A forward demeaning transformation for a dynamic count panel data model*

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Abstract
In this note, a forward demeaning transformation is proposed for the linear feedback model with explanatory variables being strictly exogenous on count panel data. This transformation is analogous to that proposed by Arellano and Bover (1995) for the ordinary dynamic panel data model.

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Keywords: forward demeaning; linear feedback model; strictly exogenous explanatory variables; count panel data

1. Introduction
Some consistent estimators are proposed for the linear feedback model (LFM hereafter) advocated by Blundell et al. (2002) on count panel data (e.g. Blundell et al., 1999 and 2002 and Kitazawa, 2007, 2009a and 2009b). In this paper, another consistent estimator is proposed by using the forward demeaning transformation similar to that proposed by Arellano and Bover (1995) for the ordinary dynamic panel data model. This estimator is consistent for the case of strictly exogenous explanatory variables.

The rest of the paper is as follows. Section 2 presents the model, transformation and moment conditions. Section 3 reports some Monte Carlo evidences for the GMM estimator (proposed by Hansen, 1982) using the moment conditions. Section 4 concludes.

2. Model, transformation and moment conditions
The LFM is written as follows:

$$y_{it}=\gamma y_{i,t-1}+\exp(\beta x_{it}+\eta_i)+v_{it} \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \tag{2.1}$$

where i denotes the individual unit with $i=1,\ldots,N$, t denotes the time period and it is assumed that T is fixed and $N\rightarrow\infty$. The observable variables y_{it} and x_{it} are the non-negative integer-valued dependent variables and the ordinary real-valued explanatory variables respectively, while the unobservable variables η_i and v_{it} are the individual fixed effect and the disturbance respectively. When x_{it} is strictly exogenous, it is assumed that

$$E[v_{it}\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^T]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \tag{2.2}$$

where $v_i^{t-1}=(v_{i1},\ldots,v_{i,t-1})$ and $x_i^T=(x_{i1},\ldots,x_{iT})$. For convenience, equations (2.1) are rewritten as

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$$y_{it}=\gamma y_{i,t-1}+u_{it} \ , \qquad \text{for} \quad t=2,\ldots,T \ , \qquad (2.3)$$

$$u_{it}=\phi_i \mu_{it}+v_{it} \ , \qquad \text{for} \quad t=2,\ldots,T \ , \qquad (2.4)$$

where $\phi_i=\exp(\eta_i)$ and $\mu_{it}=\exp(\beta x_{it})$. It should be noted that $u_{it}=y_{it}-\gamma y_{i,t-1}$ are plugged into the equations to be hereafter described.

Only for the case where γ is set to be zero, the within group mean scaling estimator (hereafter WG estimator) for β is consistent, in the setting above.[1] That is, the WG estimator for the set of γ and β is inconsistent in the setting above, as suggested by Blundell et al. (2002). This implies that the moment conditions used in the WG estimator are not valid for the specification (2.1) with (2.2).

From now on, the forward demeaning transformation is derived for the LFM (2.1) with (2.2) and the moment conditions are proposed using it. From (2.4), the following equations are obtained:

$$u_{it}^*=\phi_i \mu_{it}^*+v_{it}^* \ , \qquad \text{for} \quad t=2,\ldots,T \ , \qquad (2.5)$$

where $u_{it}^*=(1/(T-t+1))\sum_{s=t}^{T} u_{is}$, $\mu_{it}^*=(1/(T-t+1))\sum_{s=t}^{T} \mu_{is}$ and $v_{it}^*=(1/(T-t+1))\sum_{s=t}^{T} v_{is}$. Solving (2.5) with respect to ϕ_i gives

$$\phi_i=(u_{it}^*-v_{it}^*)/\mu_{it}^* \ , \qquad \text{for} \quad t=2,\ldots,T \ . \qquad (2.6)$$

Accordingly, from (2.2), (2.4) and (2.6), it follows that

$$E[(u_{it}-\mu_{it}((u_{it}^*-v_{it}^*)/\mu_{it}^*))\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^T]=0 \ , \quad \text{for} \quad t=2,\ldots,T \ . \qquad (2.7)$$

