Some additional moment conditions for a dynamic count panel data model*

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Abstract

This paper proposes some additional moment conditions for the linear feedback model formulated in count panel data model, proposed by Blundell et al. (2002). It is shown that the moment conditions based on the quasi-differenced transformation proposed by Chamberlain (1992) and Wooldridge (1997) and some additional moment conditions are derived by using a new operation which the assumptions for disturbances underlie. Two kinds of the additional moment conditions are conceptually equivalent to those proposed by Windmeijer (2000) and Crépon and Duguet (1997) in some regards. Some GMM estimators are constructed using these moment conditions. The small sample performances for the GMM estimators are investigated with some Monte Carlo experiments and it is shown that the GMM estimators perform well when using the additional moment conditions, with minor exceptions.

Keywords: count panel data, linear feedback model, implicit operation, moment conditions, generalized method of moments, Monte Carlo experiments

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1 Introduction

Since the pioneering work conducted by Hausman et al. (1984), various models and estimators are proposed for the purpose of dealing with count panel data (CPD).

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In many cases, the count panel data model is discussed under the assumption that the number of time periods is small and the cross-sectional size is large and therefore asymptotics of the estimators for the model rely on the cross-sectional size. In addition, the presence of the multiplicative fixed effect (whose logarithm is often considered to be correlated with the explanatory variables) makes it difficult to estimate the parameters of interest consistently. At the time of foundation of the count panel data model, the model incorporates no dynamics and therefore the Poisson conditional maximum likelihood estimator (CMLE) proposed by Hausman et al. (1984) is used as a consistent estimator under the assumption that the explanatory variables are strictly exogenous. The invention of the quasi-differenced transformation conducted by Chamberlain (1992) and Wooldridge (1997) enables the researchers to implement the consistent estimation of the count panel data model under the assumption of predetermined explanatory variables by the assistance of the generalized method of moments (GMM) estimator proposed by Hansen (1982), although the small sample performance of this GMM estimator is not amply investigated in the framework of econometric theory as well as in the framework of Monte Carlo study.[1] Using the count panel data without dynamics, some empirical applications using the GMM based on the quasi-differenced transformation are conducted by Cincera (1997), Crépon and Duguet (1997), Montalvo (1997), Blundell et al. (1999), and Kim and Marschke (2005), with the intention of estimating the patent (and/or innovation) production function.

Besides the implementation of the empirical applications above, an attractive model incorporating a dynamics into the count panel data model is proposed by Blundell, Griffith, and Windmeijer (2002), denoted by BGW hereafter, with the aim of spelling over the possibility that the past count dependent variables have an influence on the current count dependent variable. The linear feedback model (LFM) grows out of the integer valued autoregressive model for the time series Poisson count model developed by Al-Osh and Alzaid (1987), McKenzie (1988), Alzaid and Al-Osh (1990), and Jin-Guan and Yuan (1991). In the LFM, the lagged dependent variables are included as additive regressors and therefore the problems associated with the explosive dependent variables or the treatment of the zero valued dependent variables can be circumvented.[2] However, the estimators proposed until now for the LFM are not satisfactory for the case where the number of individ-

[1] It is likely that the Monte Carlo study carried out by Windmeijer (2006) suggests the unattractiveness of the small sample property of this GMM estimator for the persistent explanatory variables.

[2] The alternative models incorporating the dynamics are proposed by Crépon and Duguet (1997) and Blundell et al. (1999).

uals is large but the number of time periods is small and therefore the asymptotics for the estimators rely on the number of individuals. The Level estimator and the within group (WG) estimator are inconsistent and therefore the presence of the unignorable deviations endemic to the inconsistent estimators is founded as shown in the Monte Carlo study by BGW. The generalized method of moments (GMM) estimator based on the quasi-differenced transformation, which is an application of the estimator proposed by Chamberlain (1992) and Wooldridge (1997) to the LFM, is consistent but its small sample property is not favorable presumably due to the problem of the weak instruments, which is shown in the Monte Carlo study by BGW. The pre-sample mean (PSM) estimator proposed by Blundell et al. (1999) and BGW is consistent and it has a good to excellent small sample property as long as the relatively long history of the dependent variable is available for each individual, the fixed effect composing the explanatory variable is proportional to the fixed effect in the regression for each individual, and the (finite) moment generating functions of the disturbance terms composing the explanatory variables are equal over time and for all individuals, but it is not necessarily probable that these situations are arranged for many empirical applications. However, in late years, challenges of empirical studies applying the GMM and/or PSM estimators to the LFM are conducted where the data is used with large sizes of cross-section and/or the results are examined with circumspection: In the investigation of factors inducing the innovation of firms, Salomon and Shaver (2005) find that exporting is positively associated with the product innovation and the patent application and Uchida and Cook (2007) obtain the result that the competition has a positive effect on innovation activities, while in the investigation of the relationship between the outward foreign direct investment (FDI) by firms and productivity, Domijan et al. (2007) find that more productive firms are more likely to invest in foreign affiliates.[3]

In this paper, some moment conditions for estimating the LFM consistently are derived based on the structure of variance and covariance originating from the usage of the conditional expectations for the disturbances and then the GMM estimators are constructed using these moment conditions. The method of deriving the moment conditions is referred to as the implicit operation. The two types of the conditional expectations are used: that for the case of predetermined explanatory variables and that for the case of strictly exogenous explanatory variables. These moment conditions are composed of the moment conditions based on the quasi-differenced transformations and the additional moment conditions akin to those for the the

[3]At the vanguard of these empirical studies, some applications of the GMM estimator to the LFM whose form is different from that of BGW were carried out by Cincera (1997) and Blundell et al. (1999) in 90's.

simple dynamic panel data model (DPDM) proposed by Ahn (1990) and Ahn and Schmidt (1995) in a conceptual basis. Although the moment conditions based on the quasi-differenced transformations in this paper are conceptually equivalent to the moment conditions proposed by Chamberlain (1992) and Wooldridge (1997) and the additional moment conditions proposed in this paper are conceptually equivalent to the additional moment conditions proposed by Windmeijer (2000) and Crépon and Duguet (1997), they are elaborated in the sense that moment conditions can vary with the case whether the explanatory variable is predetermined or strictly exogenous. Further, assuming the stationarity of the dependent variables and the explanatory variables generates some moment conditions for estimating the LFM consistently, and assuming the equidispersion of the dependent variables generates some moment conditions for estimating the LFM consistently as well. The process of deriving the moment conditions for the LFM has its origin in that for the simple DPDM proposed by Ahn (1990) and Ahn and Schmidt (1995) and is decorated with some improvements in order that all of the available moment conditions shall be able to be exploited for the LFM. It is shown in some Monte Carlo experiments that the GMM estimators constructed by additionally using the moment conditions newly proposed in this paper perform well, compared with the GMM estimator based only on the conventional quasi-differenced transformation.

The rest of the paper is organized as follows. Section 2 states one type of the implicit operation of deriving the moment conditions for the simple dynamic panel data model (DPDM), which is slightly different from the method proposed by Ahn (1990) and Ahn and Schmidt (1995). Then, in section 3, the implicit operation is applied to the linear feedback model for count panel data and the valid moment conditions are derived for the four cases of predetermined explanatory variables, strictly exogenous variables, mean-stationary dependent variables, and equidispersion. In section 4, the GMM estimators are constructed using these moment conditions. In section 5, some Monte Carlo experiments are carried out with the intention of investigating the small sample performances of the GMM estimators proposed newly and comparing them with the conventional estimators proposed until now. Section 6 concludes.

2 Moment conditions for simple DPDM

The moment conditions for estimating the parameter of interest for the simple dynamic panel data model (DPDM) are constructed based on the structures of covariances on the disturbances, the initial dependent variable, and the individual specific effect. This section introduces two methods of deriving the moment conditions for the simple DPDM. One is the conventional method in line with Ahn (1990) and Ahn and Schmidt (1995) and another is a new method (i.e. the implicit operation) proposed in this paper. In the framework of the DPDM, the implicit operation is described and compared with the conventional method, as a preparation for the applications to the linear feedback model (LFM) in count panel data (CPD).

2.1 Simple DPDM

The simple DPDM is written as

$$y_{it} = \gamma y_{i,t-1} + \eta_i + v_{it}, \quad \text{for } t = 2, \ldots, T, \tag{2.1.1}$$

where the subscript i denotes the individual unit with $i = 1, \ldots, N$, t denotes the time period, y_{it} is the dependent variable for individual i at time t, η_i is the individual specific effect for individual i, v_{it} is the disturbance for individual i at time t, and the parameter of interest is γ. The discussion is conducted for the case where $N \to \infty$ and T is fixed.

Equation (2.1.1) is also rewritten in an easily-viewable form as follows:

$$y_{it} = \gamma y_{i,t-1} + u_{it}, \quad \text{for } t = 2, \ldots, T, \tag{2.1.2}$$

$$u_{it} = \eta_i + v_{it}, \quad \text{for } t = 2, \ldots, T. \tag{2.1.3}$$

Here, it is assumed that the following conditional moment conditions hold for v_{it} in (2.1.1):

$$E[v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}] = 0, \quad \text{for } t = 2, \ldots, T, \tag{2.1.4}$$

where $v_i^{t-1} = (v_{i1}, \ldots, v_{i,t-1})$ with v_{i1} being the empty set for the sake of convenience. The implication of (2.1.4) is that the DPDM (2.1.1) is written as the implicit form.

2.2 Conventional operation

The moment conditions for consistently estimating γ in (2.1.1) are constructed based on the covariance structures among v_{it}, between v_{it} and η_i, and between v_{it} and y_{i1}. Multiplying both sides of (2.1.4) by the variables in the information set $(y_{i1}, \eta_i, v_i^{t-1})$

without any transformations generates the following conditional moment conditions:

$$E[y_{i1}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}] = 0, \tag{2.2.1}$$
$$E[v_{is}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}] = 0, \quad \text{for } 2 \leq s \leq t-1, \tag{2.2.2}$$
$$E[\eta_i v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}] = 0. \tag{2.2.3}$$

Applying the law of total expectation to (2.2.1), (2.2.2), and (2.2.3), the following unconditional moment conditions are generated:

$$E[y_{i1}v_{it}] = 0, \tag{2.2.4}$$
$$E[v_{is}v_{it}] = 0, \quad \text{for } 2 \leq s \leq t-1, \tag{2.2.5}$$
$$E[\eta_i v_{it}] = 0. \tag{2.2.6}$$

The unconditional moment conditions (2.2.4), (2.2.5), and (2.2.6) are nothing short of the standard assumptions employed in Ahn (1990) and Ahn and Schmidt (1995). The unconditional moment conditions (2.2.4), (2.2.5), and (2.2.6) say that y_{i1} and v_{it} are uncorrelated, that v_{it} are serially uncorrelated, and that η_i and v_{it} are uncorrelated, respectively. In this paper, the unconditional moment conditions (2.2.4), (2.2.5), and (2.2.6) are specially called the explicit standard assumptions. The dynamic panel data model is discussed starting from the explicit standard assumptions in many literatures.

