1^2 + 2e_1e_2 + e_2^2}{4} - e_1e_2 \\ &= \frac{e_1^2 - 2e_1e_2 + e_2^2}{4} \end{aligned}$$



$$= \frac{(e_1 - e_2)^2}{4}.$$

Thus, if  $e_1 > 0$  and  $e_2 > 0$ , then the following inequality holds:

$$\frac{e_1 + e_2}{2} > \sqrt{e_1 e_2}. \quad (7)$$

When both sides of equation (7) are squared, this gives,

$$e_1^2 + e_2^2 > 2e_1 e_2. \quad (8)$$

Since  $\text{Sgn}\{e_1 e_2 - (e_1^2 + e_2^2)\} < 0$ , we have the result as desired.

Q.E.D.

Following from Proposition 1, the total emission quantity realised under a Cournot equilibrium in a duopoly market is a decreasing function of ambient charges.

#### **4. Conclusion**

This paper uses oligopoly theory to examine the effectiveness of ambient charges, a policy measure for controlling industrial non-point source pollution.

Previous studies in the field of experimental economics are in agreement on the efficacy of ambient charges. By contrast, Ganguli and Raju (2012) argue from the perspective of oligopoly theory that ambient charges have perverse effects in the context of Bertrand competition. This paper has examined the effects of ambient charges in the context of Cournot competition, and supports the results gleaned from experimental economics, making clear that ambient charges remain an effective policy measure.

This paper has not touched on the issue of what choices firms make when it comes to environmental technology. Ganguli and Raju (2012) argue that in the context of Bertrand competition, policy effects remain perverse when firms are allowed to choose their environmental technology.

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