Since $E[(\mu_{it}/\mu_{it}^*)\,v_{it}^*\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^T]=0$ (after using the law of iterated expectations), equations (2.7) result in

$$E[(u_{it}-\mu_{it}(u_{it}^*/\mu_{it}^*))\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^T]=0 \ , \qquad \text{for} \quad t=2,\ldots,T \ . \qquad (2.8)$$

Noting that equation (2.8) for $t=T$ holds irrespective of any set of values of γ and β , the conditional moment conditions (2.8) give the following $(T-2)(T-1)/2$ and $(T-2)T$ unconditional moment conditions for estimating γ and β consistently:

$$E[y_{is}(u_{it}-\mu_{it}(u_{it}^*/\mu_{it}^*))]=0 \ , \quad \text{for} \quad s=1,\ldots,t-1 \ ; \quad t=2,\ldots,T-1 \ , \qquad (2.9)$$

$$E[x_{is}(u_{it}-\mu_{it}(u_{it}^*/\mu_{it}^*))]=0 \ , \quad \text{for} \quad s=1,\ldots,T \ ; \quad t=2,\ldots,T-1 \ . \qquad (2.10)$$

It is possible that the consistent GMM estimator for the set of γ and β in the model (2.1) with (2.2) is constructed by using the moment conditions (2.9) and (2.10). This estimator is referred

1 The origins of the WG estimator can be traced to the ordinary and conditional maximum likelihood estimators assuming the Poisson distribution. The latter is proposed by Hausman et al. (1984). Blundell et al. (2002) and Lancaster (2002) pin down the identity of both estimators and further the formers show that both estimators result in the WG estimator requiring no distributional assumption (see also Windmeijer, 2008).

to as the "GMM(fdm) estimator" in this paper. The forward demeaning transformation $u_{it}-\mu_{it}(u^*_{it}/\mu^*_{it})$ is analogous to the forward orthogonal deviations transformation proposed by Arellano and Bover (1995) for the ordinary dynamic panel data model.

Just for the record, Kitazawa (2007) proposes other types of moment conditions associated with the specification (2.1) with (2.2).

3. Monte Carlo

The small sample performance of the GMM(fdm) estimator is investigated by using some Monte Carlo experiments. In the experiments, the Level estimator (which ignores the fixed effect), the WG estimator, the GMM(qd) estimator (which is proposed by Blundell et al., 2002 for the LFM and uses the quasi-differencing transformation proposed by Chamberlain, 1992 and Wooldridge, 1997) and the PSM (pre-sample mean) estimator proposed by Blundell et al. (1999 and 2002) are used as controls. An econometric software TSP 4.5 is used (see Hall and Cummins, 2006).

The DGP (data generating process) is as follows:

$$y_{it} \sim Poisson(\gamma y_{i,t-1} + \exp(\beta x_{it} + \eta_i)) \quad ,$$

$$y_{i,-TG+1} \sim Poisson(\exp(\beta x_{i,-TG+1} + \eta_i)) \quad ,$$

$$x_{it} = \rho x_{i,t-1} + \tau \eta_i + \varepsilon_{it} \quad ,$$

$$x_{i,-TG+1} = (1/(1-\rho))\tau \eta_i + (1/(1-\rho^2)^{(1/2)})\varepsilon_{i,-TG+1} \quad ,$$

$$\eta_i \sim N(0, \sigma^2_\eta) \quad ; \qquad \varepsilon_{it} \sim N(0, \sigma^2_\varepsilon) \quad ,$$

where $t=-TG+1,\ldots,-1,0,1,\ldots,T$ with TG being the number of pre-sample periods to be generated. In the DGP, values are set to the parameters γ , β , ρ , τ , σ^2_η and σ^2_ε . The experiments are carried out with $TG=50$, the cross-sectional sizes $N=100$, 500 and 1000 , the numbers of periods used for the estimation $T=4$ and 8 and the number of replications $NR=1000$. This DGP setting is the same as that of Blundell et al. (2002) and satisfies the assumptions (2.2).