Here, the consistent estimation of γ using the moment conditions (2.2.4), (2.2.5), and (2.2.6) is impossible, because v_{it} for $t = 2, \ldots, T$ and η_i are unobservable. Accordingly, it is necessary to construct the unconditional moment conditions using the observable variables only, in order to estimate the parameter of interest γ consistently, where the observable variables are defined as the variables written in terms of data and parameters of interest. By replacing the unobservable variable v_{it} in (2.2.4) and (2.2.5) with the observable variables u_{it}, the following unconditional moment conditions (i.e. the observable analogues) are generated:

$$E[y_{i1}u_{it}] = E[y_{i1}\eta_i], \tag{2.2.7}$$
$$E[u_{is}u_{it}] = E[\eta_i^2], \quad \text{for } 2 \leq s \leq t-1, \tag{2.2.8}$$

where (2.2.6) is additionally used to obtain (2.2.8). Then, the consistent estimation of γ is conducted by utilizing the restrictions holing among the observable analogues (2.2.7) and (2.2.8) corresponding to (2.2.4) and (2.2.5) respectively. That is, by solving the relationships holding among $E[y_{i1}u_{it}]$ for $t = 2, \ldots, T$ and $E[u_{is}u_{it}]$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$, the moment conditions for estimating γ consistently can be obtained. For now, it is advisable to note that by using the equality between $E[y_{i1}u_{it}]$ and $E[y_{i1}u_{i,t-1}]$, the equality between $E[u_{is}u_{it}]$ and $E[u_{is}u_{i,t-1}]$, and the equality between $E[u_{i,s-1}u_{it}]$ and $E[u_{is}u_{it}]$, the following three types of the moment conditions are constructed respectively:

$$\begin{aligned} E[y_{i1}\Delta u_{it}] &= 0, && && (2.2.9)\\ E[u_{is}\Delta u_{it}] &= 0, && \text{for } 2 \le s \le t-2, && (2.2.10)\\ E[\Delta u_{is}u_{it}] &= 0, && \text{for } 3 \le s \le t-1, && (2.2.11) \end{aligned}$$

where $\Delta u_{it} = u_{it} - u_{i,t-1}$.

However, the moment conditions (2.2.9), (2.2.10), and (2.2.11) are reformulated more concisely. Firstly, creating the recursive equation

$$E[y_{is}\Delta u_{it}] = \gamma E[y_{i,s-1}\Delta u_{it}] + E[u_{is}\Delta u_{it}], \quad \text{for } 2 \le s \le t-2, \tag{2.2.12}$$

from (2.1.2) for $t = s$ and then applying the initial condition (2.2.9) and the innovation (2.2.10) to (2.2.12), the moment conditions (2.2.9) and (2.2.10) are in toto reformulated as the following $(T-1)(T-2)/2$ moment conditions:

$$E[y_{is}\Delta u_{it}] = 0, \quad \text{for } s = 1, \ldots, t-2;\ t = 3, \ldots, T, \tag{2.2.13}$$

which are linear with respect to the parameter of interest γ. That is, the moment conditions (2.2.10) nonlinear with respect to γ are replaced by the linear moment conditions (2.2.13) for $2 \le s \le t-2$ linear with respect to γ. Next, allowing for the fact that some moment conditions in (2.2.11) are redundant when condensedly prescribing the relationships holding among $E[u_{is}u_{it}]$ for $t \ne s$, the moment conditions ruling out the redundancies in (2.2.11) are as follows:

$$E[\Delta u_{i,t-1}u_{it}] = 0, \quad \text{for } t = 4, \ldots, T, \tag{2.2.14}$$

whose number is $T-3$. That is, since the moment conditions (2.2.13) represent the equalities holding between $E[y_{i1}u_{it}]$ and $E[y_{i1}u_{i,t-1}]$ for $t = 3, \ldots, T$ and between $E[u_{is}u_{it}]$ and $E[u_{is}u_{i,t-1}]$ for $s = 2, \ldots, t-2$ and $t = 4, \ldots, T$ and the moment conditions (2.2.14) represent the equalities holding between $E[u_{i,t-2}u_{it}]$ and $E[u_{i,t-1}u_{it}]$ for $t = 4, \ldots, T$, the other equalities holding among $E[u_{is}u_{it}]$ for $t \ne s$ can be confirmed immediately, only using (2.2.13) and (2.2.14).

As a result, the condensed full set of the moment conditions for estimating γ consistently for the simple DPDM (2.1.1) under the assumptions (2.1.4) is composed of (2.2.13) and (2.2.14).

This operation is in line with that of Ahn (1990) and Ahn and Schmidt (1995). The unconditional moment conditions (2.2.13) are proposed by Holtz-Eakin et al. (1988) and Arellano and Bond (1991), which are called the standard moment conditions, while the unconditional moment conditions (2.2.14) are proposed by Ahn (1990) and Ahn and Schmidt (1995), which are called the additional nonlinear moment conditions.

2.3 Implicit operation

The moment conditions (2.2.13) and (2.2.14) can be obtained, starting from the assumptions (2.2.1), (2.2.2), and (2.2.3) without using the explicit standard assumptions (2.2.4), (2.2.5), and (2.2.6). In this paper, the conditional moment conditions (2.2.1), (2.2.2), and (2.2.3) are called the implicit standard assumptions, in contrast to the explicit standard assumptions. The conditional moment conditions (2.2.1), (2.2.2), and (2.2.3) say that y_{it} and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1})$, that v_{is} for $s < t$ and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1})$, and that η_i and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1})$, respectively. Different from the conventional operation, the observable analogues in this case are designed for the conditional moment conditions (2.2.1) and (2.2.2) by replacing the unobservable variable v_{it} in the conditional moment conditions (2.2.1) and (2.2.2) with the observable variable u_{it}. The observable analogues for (2.2.1) and (2.2.2) are as follows respectively:

$$E[y_{i1}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}] = y_{i1}\eta_i, \tag{2.3.1}$$

$$E[u_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}] = \eta_i^2 + v_{is}\eta_i, \quad \text{for } 2 \le s \le t-1, \tag{2.3.2}$$

where (2.2.3) is additionally used to obtain (2.3.2). It should be noted that the observable analogues (2.3.1) and (2.3.2) are able to be transformed into various forms by using any transformations of the variables in the information set $(y_{i1}, \eta_i, v_i^{t-1})$. Applying the law of total expectation to (2.3.1) and (2.3.2) generates (2.2.7) and (2.2.8), where (2.2.6) for $t = s$ is utilized. After these procedures, the same procedures as in pervious subsection are implemented in order to derive the condensed full set of the moment conditions for estimating γ consistently composed of (2.2.13) and (2.2.14).

The concept of the implicit operation is that after making the observable analogues conditional on the information set from the implicit standard assumptions, the valid unconditional moment conditions are constructed from the relationships holding among the observable analogues transformed using the transformed variables in the information set. For the case of the DPDM, the implicit operation generates the same results as the conventional operation and makes no significant contribution. However, the implicit operation can be a powerful tool for constructing the valid moment conditions, as will be seen in next section.

3 Moment conditions for LFM in CPD

In this section, some moment conditions are derived for the simple linear feedback model (LFM) in count panel data (CPD), by using the implicit operation introduced in previous section. The idea by Ahn (1990) and Ahn and Schmidt (1995) (which is used for the DPDM) is woven into the derivation of the moment conditions, in the sense that the moment conditions are constructed on the basis of the assumptions on the structures of covariances between the initial dependent varable and the disturbances, among the disturbances, and between the explanatory variables and the disturbances and on the structures of variances of the disturbances.

3.1 LFM for CPD

The following simple linear feedback model (LFM) for count panel data (CPD) is considered here:

$$y_{it} = \gamma y_{i,t-1} + \exp(\beta x_{it} + \eta_i) + v_{it}, \quad \text{for } t = 2, \ldots, T, \tag{3.1.1}$$

where the subscript i denotes the individual unit with $i = 1, \ldots, N$, t denotes the time period, y_{it} is the count dependent variable whose value is zero or positive integer for individual i at time t, x_{it} is the explanatory variable for individual i at time t, η_i is the individual specific effect for individual i, v_{it} is the disturbance for individual i at time t, and the parameters of interest are γ and β. The discussion is conducted for the case where $N \to \infty$ and T is fixed.

Equation (3.1.1) is also rewritten in an easily-viewable form as follows:

$$y_{it} = \gamma y_{i,t-1} + u_{it}, \quad \text{for } t = 2, \ldots, T, \tag{3.1.2}$$

$$u_{it} = \phi_i \mu_{it} + v_{it}, \quad \text{for } t = 2, \ldots, T, \tag{3.1.3}$$

where $\phi_i = \exp(\eta_i)$ and $\mu_{it} = \exp(\beta x_{it})$.

For v_{it} in equation (3.1.1), it is pertinent that when x_{it} is predetermined, the following assumptions are made:

$$E[v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = 0, \quad \text{for } t = 2, \dots, T, \tag{3.1.4}$$

where $v_i^{t-1} = (v_{i1}, \dots, v_{i,t-1})$ with v_{i1} being the empty set unless otherwise noted and $x_i^t = (x_{i1}, \dots, x_{it})$, while it is pertinent that when x_{it} is strictly exogenous, the following assumptions are made:

$$E[v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] = 0, \quad \text{for } t = 2, \dots, T, \tag{3.1.5}$$

where $x_i^T = (x_{i1}, \dots, x_{iT})$. The implication of (3.1.4) and (3.1.5) is that the LFM is written as the implicit form.

