Average elasticity in the framework of the fixed effects logit model*

Yoshitsugu Kitazawa†

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Abstract
This note proposes the average elasticity of the logit probabilities with respect to the exponential functions of explanatory variables in the framework of the fixed effects logit model. The average elasticity is able to be calculated using the consistent estimators of parameters of interest and the average of binary dependent variables, regardless of the fixed effects.

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1. Introduction

With the aim of consistently estimating the parameters of interest after ruling out the fixed effects for the case with time dimension being fixed and cross-sectional size being large in the fixed effects logit model, an conditional maximum likelihood estimator (CMLE) and an estimator using the generalized method of moments (GMM) advocated by Hansen (1982) are proposed by Chamberlain (1980) and Kitazawa (2010), respectively.[1] However, since the fixed effect is not consistently estimated using both estimators, the marginal effect is not obtained.[2] In my best knowledge, it seems that no appropriate index measuring the effect of the change of the explanatory variable are developed for the case of the fixed effects logit model with time dimension being fixed.

This note proposes an index measuring the effect of the change of the explanatory variable on the change of the logit probability with which the binary dependent variable takes one. The average elasticity of the probability with respect to exponential of the explanatory variable is able to be calculated without relation to the fixed effects.

2. Average elasticity

The formula calculating the average elasticity of the logit probability with respect to exponential of the explanatory variable (with other variables held constant) is proposed for the case with two explanatory variables. The formula can be simply expanded for the case with multiple explanatory variables.

According to Kitazawa (2010), the fixed effects logit model with two explanatory variables is defined by using the following implicit form:

$$y_{it} = p_{it} + v_{it} \ , \qquad \text{for} \quad t = 1, \ldots, T \ , \qquad (1)$$

$$p_{it} = \exp(\psi_i + \delta w_{it} + \beta x_{it}) / (1 + \exp(\psi_i + \delta w_{it} + \beta x_{it})) \ , \qquad \text{for} \quad t = 1, \ldots, T \ , \qquad (2)$$

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† Correspondence: Faculty of Economics, Kyushu Sangyo university, Matsukadai 2-3-1, Higashi-ku, Fukuoka, Japan. E-mail: kitazawa@ip.kyusan-u.ac.jp

1 The origin of the CMLE proposed by Chamberlain (1980) is Rasch (1960), (1961).

2 Another solution to the incidental parameters problem presented by Neyman and Scott (1948) is the bias reduction estimation. Although it seems to be said that the (inconsistent) bias reduction estimators (see Arellano and Hahn, 2007, and Hsiao, 2010, for their reviews) are available to obtain the average marginal effect in moderately long panel, it is questionable whether they display their force in short panel.

where the probability with which the binary dependent variable y_{it} takes one is constructed by using the fixed effect Ψ_i, the parameters of interest δ and β, and the real-valued explanatory variables w_{it} and x_{it}.[3] The subscripts i and t denote the individual and the time, respectively. It is assumed that number of individuals $N\rightarrow\infty$, while number of time periods T is fixed. Under the assumption that

$$E[v_{it}|v_i^{t-1},\psi_i,x_i^{t+1},w_i^{t+1}]=0 \quad , \qquad \text{for} \quad t=1,\ldots,T \quad , \tag{3}$$

the following one set of moment conditions are able to be constructed according to Kitazawa (2010):

$$E[\Delta w_{it}(\Delta y_{it}-\tanh((\delta\Delta w_{it}+\beta\Delta x_{it})/2)(\Delta y_{it})^2)]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \tag{4}$$

$$E[\Delta x_{it}(\Delta y_{it}-\tanh((\delta\Delta w_{it}+\beta\Delta x_{it})/2)(\Delta y_{it})^2)]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \tag{5}$$

where Δ is the first-differencing operator. The GMM estimation jointly using the moment conditions (4) and (5) gives birth to the consistent estimators $\hat{\delta}$ and $\hat{\beta}$ for δ and β, respectively.[4]

The new index is constructed from now on. With $W_{it}=\exp(w_{it})$, the elasticity of the probability p_{it} with respect to the positive-valued variable W_{it} (with both Ψ_i and $X_{it}(=\exp(x_{it}))$ held constant) is defined as follows:

$$\eta_{it}^W=(\partial p_{it}/\partial W_{it})(W_{it}/p_{it})=\delta(1-p_{it}) \quad , \qquad \text{for} \quad t=1,\ldots,T \quad . \tag{6}$$