The Monte Carlo results are exhibited in Tables 1-4, where the different types of settings of parameter values and T in Blundell et al. (2002) are used. The endemic upward and downward biases are found in the Level and WG estimators, which are the reflection of the inconsistency, while the PSM estimator behaves well as the pre-sample length used elongates, because some assumptions needed for the consistent PSM estimation are satisfied in this DGP setting (see Blundell et al., 2002 and Kitazawa, 2007). The instruments used for the GMM estimators in these tables are curtailed so that for the GMM(qd) estimator only the past dependent variables (y_{it}) dated $t-2$ and the past explanatory variables (x_{it}) dated $t-1$ and $t-2$ are used as the instruments for the quasi-differenced equations dated t , while for the GMM(fdm) estimator the past dependent variables dated $t-2$ and before are not used as the instruments for the forward demeaned equations dated t . It can be seen that the performances of the consistent GMM(qd) estimator (which uses the instruments valid for the case of predetermined explanatory variables) are poor in small samples, while those of the GMM(fdm) estimator improve dramatically, except for the rmse for β with $N=100$ and 500 for $T=4$ in Table 1.

4. Conclusion

This note proposed a forward demeaning transformation for the linear feedback model with explanatory variables being strictly exogenous on count panel data. Some Monte Carlo experiments showed that the GMM estimator based on the forward demeaning transformation behaves better than the conventional quasi-differencing GMM estimator in the DGP setting of strictly exogenous explanatory variables.

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Table 1. **Monte Carlo results for LFM, T=4**

(Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5$; $\beta = 0.5$; $\rho = 0.5$; $\tau = 0.1$; $\sigma_\eta^2 = 0.5$; $\sigma_\varepsilon^2 = 0.5$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.259	0.267	0.275	0.277	0.277	0.278
	β	0.543	0.642	0.570	0.633	0.555	0.567
WG	γ	−0.454	0.464	−0.446	0.448	−0.447	0.448
	β	−0.261	0.272	−0.261	0.264	−0.261	0.262
GMM(qd)	γ	−0.281	0.415	−0.108	0.166	−0.063	0.114
	β	−0.246	0.377	−0.126	0.224	−0.075	0.175
GMM(fdm)	γ	−0.163	0.268	−0.039	0.114	−0.021	0.078
	β	−0.022	0.516	0.000	0.275	−0.006	0.105
PSM	γ(4)	0.136	0.158	0.160	0.166	0.162	0.165
	β(4)	0.198	0.316	0.214	0.243	0.211	0.221
	γ(8)	0.108	0.132	0.128	0.135	0.130	0.134
	β(8)	0.141	0.227	0.154	0.177	0.153	0.162
	γ(25)	0.048	0.092	0.063	0.075	0.066	0.072
	β(25)	0.062	0.152	0.065	0.088	0.066	0.076
	γ(50)	0.023	0.085	0.036	0.053	0.038	0.047
	β(50)	0.036	0.135	0.035	0.064	0.036	0.050

Notes: (1) The number of replications is 1000. (2) The instrument sets for GMM estimators include no time dummies. (3) The replications where no convergence of the estimations is achieved are eliminated when calculating the values of the Monte Carlo statistics. Their rates are fairly small. (4) The individuals where the pre-sample means are zero are eliminated in each replication when estimating the parameters of interest using the PSM estimator. The number of these individuals is fairly small for each replication. (5) Although there may be a few replications where the Level and PSM estimators generate the estimates of γ and β with their absolute values exceeding 10, these replications are eliminated when calculating the values of the Monte Carlo statistics. (6) The values of the Monte Carlo statistics are obtained using the true values of γ and β as the starting values in the optimization for each replication. The values of the statistics obtained using the true values are not much different from those obtained using two different types of the starting values, relative to the values of the statistics.