3.2 Case of predetermined explanatory variables

In this case, the assumptions (3.1.4) are assumed. Multiplying both sides of (3.1.4) by the variables in the information set conditioning v_{it} (i.e. $(y_{i1}, \eta_i, v_i^{t-1}, x_i^t)$) without any transformations generates the following conditional moment conditions

$$\begin{aligned}
E[y_{i1}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= 0, && (3.2.1)\\
E[v_{is}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= 0, \quad \text{for } 2 \le s \le t-1, && (3.2.2)\\
E[x_{is}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= 0, \quad \text{for } 1 \le s \le t, && (3.2.3)\\
E[\eta_i v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= 0. && (3.2.4)
\end{aligned}$$

From (3.2.1), (3.2.2), (3.2.3), and (3.2.4), the following unconditional moment conditions are generated according to the manner of the conventional operation:

$$\begin{aligned}
E[y_{i1}v_{it}] &= 0, && (3.2.5)\\
E[v_{is}v_{it}] &= 0, \quad \text{for } 2 \le s \le t-1, && (3.2.6)\\
E[x_{is}v_{it}] &= 0, \quad \text{for } 1 \le s \le t, && (3.2.7)\\
E[\eta_i v_{it}] &= 0. && (3.2.8)
\end{aligned}$$

The unconditional moment conditions (3.2.5), (3.2.6), (3.2.7), and (3.2.8) imply the explicit standard assumptions in the LFM. The unconditional moment conditions (3.2.5), (3.2.6), (3.2.7), and (3.2.8) say that y_{i1} and v_{it} are uncorrelated unconditionally, that v_{it} is serially uncorrelated unconditionally, that x_{is} for $s = 1, \dots, t$ and v_{it} are uncorrelated unconditionally, and that η_i and v_{it} are uncorrelated unconditionally, respectively. It is impossible to conduct the consistent estimation of γ and

β directly using the explicit standard assumptions, because v_{it} for $t = 2, \ldots, T$ and η_i are unobservable. By replacing the unobservable variable v_{it} in (3.2.5), (3.2.6), and (3.2.7) with the observable variable u_{it}, the following observable analogues are generated:

$$
\begin{aligned}
E[y_{i1}u_{it}] &= E[y_{i1}\phi_i\mu_{it}], && (3.2.9)\\
E[u_{is}u_{it}] &= E[\phi_i^2\mu_{is}\mu_{it} + v_{is}\phi_i\mu_{it}], \quad \text{for } 2 \leq s \leq t-1, && (3.2.10)\\
E[x_{is}u_{it}] &= E[x_{is}\phi_i\mu_{it}], \quad \text{for } 1 \leq s \leq t, && (3.2.11)
\end{aligned}
$$

which correspond to the unconditional moment conditions (3.2.5), (3.2.6), and (3.2.7) respectively. The observable analogue (3.2.10) is derived by using $E[v_{it}\phi_i\mu_{is}] = 0$ for $2 \leq s \leq t$ which stems from the assumptions (3.1.4), instead of (3.2.8).

However, the relationships are not found among $E[y_{i1}u_{it}]$ for $t = 2, \ldots, T$, $E[u_{is}u_{it}]$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$, and $E[x_{is}u_{it}]$ for $s = 1, \ldots, t$ and $t = 2, \ldots, T$, as is clear from the moment conditions (3.2.9), (3.2.10), and (3.2.11). That is, neither the equality nor any one of the other appropriate relationships after ruling out the unobservable variables η_i and v_{it} holds between $E[y_{i1}u_{it}]$ and $E[y_{i1}u_{i,t-1}]$, between $E[u_{is}u_{it}]$ and $E[u_{is}u_{i,t-1}]$, between $E[u_{is}u_{it}]$ and $E[u_{i,s-1}u_{it}]$, between $E[x_{is}u_{it}]$ and $E[x_{is}u_{i,t-1}]$, and between $E[x_{is}u_{it}]$ and $E[x_{i,s-1}u_{it}]$. Accordingly, it is impossible to construct the unconditional moment conditions for estimating γ and β consistently in the framework of the conventional operation. It will be seen from the descriptions below that the implicit operation provides a powerful means of solving this problem and deriving the unconditional moment conditions for estimating γ and β consistently in the LFM for the CPD.

From now on, the implicit operation begins from the implicit standard assumptions (3.2.1), (3.2.2), (3.2.3), and (3.2.4). The conditional moment conditions (3.2.1), (3.2.2), (3.2.3), and (3.2.4) say that y_{i1} and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^t)$ (i.e. the information set corresponding to the case of predetermined explanatory variables), that v_{is} for $s = 2, \ldots, t-1$ and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^t)$, that x_{is} for $s = 1, \ldots, t$ and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^t)$, and that η_i and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^t)$, respectively. By replacing the unobservable variable v_{it} in the conditional moment conditions (3.2.1), (3.2.2), and (3.2.3) with the observable variable u_{it}, the the following observable analogues for (3.2.1), (3.2.2), and (3.2.3) are generated:

$$
\begin{aligned}
E[y_{i1}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= y_{i1}\phi_i\mu_{it}, & (3.2.12)\\
E[u_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= \phi_i^2\mu_{is}\mu_{it} + v_{is}\phi_i\mu_{it}, \quad \text{for } 2 \leq s \leq t-1, & (3.2.13)\\
E[x_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= x_{is}\phi_i\mu_{it}, \quad \text{for } 1 \leq s \leq t, & (3.2.14)
\end{aligned}
$$

where the observable analogues (3.2.12), (3.2.13), and (3.2.14) correspond to the conditional moment conditions (3.2.1), (3.2.2), and (3.2.3), respectively. The observable analogue (3.2.13) is derived by using $E[v_{it}\phi_i\mu_{is} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = 0$ for $2 \leq s \leq t$ which stems from the assumption (3.1.4), instead of (3.2.4).

One of the advantages inherent to the implicit operation is that some properties of conditional expectations are exploitable with effect for the observable analogues (3.2.12), (3.2.13), and (3.2.14). After the transformations of the observable analogues (3.2.12), (3.2.13), and (3.2.14) by using the properties of conditional expectations, the moment conditions for consistently estimating γ and β are obtained. The idea used here is that the relationships holding among $y_{i1}u_{it}$ for $t = 2, \ldots, T$, $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$, and $x_{is}u_{it}$ for $s = 1, \ldots, t$ and $t = 2, \ldots, T$ weighted with the appropriate transformations of the explanatory variables are solved through the intermediary of the unconditional expectation operator. The concrete details are described hereinafter.

From the properties of conditional expectations, multiplying both sides of equations (3.2.12), (3.2.13), and (3.2.14) by $\mu_{i,t-1}/\mu_{it}$ generates

$$
\begin{aligned}
E[y_{i1}(\mu_{i,t-1}/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= y_{i1}\phi_i\mu_{i,t-1}, & (3.2.15)\\
E[u_{is}(\mu_{i,t-1}/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= \phi_i^2\mu_{is}\mu_{i,t-1} + v_{is}\phi_i\mu_{i,t-1}, & (3.2.16)\\
& \qquad \text{for } 2 \leq s \leq t-1, &\\
E[x_{is}(\mu_{i,t-1}/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= x_{is}\phi_i\mu_{i,t-1}, & (3.2.17)\\
& \qquad \text{for } 1 \leq s \leq t. &
\end{aligned}
$$

Firstly, the relationship through the intermediary of the unconditional expectation operator is solved between $y_{i1}u_{it}$ weighted with $\mu_{i,t-1}/\mu_{it}$ and $y_{i1}u_{i,t-1}$. That is, the relationship between the transformed $E[y_{i1}u_{it}]$ (i.e. $E[y_{i1}(\mu_{i,t-1}/\mu_{it})u_{it}]$) and $E[y_{i1}u_{i,t-1}]$ is solved. Applying the law of total expectation to (3.2.12) and (3.2.15) generates

$$
\begin{aligned}
E[y_{i1}u_{it}] &= E[y_{i1}\phi_i\mu_{it}], & (3.2.18)\\
E[y_{i1}(\mu_{i,t-1}/\mu_{it})u_{it}] &= E[y_{i1}\phi_i\mu_{i,t-1}]. & (3.2.19)
\end{aligned}
$$

Subtracting (3.2.18) at $t = t-1$ from (3.2.19) generates (for $t = 3, \ldots, T$)

$$E[y_{i1}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] = 0. \tag{3.2.20}$$

Secondly, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{is}u_{it}$ weighted with $\mu_{i,t-1}/\mu_{it}$ and $u_{is}u_{i,t-1}$. That is, the relationship between the transformed $E[u_{is}u_{it}]$ (i.e. $E[u_{is}(\mu_{i,t-1}/\mu_{it})u_{it}]$) and $E[u_{is}u_{i,t-1}]$ is solved. Applying the law of total expectation to (3.2.13) and (3.2.16) generates

$$\begin{aligned} E[u_{is}u_{it}] &= E[\phi_i^2\mu_{is}\mu_{it} + v_{is}\phi_i\mu_{it}], \quad \text{for } 2 \leq s \leq t-1, & (3.2.21) \\ E[u_{is}(\mu_{i,t-1}/\mu_{it})u_{it}] &= E[\phi_i^2\mu_{is}\mu_{i,t-1} + v_{is}\phi_i\mu_{i,t-1}], & (3.2.22) \\ & \quad \text{for } 2 \leq s \leq t-1. & \end{aligned}$$

Subtracting (3.2.21) at $t = t-1$ from (3.2.22) generates (for $t = 4, \ldots, T$)

$$E[u_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] = 0, \quad \text{for } 2 \leq s \leq t-2. \tag{3.2.23}$$

At this stage, creating the recursive equation

$$\begin{aligned} E[y_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] &= \gamma E[y_{i,s-1}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] \\ &\quad + E[u_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})], \quad (3.2.24) \\ &\quad \text{for } 2 \leq s \leq t-2, \end{aligned}$$

from (3.1.2) for $t = s$ and then applying the initial condition (3.2.20) and the innovation (3.2.23) to (3.2.24) according to the same manner as conducted in subsection 2.2, the moment conditions (3.2.20) and (3.2.23) are in toto reformulated as

$$E[y_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] = 0, \quad \text{for } s = 1, \ldots, t-2; \ t = 3, \ldots, T. \tag{3.2.25}$$

The number of the moment conditions (3.2.25) is $(T-2)(T-1)/2$. It can be seen that the order reduction with respect to γ is conducted in the moment conditions (3.2.25) for $2 \leq s \leq t-2$ which are the replacement of (3.2.23).