Under the assumption that $N\rightarrow\infty$, the overall average elasticity of p_{it} with respect to W_{it} is calculated with the following formula:

$$\bar{\eta}_A^W=\hat{\delta}(1-\bar{y}_A) \quad , \tag{7}$$

where $\hat{\delta}$ is the consistent estimator for δ such that $\mathrm{plim}_{N\rightarrow\infty}\hat{\delta}=\delta$, and $\bar{y}_A=T^{-1}N^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N}y_{it}$. Since p_{it} is the probability and $E[v_{it}]=0$ (and accordingly variances of v_{it} are finite), it can be seen that $\mathrm{plim}_{N\rightarrow\infty}\bar{\eta}_A^W=\delta(1-\phi_A)$, if $\phi_A=\lim_{N\rightarrow\infty}(T^{-1}N^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N}E[p_{it}])$ (which is referred to as the average logit probability in this paper).[5] Similarly, the overall elasticity of the probability p_{it} with respect to X_{it} is defined as follows:

3 The implicit form is also used by Blundell et al. (2002) in count panel data.

4 It should be noted that for $T=2$, the GMM estimator using the moment conditions (4) and (5) is equivalent to the CMLE proposed by Chamberlain (1980) (see Kitazawa, 2010), while for $T>2$, the CMLE elaborately designed by Chamberlain (1980) is inconsistent under the assumption (3).

5 Just in case, it is assumed that both $E[p_{it}]$ and $\mathrm{Var}[p_{it}](<\infty)$ exist for each i and t. However, I think that it seems that this assumption is satisfied in any case.

$$\bar{\eta}_A^X = \hat{\beta}(1-\bar{y}_A) \quad , \tag{8}$$

where $\hat{\beta}$ is the consistent estimator for β such that $\text{plim}_{N\to\infty}\hat{\beta}=\beta$.

In addition, the cross-section average elasticity for a specific time period and the group average elasticity for a group (e.g. a gender) are able to be calculated as follows: The formula calculating the cross-section average of p_{it} with respect to W_{it} for period t is

$$\bar{\eta}_t^W = \hat{\delta}(1-\bar{y}_t) \quad , \tag{9}$$

where $\bar{y}_t = N^{-1}\sum_{i=1}^{N} y_{it}$, while that calculating the group average elasticity for group G in population is

$$\bar{\eta}_G^W = \hat{\delta}(1-\bar{y}_G) \quad , \tag{10}$$

where $\bar{y}_G = T^{-1} N_G^{-1} \sum_{t=1}^{T} \sum_{i=1}^{N_G} y_{it}$ with N_G being number of individual units belonging to group G .

References

Arellano M., Hahn J., 2007. Understanding bias in nonlinear panel models: some recent developments, In *Advances in Economics and Econometrics*, Blundell R., Newey W., Persson T. (eds). Cambridge University Press: Cambridge.

Blundell, R., Griffith, R., Windmeijer, F., 2002. Individual effects and dynamics in count data models. *Journal of Econometrics* **108**, 113-131.

Chamberlain, G., 1980. Analysis of covariance with qualitative data. *Review of Economic Studies* **47**, 225-238.

Hansen, L.P., 1982. Large sample properties of generalized method of moments estimators. *Econometrica* **50**, 1029-1054.

Hsiao, C., 2010. Longitudinal data analysis, In *Microeconometrics*, Durlauf, S.N., Blume, E.B. (eds). Palgrave and Macmillan.

Kitazawa, Y., 2010. A hyperbolic transformation for a fixed effects logit model. *Kyushu Sangyo University, Faculty of Economics, Discussion Paper*, No. **43**.

Neyman, J., Scott, E.L., 1948. Consistent estimates based on partially consistent observations, Econometrica, Vol. 16, pp. 1-32.

Rasch, G., 1960. *Probabilistic Models for Some Intelligence and Attainment Tests*, The Danish Institute for Educational Research. (Japanese version of expanded edition (1980) with foreword and afterword by B.D. Wright. Chicago: The University of Chicago Press, entitled *Shinri Tesuto no Kakuritsu Moderu*, the translation supervised by Y. Uchida, Nagoya Daigaku Shuppankai, 1985).

Rasch, G., 1961. On general laws and the meaning of measurement in psychology, *Preceeding of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* **4**. 321-333.