Table 2. **Monte Carlo results for LFM, T=8**

(Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5$; $\beta = 0.5$; $\rho = 0.5$; $\tau = 0.1$; $\sigma_\eta^2 = 0.5$; $\sigma_\varepsilon^2 = 0.5$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.263	0.268	0.275	0.276	0.277	0.277
	β	0.538	0.592	0.552	0.565	0.554	0.560
WG	γ	−0.189	0.197	−0.183	0.185	−0.184	0.185
	β	−0.128	0.142	−0.126	0.129	−0.127	0.128
GMM(qd)	γ	−0.237	0.273	−0.075	0.094	−0.043	0.061
	β	−0.238	0.270	−0.104	0.131	−0.059	0.088
GMM(fdm)	γ	−0.085	0.126	−0.026	0.054	−0.016	0.035
	β	−0.057	0.117	−0.017	0.059	−0.010	0.040
PSM	γ(4)	0.145	0.155	0.163	0.166	0.164	0.166
	β(4)	0.192	0.229	0.212	0.221	0.213	0.219
	γ(8)	0.116	0.127	0.132	0.135	0.133	0.135
	β(8)	0.140	0.174	0.155	0.164	0.157	0.162
	γ(25)	0.058	0.077	0.068	0.073	0.069	0.072
	β(25)	0.061	0.098	0.068	0.078	0.069	0.075
	γ(50)	0.029	0.059	0.039	0.047	0.040	0.044
	β(50)	0.030	0.076	0.037	0.049	0.038	0.045

Notes: See notes in Table 1.

Table 3. **Monte Carlo results for LFM, T=8**

(Situation of considerably persistent y_{it} and x_{it})

$\gamma = 0.7$; $\beta = 1$; $\rho = 0.9$; $\tau = 0$; $\sigma_{\eta}^{2} = 0.5$; $\sigma_{\varepsilon}^{2} = 0.05$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.170	0.174	0.180	0.181	0.183	0.183
	β	0.421	0.669	0.425	0.471	0.428	0.455
WG	γ	−0.251	0.258	−0.245	0.246	−0.244	0.245
	β	−0.369	0.403	−0.367	0.373	−0.367	0.371
GMM(qd)	γ	−0.361	0.415	−0.109	0.143	−0.060	0.084
	β	−0.696	0.880	−0.412	0.595	−0.272	0.406
GMM(fdm)	γ	−0.146	0.185	−0.057	0.082	−0.033	0.055
	β	−0.210	0.409	−0.105	0.206	−0.066	0.148
PSM	γ(4)	0.115	0.125	0.133	0.136	0.137	0.139
	β(4)	0.046	0.460	0.070	0.289	0.066	0.167
	γ(8)	0.105	0.115	0.122	0.125	0.126	0.127
	β(8)	0.011	0.360	0.025	0.165	0.024	0.119
	γ(25)	0.076	0.089	0.091	0.094	0.094	0.096
	β(25)	−0.017	0.206	−0.004	0.100	−0.004	0.074
	γ(50)	0.056	0.073	0.069	0.073	0.072	0.074
	β(50)	−0.009	0.182	−0.001	0.081	−0.001	0.060

Notes: See notes in Table 1.

Table 4. **Monte Carlo results for LFM, T=8**

(Situation of considerably persistent y_{it} and extremely persistent x_{it})

$\gamma = 0.7$; $\beta = 1$; $\rho = 0.95$; $\tau = 0$; $\sigma_{\eta}^{2} = 0.5$; $\sigma_{\varepsilon}^{2} = 0.015$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.175	0.178	0.183	0.184	0.184	0.184
	β	0.244	0.524	0.250	0.322	0.234	0.272
WG	γ	−0.274	0.280	−0.272	0.273	−0.271	0.272
	β	−0.367	0.469	−0.360	0.380	−0.363	0.373
GMM(qd)	γ	−0.449	0.511	−0.138	0.189	−0.070	0.108
	β	−0.746	1.378	−0.588	1.148	−0.379	0.829
GMM(fdm)	γ	−0.201	0.243	−0.087	0.116	−0.056	0.079
	β	−0.262	0.498	−0.131	0.271	−0.100	0.214
PSM	γ(4)	0.112	0.122	0.128	0.131	0.131	0.132
	β(4)	−0.205	0.406	−0.185	0.250	−0.196	0.223
	γ(8)	0.101	0.111	0.116	0.119	0.118	0.119
	β(8)	−0.248	0.385	−0.231	0.270	−0.240	0.258
	γ(25)	0.074	0.087	0.087	0.091	0.089	0.091
	β(25)	−0.236	0.332	−0.224	0.248	−0.233	0.245
	γ(50)	0.058	0.073	0.070	0.074	0.071	0.073
	β(50)	−0.173	0.277	−0.165	0.190	−0.173	0.184

Notes: See notes in Table 1.