Thirdly, the relationship through the intermediary of the unconditional expectation operator is solved between $x_{is}u_{it}$ weighted with $\mu_{i,t-1}/\mu_{it}$ and $x_{is}u_{i,t-1}$. That is, the relationship between the transformed $E[x_{is}u_{it}]$ (i.e. $E[x_{is}(\mu_{i,t-1}/\mu_{it})u_{it}]$) and $E[x_{is}u_{i,t-1}]$ is solved. Applying the law of total expectation to (3.2.14) and (3.2.17) generates

$$
\begin{aligned}
E[x_{is}u_{it}] &= E[x_{is}\phi_i\mu_{it}], \quad \text{for } 1 \le s \le t, && (3.2.26)\\
E[x_{is}(\mu_{i,t-1}/\mu_{it})u_{it}] &= E[x_{is}\phi_i\mu_{i,t-1}], \quad \text{for } 1 \le s \le t. && (3.2.27)
\end{aligned}
$$

Subtracting (3.2.26) at $t = t-1$ from (3.2.27) generates

$$
E[x_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] = 0, \quad \text{for } s = 1, \dots, t-1;\ t = 3, \dots, T. \qquad (3.2.28)
$$

The number of the moment conditions (3.2.28) is $(T-1)T/2 - 1$. The moment conditions (3.2.28) are the moment conditions proposed by Chamberlain (1991) and Wooldridge (1997) for a count panel data model without lagged dependent variables, while the moment conditions (3.2.25) are their extension to the application to the LFM in BGW.

Finally, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{is}u_{it}$ weighted with $(\mu_{i,s-1}/\mu_{is})(1/\mu_{it})$ and $u_{i,s-1}u_{it}$ weighted with $1/\mu_{it}$. That is, the relationship between the transformed $E[u_{is}u_{it}]$ (i.e. $E[(\mu_{i,s-1}/\mu_{is})u_{is}(1/\mu_{it})u_{it}]$) and the transformed $E[u_{i,s-1}u_{it}]$ (i.e. $E[u_{i,s-1}(1/\mu_{it})u_{it}]$) is solved. Multiplying both sides of (3.2.13) with $1/\mu_{it}$ generates

$$
E[u_{is}(1/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = \phi_i^2\mu_{is} + v_{is}\phi_i, \quad \text{for } 2 \le s \le t-1. \qquad (3.2.29)
$$

Further, multiplying both sides of (3.2.29) by $\mu_{i,s-1}/\mu_{is}$ generates

$$
\begin{aligned}
E[(\mu_{i,s-1}/\mu_{is})u_{is}(1/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] &= \phi_i^2\mu_{i,s-1} + v_{is}\phi_i(\mu_{i,s-1}/\mu_{is}),\\
&\qquad \text{for } 3 \le s \le t-1. \qquad (3.2.30)
\end{aligned}
$$

Applying the law of total expectation to (3.2.29) and (3.2.30) generates

$$
\begin{aligned}
E[u_{is}(1/\mu_{it})u_{it}] &= E[\phi_i^2\mu_{is}], \quad \text{for } 2 \le s \le t-1, && (3.2.31)\\
E[(\mu_{i,s-1}/\mu_{is})u_{is}(1/\mu_{it})u_{it}] &= E[\phi_i^2\mu_{i,s-1}], \quad \text{for } 3 \le s \le t-1, && (3.2.32)
\end{aligned}
$$

where $E[v_{is}\phi_i] = 0$ for $2 \le s \le T$ and $E[v_{is}\phi_i(\mu_{i,s-1}/\mu_{is})] = 0$ for $3 \le s \le T$ are used for the derivation of (3.2.31) and (3.2.32) respectively, both of which stem from (3.1.4) for $t = s$. Subtracting (3.2.31) at $s = s-1$ from (3.2.32) generates (for $t = 4, \dots, T$)

$$
E[((\mu_{i,s-1}/\mu_{is})u_{is} - u_{i,s-1})(1/\mu_{it})u_{it}] = 0, \quad \text{for } 3 \le s \le t-1. \qquad (3.2.33)
$$

The moment conditions (3.2.23) and (3.2.33) represent a full set of the relationships holding among $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$ through the intermediary of the unconditional expectation operator after weighting $u_{is}u_{it}$ with appropriate transformations of explanatory variables, although some of the moment conditions are redundant. Accordingly, the moment conditions ruling out the redundancies in (3.2.33):

$$E[((\mu_{i,t-2}/\mu_{i,t-1})u_{i,t-1} - u_{i,t-2})(1/\mu_{it})u_{it}] = 0, \quad \text{for } t = 4, \ldots, T, \qquad (3.2.34)$$

whose number is $T-3$, and the moment conditions (3.2.23) are a condensed full set of the moment conditions representing the relationships among $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$. That is, since the moment conditions (3.2.23) are obtained after solving the relationships between $u_{is}u_{it}$ (weighted with $\mu_{i,t-1}/\mu_{it}$) and $u_{is}u_{i,t-1}$ for $s = 2, \ldots, t-2$ and $t = 4, \ldots, T$, while the moment conditions (3.2.34) are obtained after solving the relationships between $u_{i,t-1}u_{it}$ (weighted with $(\mu_{i,t-2}/\mu_{i,t-1})(1/\mu_{it})$) and $u_{i,t-2}u_{it}$ (weighted with $1/\mu_{it}$) for $t = 4, \ldots, T$, the moment conditions (3.2.23) and (3.2.34) are the condensed full set in the sense that the other relationships among $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$ are indirectly traced based on the trunk connections by exploiting (3.2.23) and (3.2.34), where the appropriate transformations of the explanatory variables and the unconditional expectation operator are used on an as-needed basis.

No relationship between the transformed $x_{is}u_{it}$ and $x_{i,s-1}u_{it}$ was found through the intermediary of the unconditional expectation operator.

The set of moment conditions (3.2.34) is a restrictive variant of the set of the moment conditions (2) proposed by Windmeijer (2000), which is obtained under the situation where the model has not the lagged dependent variables, the disturbance is multiplicative, and the explanatory variable is allowed to be endogenous.[4]

Eventually, noting that the moment conditions (3.2.20) and (3.2.23) are in toto reformulated into (3.2.25) and by adding the relationships holding between $x_{is}u_{it}$ weighted with $\mu_{i,t-1}/\mu_{it}$ and $x_{is}u_{i,t-1}$ for $s = 1, \ldots, t-1$ and $t = 3, \ldots, T$ through the intermediary of the unconditional expectation operator, the condensed full set

[4]The alternative of the set of the moment conditions (3.2.34) in accordance with the manner of Windmeijer (2000) is

$$E[((1/\mu_{i,t-1})u_{i,t-1} - (1/\mu_{i,t-2})u_{i,t-2})(1/\mu_{it})u_{it}] = 0, \quad \text{for } t = 4, \ldots, T.$$

These moment conditions hold for the case of predetermined explanatory variables, but the moment conditions (3.2.34) do not hold under the assumptions proposed by Windmeijer (2000). In this sense, the moment conditions (3.2.34) are more restrictive than those constructed by Windmeijer (2000).

of the moment conditions is (3.2.25), (3.2.28), and (3.2.34) when the assumptions (3.1.4) hold which imply that the explanatory variable x_{it} is predetermined.

3.3 Case of strictly exogenous explanatory variables

In this case, the assumptions (3.1.5) are assumed. Multiplying both sides of (3.1.5) by the variables in the information set conditioning v_{it} (i.e. $(y_{i1}, \eta_i, v_i^{t-1}, x_i^T)$) without any transformations generates the following conditional moment conditions:

$$
\begin{aligned}
E[y_{i1}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= 0, && (3.3.1)\\
E[v_{is}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= 0, \quad \text{for } 2 \leq s \leq t-1, && (3.3.2)\\
E[x_{is}v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= 0, \quad \text{for } 1 \leq s \leq T, && (3.3.3)\\
E[\eta_i v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= 0. && (3.3.4)
\end{aligned}
$$

From (3.3.1), (3.3.2), (3.3.3), and (3.3.4), the following unconditional moment conditions are generated according to the manner of the conventional operation:

$$
\begin{aligned}
E[y_{i1}v_{it}] &= 0, && (3.3.5)\\
E[v_{is}v_{it}] &= 0, \quad \text{for } 2 \leq s \leq t-1, && (3.3.6)\\
E[x_{is}v_{it}] &= 0, \quad \text{for } 1 \leq s \leq T, && (3.3.7)\\
E[\eta_i v_{it}] &= 0. && (3.3.8)
\end{aligned}
$$

The explicit standard assumptions (3.3.5), (3.3.6), (3.3.7), and (3.3.8) say that y_{i1} and v_{it} are uncorrelated unconditionally, that v_{it} is serially uncorrelated unconditionally, that x_{is} for $s = 1, \ldots, T$ and v_{it} are uncorrelated unconditionally, and that η_i and v_{it} are uncorrelated unconditionally, respectively. The case of strictly exogenous explanatory variables is characterized by the unconditional moment condition (3.3.7). It is impossible to conduct the consistent estimation of γ and β directly using the explicit standard assumptions due to the presence of the unobservable variables v_{it} and η_i. By replacing the unobservable variable v_{it} in (3.3.5), (3.3.6), and (3.3.7) with the observable variable u_{it}, the following observable analogues are generated:

$$
\begin{aligned}
E[y_{i1}u_{it}] &= E[y_{i1}\phi_i\mu_{it}], && (3.3.9)\\
E[u_{is}u_{it}] &= E[\phi_i^2\mu_{is}\mu_{it}], \quad \text{for } 2 \leq s \leq t-1, && (3.3.10)\\
E[x_{is}u_{it}] &= E[x_{is}\phi_i\mu_{it}]. \quad \text{for } 1 \leq s \leq T, && (3.3.11)
\end{aligned}
$$

which correspond to the unconditional moment conditions (3.3.5), (3.3.6), and (3.3.7), respectively. The observable analogue (3.3.10) is derived by using $E[v_{it}\phi_i\mu_{is}] = 0$ for $2 \leq s \leq T$ which stems from the assumptions (3.1.5), instead of (3.3.8).

As is similar to the case of predetermined explanatory variables, no appropriate relationship is found among $E[y_{i1}u_{it}]$ for $t = 2, \ldots, T$, $E[u_{is}u_{it}]$ for $2 \leq s \leq t-1$ and $t = 3, \ldots, T$, and $E[x_{is}u_{it}]$ for $1 \leq s \leq T$ and $t = 2, \ldots, T$, as is clear from the moment conditions (3.3.9), (3.3.10), and (3.3.11). Accordingly, the implicit operation as described below is also needed to obtain the unconditional moment conditions for consistently estimating γ and β in this case.

As is similar to the case of predetermined explanatory variables, the implicit operation begins from the implicit standard assumptions (3.3.1), (3.3.2), (3.3.3), and (3.3.4), which say that y_{i1} and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^T)$ (i.e. the information set corresponding to the case of strictly exogenous explanatory variables), that v_{is} for $s = 2, \ldots, t-1$ and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^T)$, that x_{is} for $s = 1, \ldots, T$ and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^T)$, and that η_i and v_{it} are uncorrelated conditional on $(y_{i1}, \eta_i, v_i^{t-1}, x_i^T)$, respectively. By replacing the unobservable variable v_{it} in the conditional moment conditions (3.3.9), (3.3.10), and (3.3.11) with the observable variable u_{it}, the the following observable analogues for (3.3.1), (3.3.2), and (3.3.3) are generated:

$$
\begin{aligned}
E[y_{i1}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= y_{i1}\phi_i\mu_{it}, & (3.3.12)\\
E[u_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= \phi_i^2\mu_{is}\mu_{it} + v_{is}\phi_i\mu_{it}, \quad \text{for } 2 \leq s \leq t-1, & (3.3.13)\\
E[x_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= x_{is}\phi_i\mu_{it}, \quad \text{for } 1 \leq s \leq T, & (3.3.14)
\end{aligned}
$$

which correspond to the conditional moment conditions (3.3.1), (3.3.2), and (3.3.3), respectively. The observable analogue (3.3.13) is derived by using $E[v_{it}\phi_i\mu_{is} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] = 0$ for $2 \leq s \leq T$ which stems from the assumptions (3.1.5), instead of (3.3.4).

Along the lines of the implicit operation in previous subsection, the properties of conditional expectations are applied to the conditional moment conditions (3.3.12), (3.3.13), and (3.3.14) with the intention of obtaining the moment conditions for consistently estimating γ and β. The idea used here is that the relationships holding among $y_{i1}u_{it}$ for $t = 2, \ldots, T$, $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$, and $x_{is}u_{it}$ for $s = 1, \ldots, T$ and $t = 2, \ldots, T$ weighted with the appropriate transformations of the explanatory variables are solved through the intermediary of the unconditional expectation operator.

Since the explanatory variable x_{it} is strictly exogenous, multiplying both sides of equations (3.3.12), (3.3.13), and (3.3.14) by $\mu_{i,t+1}/\mu_{it}$ can generates

$$\begin{aligned}
E[y_{i1}(\mu_{i,t+1}/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= y_{i1}\phi_i\mu_{i,t+1}, &(3.3.15)\\
E[u_{is}(\mu_{i,t+1}/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= \phi_i^2\mu_{is}\mu_{i,t+1} + v_{is}\phi_i\mu_{i,t+1}, &(3.3.16)\\
&\quad \text{for } 2 \leq s \leq t-1,\\
E[x_{is}(\mu_{i,t+1}/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= x_{is}\phi_i\mu_{i,t+1}, &(3.3.17)\\
&\quad \text{for } 1 \leq s \leq T,
\end{aligned}$$

where it should be noted that these forms of moment conditions cannot be obtained when the explanatory variable x_{it} is predetermined.

Firstly, the relationship through the intermediary of the unconditional expectation operator is solved between $y_{i1}u_{it}$ and $y_{i1}u_{i,t-1}$ weighted with $\mu_{it}/\mu_{i,t-1}$. That is, the relationship between $E[y_{i1}u_{it}]$ and the transformed $E[y_{i1}u_{i,t-1}]$ (i.e. $E[y_{i1}(\mu_{it}/\mu_{i,t-1})u_{i,t-1}]$) is solved. Applying the law of total expectation to (3.3.12) and (3.3.15) generates

$$\begin{aligned}
E[y_{i1}u_{it}] &= E[y_{i1}\phi_i\mu_{it}], &(3.3.18)\\
E[y_{i1}(\mu_{i,t+1}/\mu_{it})u_{it}] &= E[y_{i1}\phi_i\mu_{i,t+1}]. &(3.3.19)
\end{aligned}$$

Subtracting (3.3.19) at $t = t-1$ from (3.3.18) generates (for $t = 3, \ldots, T$)

$$E[y_{i1}(u_{it} - (\mu_{it}/\mu_{i,t-1})u_{i,t-1})] = 0. \qquad (3.3.20)$$

Secondly, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{is}u_{it}$ and $u_{is}u_{i,t-1}$ weighted with $\mu_{it}/\mu_{i,t-1}$. That is, the relationship between $E[u_{is}u_{it}]$ and the transformed $E[u_{is}u_{i,t-1}]$ (i.e. $E[u_{is}(\mu_{it}/\mu_{i,t-1})u_{i,t-1}]$) is solved. Applying the law of total expectation to (3.3.13) and (3.3.16) generates

$$\begin{aligned}
E[u_{is}u_{it}] &= E[\phi_i^2\mu_{is}\mu_{it}], \quad \text{for } 2 \leq s \leq t-1, &(3.3.21)\\
E[u_{is}(\mu_{i,t+1}/\mu_{it})u_{it}] &= E[\phi_i^2\mu_{is}\mu_{i,t+1}], \quad \text{for } 2 \leq s \leq t-1, &(3.3.22)
\end{aligned}$$

where $E[v_{is}\phi_i\mu_{it}] = 0$ for $2 \leq s \leq T$ stemming from (3.1.5) is used for the derivation of (3.3.21) and (3.3.22). Subtracting (3.3.22) at $t = t-1$ from (3.3.21) generates (for $t = 4, \ldots, T$)

$$E[u_{is}(u_{it} - (\mu_{it}/\mu_{i,t-1})u_{i,t-1})] = 0, \quad \text{for } 2 \leq s \leq t-2. \qquad (3.3.23)$$

According to the same manner of creating the recursive equation as in previous subsection, the moment conditions (3.3.20) and (3.3.23) are in toto reformulated as

$$E[y_{is}(u_{it} - (\mu_{it}/\mu_{i,t-1})u_{i,t-1})] = 0, \quad \text{for } s = 1, \dots, t-2;\ t = 3, \dots, T. \tag{3.3.24}$$

The number of the moment conditions (3.3.24) is $(T-2)(T-1)/2$.

Thirdly, the relationship through the intermediary of the unconditional expectation operator is solved between $x_{is}u_{it}$ and $x_{is}u_{i,t-1}$ weighted with $\mu_{it}/\mu_{i,t-1}$. That is, the relationship between $E[x_{is}u_{it}]$ and the transformed $E[x_{is}u_{i,t-1}]$ (i.e. $E[x_{is}(\mu_{it}/\mu_{i,t-1})u_{i,t-1}]$) is solved. Applying the law of total expectation to (3.3.14) and (3.3.17) generates

$$\begin{aligned} E[x_{is}u_{it}] &= E[x_{is}\phi_i\mu_{it}], \quad \text{for } 1 \le s \le T, & (3.3.25)\\ E[x_{is}(\mu_{i,t+1}/\mu_{it})u_{it}] &= E[x_{is}\phi_i\mu_{i,t+1}], \quad \text{for } 1 \le s \le T. & (3.3.26) \end{aligned}$$

Subtracting (3.3.26) at $t = t-1$ from (3.3.25) generates

$$E[x_{is}(u_{it} - (\mu_{it}/\mu_{i,t-1})u_{i,t-1})] = 0, \quad \text{for } s = 1, \dots, T;\ t = 3, \dots, T. \tag{3.3.27}$$

The number of the moment conditions (3.3.27) is $(T-2)T$.

It should be noted that for the case of strictly exogenous explanatory variables the moment conditions (3.2.25) and (3.2.28) are valid, while for the case of predetermined explanatory variables the moment conditions (3.3.24) are not valid and the moment conditions (3.3.27) are not also valid even when the instrument variables x_{is} for $s = 1, \dots, t$ are used for the transformed equation $u_{it} - (\mu_{it}/\mu_{i,t-1})u_{i,t-1}$ in (3.3.27).

Finally, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{is}u_{it}$ and $u_{i,s-1}u_{it}$ weighted with $\mu_{is}/\mu_{i,s-1}$. That is, the relationship between $E[u_{is}u_{it}]$ and the transformed $E[u_{i,s-1}u_{it}]$ (i.e. $E[(\mu_{is}/\mu_{i,s-1})u_{i,s-1}u_{it}]$) is solved. Multiplying both sides of (3.3.13) by $\mu_{i,s+1}/\mu_{is}$

$$\begin{aligned} E[(\mu_{i,s+1}/\mu_{is})u_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] &= \phi_i^2\mu_{i,s+1}\mu_{it} + v_{is}\phi_i(\mu_{i,s+1}/\mu_{is})\mu_{it},\\ &\quad \text{for } 2 \le s \le t-1. & (3.3.28) \end{aligned}$$

Applying the law of total expectation to (3.3.13) and (3.3.28),

$$\begin{aligned} E[u_{is}u_{it}] &= E[\phi_i^2\mu_{is}\mu_{it}], \quad \text{for } 2 \le s \le t-1, & (3.3.29)\\ E[(\mu_{i,s+1}/\mu_{is})u_{is}u_{it}] &= E[\phi_i^2\mu_{i,s+1}\mu_{it}], \quad \text{for } 2 \le s \le t-1, & (3.3.30) \end{aligned}$$

where (3.3.29) and (3.3.30) are obtained by using $E[v_{is}\phi_i\mu_{it}] = 0$ for $2 \leq s \leq T$ and $E[v_{is}\phi_i(\mu_{i,s+1}/\mu_{is})] = 0$ for $2 \leq s \leq T-1$ respectively, both of which stem from (3.1.5). Subtracting (3.3.30) at $s = s-1$ from (3.3.29) generates (for $t = 4, \dots, T$)

$$E[(u_{is} - (\mu_{is}/\mu_{i,s-1})u_{i,s-1})u_{it}] = 0, \quad \text{for } 3 \leq s \leq t-1. \tag{3.3.31}$$

It should be noted that the moment conditions (3.2.33) are valid for the case of strictly exogenous explanatory variables, while the moment conditions (3.3.31) are not valid for the case of predetermined explanatory variables.

The moment conditions (3.3.23) and (3.3.31) represent a full set of the relationships holding among $u_{is}u_{it}$ for $s = 2, \dots, t-1$ and $t = 3, \dots, T$ through the intermediary of the unconditional expectation operator after weighting $u_{is}u_{it}$ with the appropriate transformations of explanatory variables, although some of the moment conditions are redundant. The moment conditions ruling out the redundancies in (3.3.31):

$$E[(u_{i,t-1} - (\mu_{i,t-1}/\mu_{i,t-2})u_{i,t-2})u_{it}] = 0, \quad \text{for } t = 4, \dots, T, \tag{3.3.32}$$

whose number is $T-3$, and the moment conditions (3.3.23) are a condensed full set of the moment conditions representing the relationships among $u_{is}u_{it}$ for $s = 2, \dots, t-1$ and $t = 3, \dots, T$. The reason why the moment conditions (3.3.23) and (3.3.32) are the condensed full set is conceptually the same as that described in previous subsection.

No relationship between the transformed $x_{is}u_{it}$ and $x_{i,s-1}u_{it}$ was found through the intermediary of the unconditional expectation operator.

Eventually, based on the reason similar to that described in previous subsection, the condensed full set of the moment conditions is (3.3.24), (3.3.27), and (3.3.32), when the assumptions (3.1.5) hold which imply that x_{it} is strictly exogenous.

When γ is set to be zero, the set of the moment conditions (3.3.24) and (3.3.32) is conceptually equivalent to the set of the moment conditions generated after applying the law of total expectation to (22) and (23) in Crépon and Duguet (1997).[5]

[5]According to the idea of Crépon and Duguet (1997), it is conceivable that the moment conditions (3.3.24) and (3.3.32) are replaced by (3.2.25) and

$$E[((\mu_{i,t-2}/\mu_{it})u_{it} - u_{i,t-2})u_{i,t-1}] = 0, \quad \text{for } t = 4, \dots, T,$$

when both γ and β are estimated jointly. The former moment conditions are derived from the relationships stated in subsection 3.2, while the latter moment conditions are derived from the relationships holding between $u_{i,t-1}u_{it}$ weighted with $\mu_{i,t-2}/\mu_{it}$ and $u_{i,t-2}u_{i,t-1}$ through the intermediary of the unconditional expectation operator.

3.4 Case of mean-stationary dependent variables

In this case, the stationarity of the dependent variable and the explanatory variable is additively assumed for the LFM (3.1.1). The discussion is conducted, just confined to the case of predetermined explanatory variables, that is, just under the assumption (3.1.4). The moment conditions obtained under the assumption of the stationarity in the LFM, referred to as the stationarity moment conditions hereafter, are of the form different from those proposed by Arellano and Bover (1995) and discussed by Ahn and Schmidt (1995) and Blundell and Bond (1998) for the case of the DPDM.

The discussion is conducted on the stationarity of the explanatory variable assumed in this subsection. The explanatory variable x_{it} can be said to be stationary in the sense that its moment generating function conditional on the fixed effect η_i is equal over time, when the following relationships hold for x_{it}:

$$E[\exp(kx_{it}) \mid \eta_i] = E[\varphi_i(k) \mid \eta_i], \quad \text{for } t = 1, \ldots, T, \tag{3.4.1}$$

where k is any real value and $\varphi_i(k)$ is a function of k varying with individual i but being constant over time. From (3.4.1), the the following relationships among the unconditional moment conditions are obtained:

$$E[f(\eta_i)\exp(kx_{it})] = E[f(\eta_i)\varphi_i(k)], \quad \text{for } t = 1, \ldots, T, \tag{3.4.2}$$

where $f(\eta_i)$ is any function of η_i and accordingly $f(\eta_i)\varphi_i(k)$ is constant over time. Accordingly, from (3.4.2) with $k = \beta$, the following relationships are obtained:

$$E[f(\eta_i)\mu_{it}] = E[f(\eta_i)\varphi_i(\beta)], \quad \text{for } t = 1, \ldots, T. \tag{3.4.3}$$

It is seen from the descriptions above that the relationships (3.4.3) hold under the assumptions (3.4.1).

Added to the above, the assumption is imposed on the initial condition of the count dependent variable as follows. That is, it is assumed that the initial condition of the count dependent variable y_{i1} is described as

$$y_{i1} = (1/(1-\gamma))\phi_i\mu_{i1} + v_{i1}, \tag{3.4.4}$$

where $\mu_{i1} = \exp(\beta x_{i1})$ and the unobservable variable v_{i1} satisfies

$$E[v_{i1} \mid \eta_i, x_{i1}] = 0. \tag{3.4.5}$$

When the explanatory variable x_{it} is stationary in the sense that the relationships (3.4.1) hold and the initial condition of the count dependent variable y_{i1} is written as (3.4.4) with (3.4.5), the count dependent variable y_{it} is mean-stationary. This is because in the setting above,

$$E[y_{it}] = (1/(1-\gamma))E[\phi_i \varphi_i(\beta)], \quad \text{for } t = 1, \ldots, T, \tag{3.4.6}$$

which are obtained by using the driving process of (3.1.1), the relationships (3.4.3) with $f(\eta_i) = \phi_i$ being specified, and $E[v_{it}] = 0$ for $t = 1, \ldots, T$ obtained after applying the law of total expectation to (3.4.5) and (3.1.4). In other word, equation (3.4.6) implies that means of y_{it} are equal over time.

Based on the assumptions mentioned above, the moment conditions related to the stationarity of y_{it} and x_{it} are contrived.

Firstly, the stationarity moment conditions are derived with respect to the dependent variable y_{it}, when y_{it} is mean-stationary. The relationship through the intermediary of the unconditional expectation operator is solved between $y_{i1}u_{it}$ weighted with $(1-\gamma)(1/\mu_{it})$ and $u_{i2}u_{it}$ weighted with $1/\mu_{it}$ and the relationship through the intermediary of the unconditional expectation operator is solved between $u_{is}u_{it}$ weighted with $1/\mu_{it}$ and $u_{i,s-1}u_{it}$ weighted with $1/\mu_{it}$. That is, the relationships between the transformed $E[y_{i1}u_{it}]$ (i.e. $(1-\gamma)E[y_{i1}(1/\mu_{it})u_{it}]$) and the transformed $E[u_{i2}u_{it}]$ (i.e. $E[u_{i2}(1/\mu_{it})u_{it}]$) and between the transformed $E[u_{is}u_{it}]$ (i.e. $E[u_{is}(1/\mu_{it})u_{it}]$) and the transformed $E[u_{i,s-1}u_{it}]$ (i.e. $E[u_{i,s-1}(1/\mu_{it})u_{it}]$) are solved. From now on, these relationships are solved in a phased manner, in the framework of the implicit operation.

In this case, the moment conditions (3.2.12) is rewritten as

$$E[y_{i1}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = (1/(1-\gamma))\phi_i^2 \mu_{i1}\mu_{it} + v_{i1}\phi_i\mu_{it}, \tag{3.4.7}$$

by plugging (3.4.4) into (3.2.12). Multiplying both sides of (3.4.7) by $1/\mu_{it}$ generates

$$E[y_{i1}(1/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = (1/(1-\gamma))\phi_i^2 \mu_{i1} + v_{i1}\phi_i. \tag{3.4.8}$$

Applying the law of total expectation to (3.4.8),

$$E[y_{i1}(1/\mu_{it})u_{it}] = (1/(1-\gamma))E[\phi_i^2 \varphi_i(\beta)], \tag{3.4.9}$$

where the relationships (3.4.3) with $f(\eta_i) = \phi_i^2$ at $t = 1$ and the assumption (3.4.5) are used.

Further, in this case, by using (3.4.3) with $f(\eta_i) = \phi_i^2$, (3.2.31) is rewritten as

$$E[u_{is}(1/\mu_{it})u_{it}] = E[\phi_i^2 \varphi_i(\beta)], \quad \text{for } 2 \leq s \leq t-1. \tag{3.4.10}$$

Subtracting (3.4.9) multiplied by $1-\gamma$ from (3.4.10) at $s=2$, the relationship holding between $y_{i1}u_{it}$ (weighted with $(1-\gamma)(1/\mu_{it})$) and $u_{i2}u_{it}$ (weighted with $1/\mu_{it}$) through the intermediary of the unconditional expectation operator is solved as follows:

$$E[\Delta y_{i2}(1/\mu_{it})u_{it}] = 0, \tag{3.4.11}$$

where (3.1.2) for $t=2$ is used and $\Delta y_{i2} = y_{i2} - y_{i1}$.

Further, subtracting (3.4.10) at $s = s-1$ from (3.4.10), the relationship between $u_{is}u_{it}$ (weighted with $1/\mu_{it}$) and $u_{i,s-1}u_{it}$ (weighted with $1/\mu_{it}$) through the intermediary of the unconditional expectation operator is solved as follows:

$$E[\Delta u_{is}(1/\mu_{it})u_{it}] = 0, \quad \text{for } 3 \leq s \leq t-1, \tag{3.4.12}$$

where $\Delta u_{is} = u_{is} - u_{i,s-1}$.

At this stage, creating the recursive equation

$$\begin{aligned} E[\Delta y_{is}(1/\mu_{it})u_{it}] &= \gamma E[\Delta y_{i,s-1}(1/\mu_{it})u_{it}] + E[\Delta u_{is}(1/\mu_{it})u_{it}], \\ & \qquad \text{for } 3 \leq s \leq t-1, \end{aligned} \tag{3.4.13}$$

from (3.1.2) for $t = s$ and then applying the initial condition (3.4.11) and the innovation (3.4.12) to (3.4.13), the moment conditions (3.4.11) and (3.4.12) are in toto reformulated as

$$E[\Delta y_{is}(1/\mu_{it})u_{it}] = 0, \quad \text{for } 2 \leq s \leq t-1, \tag{3.4.14}$$

where $\Delta y_{is} = y_{is} - y_{i,s-1}$, for $t = 3, \ldots, T$. It can be seen that the order reduction with respect to γ is conducted in the moment conditions (3.4.14) for $3 \leq s \leq t-1$ which are the replacement of (3.4.12).

When the dependent variable and the explanatory variable satisfy the stationarity under the assumptions of predetermined explanatory variables, the set of the moment conditions (3.2.25) and (3.4.14) represents a full set of the relationships holding among $y_{i1}u_{it}$ for $t = 2, \ldots, T$ and $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$ through the intermediary of the unconditional expectation operator after weighting $y_{i1}u_{it}$ and $u_{is}u_{it}$ with the appropriate transformations of explanatory variables, although some of the moment conditions are redundant. The moment conditions ruling out the redundancies in (3.4.14):

$$E[\Delta y_{i,t-1}(1/\mu_{it})u_{it}] = 0, \quad \text{for } t = 3, \ldots, T, \tag{3.4.15}$$

whose number is $T-2$, and the moment conditions (3.2.25) are a condensed full set of the moment conditions representing the relationships holding among $y_{i1}u_{it}$ for $t = 2, \ldots, T$ and $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$. The reason why the moment conditions (3.2.25) and (3.4.15) are the condensed full set is almost similar to that described in subsection 3.2. Since the moment conditions (3.2.25) are obtained after solving the relationships between $y_{i1}u_{it}$ (weighted with $\mu_{i,t-1}/\mu_{it}$) and $y_{i1}u_{i,t-1}$ for $t = 3, \ldots, T$ and between $u_{is}u_{it}$ (weighted with $\mu_{i,t-1}/\mu_{it}$) and $u_{is}u_{i,t-1}$ for $s = 2, \ldots, t-2$ and $t = 4, \ldots, T$, while the moment conditions (3.4.15) are obtained after solving the relationships between $y_{i1}u_{i3}$ (weighted with $(1-\gamma)(1/\mu_{i3})$) and $u_{i2}u_{i3}$ (weighted with $1/\mu_{i3}$) and between $u_{i,t-1}u_{it}$ (weighted with $1/\mu_{it}$) and $u_{i,t-2}u_{it}$ (weighted with $1/\mu_{it}$) for $t = 4, \ldots, T$, the moment conditions (3.2.25) and (3.4.15) are the condensed full set in the sense that the other relationships among $y_{i1}u_{it}$ for $t = 2, \ldots, T$ and $u_{is}u_{it}$ for $s = 2, \ldots, t-1$ and $t = 3, \ldots, T$ are indirectly traced based on the trunk connections by exploiting (3.2.25) and (3.4.15).

Secondly, the stationarity moment conditions are derived with respect to the explanatory variable x_{it}, when x_{it} is stationary in the sense that its moment generating function is equal over time as implied by the assumptions (3.4.1). Two alternative types of the stationarity moment conditions with respect to x_{it} are obtained when x_{it} is predetermined as implied by the assumptions (3.1.4)

From now on, the process of deriving one type of the stationarity moment conditions with respect to x_{it} is shown. For $s \leq t$, the relationship through the intermediary of the unconditional expectation operator is solved between $x_{is}u_{it}$ weighted with $1/\mu_{it}$ and $x_{i,s-1}u_{it}$ weighted with $1/\mu_{it}$. That is, the relationship between the transformed $E[x_{is}u_{it}]$ (i.e. $E[x_{is}(1/\mu_{it})u_{it}]$) and the transformed $E[x_{i,s-1}u_{it}]$ (i.e. $E[x_{i,s-1}(1/\mu_{it})u_{it}]$) is solved. More specifically, assuming the stationarity of x_{it}, the relationship holding between the transformed $x_{is}u_{it}$ and $x_{i,s-1}u_{it}$ through the intermediary of the unconditional expectation operator is obtained, which is unable to be obtained in subsection 3.2. Multiplying both sides of (3.2.14) by $1/\mu_{it}$ generates

$$E[x_{is}(1/\mu_{it})u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = x_{is}\phi_i, \quad \text{for } 1 \leq s \leq t. \tag{3.4.16}$$

Applying the law of total expectation to (3.4.16), the following relationship is generated:

$$E[x_{is}(1/\mu_{it})u_{it}] = E[x_{is}\phi_i], \quad \text{for } 1 \leq s \leq t. \tag{3.4.17}$$

In addition, differentiating (3.4.2) with respect to k, the following relationships are obtained:

$$E[f(\eta_i)x_{it}\exp(kx_{it})] = E[f(\eta_i)\varphi_i'(k)], \quad \text{for } t = 1, \ldots, T, \tag{3.4.18}$$

where $\varphi_i'(k) = d\varphi_i(k)/dk$ and accordingly $f(\eta_i)\varphi_i'(k)$ is constant over time. From (3.4.17) and (3.4.18) with $t = s$, $k = 0$, and $f(\eta_i) = \phi_i$, the following relationship is obtained:

$$E[x_{is}(1/\mu_{it})u_{it}] = E[\varphi_i'(0)\phi_i], \quad \text{for } 1 \leq s \leq t, \tag{3.4.19}$$

where $\varphi_i'(0) = (d\varphi_i(k)/dk)|_{k=0}$. From (3.4.19), the following unconditional moment conditions are obtained (for $t = 2, \ldots, T$):

$$E[\Delta x_{is}(1/\mu_{it})u_{it}] = 0, \quad \text{for } 2 \leq s \leq t, \tag{3.4.20}$$

where $\Delta x_{is} = x_{is} - x_{i,s-1}$. When the explanatory variables satisfy the stationarity under the assumptions of predetermined explanatory variables, the set of the moment conditions (3.2.28) and (3.4.20) represents a full set of the relationships holding among $x_{is}u_{it}$ for $s = 1, \ldots, t$ and $t = 2, \ldots, T$ through the intermediary of the unconditional expectation operator after weighting $x_{is}u_{it}$ with the appropriate transformations of the explanatory variables, although some of the moment conditions are redundant.[6] The moment conditions ruling out the redundancies in (3.4.20):

$$E[\Delta x_{it}(1/\mu_{it})u_{it}] = 0, \quad \text{for } t = 2, \ldots, T, \tag{3.4.21}$$

whose number is $T-1$, and (3.2.28) are a condensed full set of the moment conditions representing the relationships among $x_{is}u_{it}$ for $s = 1, \ldots, t$ and $t = 2, \ldots, T$. Since the moment conditions (3.2.28) are obtained after solving the relationships between $x_{is}u_{it}$ (weighted with $\mu_{i,t-1}/\mu_{it}$) and $x_{is}u_{i,t-1}$ for $s = 1, \ldots, t-1$ and $t = 3, \ldots, T$, while the moment conditions (3.4.21) are obtained after solving the relationships between $x_{it}u_{it}$ weighted with $1/\mu_{it}$ and $x_{i,t-1}u_{it}$ weighted with $1/\mu_{it}$ for $t = 2, \ldots, T$, the moment conditions (3.2.28) and (3.4.21) are the condensed full set in the sense that the other relationships among $x_{is}u_{it}$ for $s = 1, \ldots, t$ and $t = 2, \ldots, T$ are indirectly traced based on the trunk connections by exploiting (3.2.28) and (3.4.21).

From here, the process of deriving another type of the stationarity moment conditions with respect to x_{it} is shown. The relationship through the intermediary of the unconditional expectation operator is solved between $x_{it}u_{it}$ and $x_{i,t-1}u_{i,t-1}$. That

[6] It should be noted that the mean-stationarity of the dependent variable is not needed to derive the moment conditions (3.4.20).

is, the relationship between $E[x_{it}u_{it}]$ and $E[x_{i,t-1}u_{i,t-1}]$ is solved. The unconditional moment condition (3.2.26) for $s = t$ is

$$E[x_{it}u_{it}] = E[x_{it}\phi_i\mu_{it}]. \tag{3.4.22}$$

From (3.4.22) and (3.4.18) with $k = \beta$ and $f(\eta_i) = \phi_i$, the following relationship is obtained:

$$E[x_{it}u_{it}] = E[\varphi_i'(\beta)\phi_i], \tag{3.4.23}$$

where $\varphi_i'(\beta) = (d\varphi_i(k)/dk)|_{k=\beta}$. Taking the first-difference of (3.4.23), the following $T-2$ unconditional moment conditions are obtained:

$$E[x_{it}u_{it} - x_{i,t-1}u_{i,t-1}] = 0, \quad \text{for } t = 3, \dots, T, \tag{3.4.24}$$

whose number is $T-2$.[7] For $t = 3, \dots, T$, the moment conditions (3.4.24) are equivalent to (3.4.21) conceptually. The moment conditions (3.4.24) are the only set of the moment conditions linear with respect to the parameters of interest in the sets constructed for the LFM until now, which are used only for the estimation of γ and hold irrespective of any values of β.

Eventually, when the dependent variable and the explanatory variable satisfy the stationarity under the assumptions of predetermined explanatory variables (that is, when (3.4.1) and (3.4.4) with (3.4.5) hold for the model (3.1.1) with (3.1.4)), it follows that one set of the moment conditions used for consistently estimating γ and β is composed of (3.2.25), (3.4.15), (3.2.28), and (3.4.21), while another set is composed of (3.2.25), (3.4.15), (3.2.28), (3.4.21) for $t = 2$, and (3.4.24).

3.5 Case of equidispersion

In the model (3.1.1), the equality between mean and variance of the dependent variable y_{it} is referred to as equidispersion, which is characteristic of the dependent variable y_{it} distributed as Poisson. The moment conditions associated with equidispersion for the LFM in CPD are derived for the following three cases: the case of predetermined explanatory variables, the case of strictly exogenous explanatory variables, and the case of stationary explanatory variables.

Firstly, the consideration is conducted for the case of predetermined explanatory variables. In the model (3.1.1) with (3.1.4), the equidispersion is additionally

[7] It should be noted that the mean-stationarity of the dependent variable is not needed to derive the moment conditions (3.4.24).

formulized as follows:

$$E[(v_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = 0, \tag{3.5.1}$$

for $t = 2, \dots, T$. For the case of predetermined explanatory variables, it is postulated that (3.5.1) is the implicit assumption added to the implicit standard assumptions (3.2.1), (3.2.2), (3.2.3), and (3.2.4). The implicit operation is implemented from now on to obtain the moment conditions for estimating γ and β consistently. Since v_{it} is unobservable and therefore $v_{it}^2 - y_{it}$ is also unobservable, the observable analogue for equation (3.5.1) is constructed by replacing v_{it} with the observable variable u_{it} as follows:

$$E[(u_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = \phi_i^2 \mu_{it}^2, \tag{3.5.2}$$

where $E[v_{it}\phi_i\mu_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = 0$ stemming from (3.1.4) is used which is a proxy for (3.2.4). Multiplying both sides of (3.5.2) by $\mu_{i,t-1}^2/\mu_{it}^2$ generates

$$E[(\mu_{i,t-1}^2/\mu_{it}^2)(u_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = \phi_i^2 \mu_{i,t-1}^2. \tag{3.5.3}$$

Applying the law of total expectation to (3.5.2) and (3.5.3), it follows that

$$\begin{aligned} E[(u_{it}^2 - y_{it})] &= E[\phi_i^2 \mu_{it}^2], & (3.5.4) \\ E[(\mu_{i,t-1}^2/\mu_{it}^2)(u_{it}^2 - y_{it})] &= E[\phi_i^2 \mu_{i,t-1}^2]. & (3.5.5) \end{aligned}$$

Subtracting (3.5.4) at $t = t-1$ from (3.5.5) generates the following unconditional moment conditions for estimating γ and β consistently:

$$E[(\mu_{i,t-1}^2/\mu_{it}^2)(u_{it}^2 - y_{it}) - (u_{i,t-1}^2 - y_{i,t-1})] = 0, \quad \text{for } t = 3, \dots, T, \tag{3.5.6}$$

whose number is $T-2$.

Next, a mention in passing is conducted when the explanatory variable x_{it} is strictly exogenous. In the model (3.1.1) with (3.1.5), the equidispersion is additionally formulized as follows:

$$E[(v_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T] = 0, \tag{3.5.7}$$

for $t = 2, \dots, T$. For the case of strictly exogenous explanatory variables, it is postulated that (3.5.7) is the implicit assumption added to the implicit standard assumptions (3.3.1), (3.3.2), (3.3.3), and (3.3.4). Based on (3.5.7) and mimicking the process of deriving (3.5.6), the following unconditional moment conditions are derived for estimating γ and β consistently:

$$E[(u_{it}^2 - y_{it}) - (\mu_{it}^2/\mu_{i,t-1}^2)(u_{i,t-1}^2 - y_{i,t-1})] = 0, \quad \text{for } t = 3, \ldots, T, \tag{3.5.8}$$

whose number is $T - 2$. It should be noted that under (3.1.5) and (3.5.7), the moment conditions (3.5.6) hold as well.

Further, when the explanatory variable x_{it} is assumed to be stationary in the sense that its moment generating function is equal over time in addition to the assumption that x_{it} is predetermined, the moment conditions associated with the equidispersion to be obtained are of the form different from (3.5.6). In this case, the relationships (3.4.1) hold for x_{it} and therefore the relationships (3.4.2) hold. Specifying that $f(\eta_i) = \phi_i^2$ and $k = 2\beta$ in (3.4.2), the following relationships are obtained:

$$E[\phi_i^2\mu_{it}^2] = E[\phi_i^2\varphi_i(2\beta)], \quad \text{for } t = 1, \ldots, T. \tag{3.5.9}$$

Accordingly, using (3.5.9), (3.5.4) is rewritten as

$$E[(u_{it}^2 - y_{it})] = E[\phi_i^2\varphi_i(2\beta)]. \tag{3.5.10}$$

Taking the first-difference of (3.5.10), the following unconditional moment conditions are obtained for estimating γ consistently:

$$E[\Delta u_{it}(u_{it} + u_{i,t-1}) - \Delta y_{it}] = 0, \quad \text{for } t = 3, \ldots, T, \tag{3.5.11}$$

whose number is $T - 2$. The moment conditions (3.5.11) hold irrespective of any values of β.

It is also possible to say that the moment conditions (3.5.6), (3.5.8), and (3.5.11) represent the relationships holding among u_{it}^2 for $t = 2, \ldots, T$ transformed with the appropriate transformations of the dependent and explanatory variables through the intermediary of the unconditional expectation operator. Accordingly, the moment conditions (3.5.6), (3.5.8), and (3.5.11) in the LFM for CPD can be regarded as those associated with the variance of the disturbance, as is the case with the intertemporal homoscedasticity moment conditions for the DPDM which are proposed by Ahn (1990) and Ahn and Schmidt (1995).

3.6 Discussion

A manipulation is needed to use any of the moment conditions (3.2.34), (3.4.15), and (3.4.21) for the estimation of γ and β in the LFM. If all values of the explanatory variable x_{it} are positive, the estimate of β using any of the moment conditions (3.2.34), (3.4.15), and (3.4.21) seems to be in danger of going to infinity. Then, the utilization of the explanatory variable with its all values being positive is a banality in the econometric analysis.[8] The breakthrough by Windmeijer (2000) is available when using the moment conditions (3.2.34), (3.4.15), and (3.4.21), where instead of x_{it}, $\tilde{x}_{it} = x_{it} - (1/(N\,T))\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}$ (i.e. the explanatory variable transformed in deviation from its overall mean) is used. It is convinced that $\tilde{x}_{it}$ contains both positive and negative values.

4 GMM estimators for LFM

Using the unconditional moment conditions derived in previous section, the GMM estimators for consistently estimating γ and β in the simple LFM (3.1.1) are constructed for the cases of predetermined explanatory variables, strictly exogenous explanatory variables, and mean-stationary dependent variables. In this section, these GMM estimators are presented.

4.1 GMM(qd) estimator

A conventional GMM estimator for the case of predetermined explanatory variables (i.e. for the model (3.1.1) with (3.1.4)) is the quasi-differenced GMM estimator proposed by Chamberlain (1992) and Wooldridge (1997). In this paper, this estimator is referred to as the GMM(qd) estimator. When the GMM(qd) estimator is constructed, the moment conditions (3.2.25) and (3.2.28) are used.

After defining the quasi-differenced transformations

$$p_{it} = (\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1}, \quad \text{for } t = 3, \ldots, T,$$

the GMM(qd) estimator for the parameter vector $\theta = [\gamma\ \beta]'$ is obtained by minimizing the following criterion function with respect to θ:

$$\left((1/N)\sum_{i=1}^{N} \epsilon_i^{qd\prime} Z_i^{qd}\right) W_N^{qd} \left((1/N)\sum_{i=1}^{N} Z_i^{qd\prime} \epsilon_i^{qd}\right), \tag{4.1.1}$$

where the column vector $\epsilon_i^{qd} = p_i$ with $p_i = [p_{i3}\ p_{i4}\ \cdots\ p_{iT}]'$ being a $(T-2)$ column

[8] If the variable $x_{it} + x_{i,t-1} - x_{i,t-2}$ contains both positive and negative values, the estimate of β using the moment conditions (3.2.34) may not be in danger of going to infinity, even if all values of x_{it} are positive. However, such a situation would be rare.

vector, the matrix

$$Z_i^{qd} = \begin{bmatrix} A_i & B_i \end{bmatrix}$$

with

$$A_i = \begin{bmatrix} y_{i1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & y_{i1} & y_{i2} & \cdots & y_{i,T-2} \end{bmatrix}$$

being a $(T-2)$ by $((T-2)(T-1)/2)$ matrix,

$$B_i = \begin{bmatrix} x_{i1} & x_{i2} & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & x_{i1} & x_{i2} & x_{i3} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & x_{i1} & x_{i2} & \cdots & x_{i,T-1} \end{bmatrix}$$

being a $(T-2)$ by $((T-1)T/2-1)$ matrix, and the weighting matrix

$$W_N^{qd} = \left((1/N) \sum_{i=1}^{N} Z_i^{qd\prime} \epsilon_i^{qd}(\tilde{\theta}_1) \epsilon_i^{qd}(\tilde{\theta}_1)' Z_i^{qd} \right)^{-1}$$

with $\epsilon_i^{qd}(\tilde{\theta}_1)$ being the vector ϵ_i^{qd} realized by incorporating an initial consistent estimate $\tilde{\theta}_1$ for θ. In this paper, the initial estimate $\tilde{\theta}_1$ for the GMM(qd) estimator is obtained by minimizing (4.1.1) with respect to θ after replacing W_N^{qd} with $\left((1/N)\sum_{i=1}^{N} Z_i^{qd\prime} Z_i^{qd}\right)^{-1}$, which is consistent for the case of predetermined explanatory variables.

For the case of predetermined explanatory variables, the GMM(qd) estimator for the parameter vector $\theta = [\gamma\ \beta]'$ is consistent. However, the Monte Carlo experiments carried out by BGW exhibit a poor small sample performance of the GMM(qd) estimator.

4.2 GMM(pr) estimator

Under the assumption of predetermined explanatory variables (i.e. for the model (3.1.1) with (3.1.4)), the consistent GMM(pr) estimator is constructed using the moment conditions (3.2.25), (3.2.28), and (3.2.34), as a GMM estimator alternative to the GMM(qd) estimator.

After defining

$$n_{it} = ((\mu_{i,t-2}/\mu_{i,t-1})u_{i,t-1} - u_{i,t-2})(1/\mu_{it})u_{it}, \quad \text{for } t = 4, \ldots, T,$$

the GMM(pr) estimator for the parameter vector $\theta = [\gamma\ \beta]'$ is obtained by minimizing the following criterion function with respect to θ:

$$\left((1/N)\sum_{i=1}^{N}\epsilon_i^{pr\prime} Z_i^{pr}\right) W_N^{pr} \left((1/N)\sum_{i=1}^{N} Z_i^{pr\prime}\epsilon_i^{pr}\right), \tag{4.2.1}$$

where the column vector $\epsilon_i^{pr} = [p_i{}'\ n_i{}']'$ with $n_i = [n_{i4}\ n_{i5}\ \cdots\ n_{iT}]'$ being a $(T-3)$ column vector, the matrix

$$Z_i^{pr} = \begin{bmatrix} A_i & B_i & O \\ O & O & I_{(T-3)} \end{bmatrix}$$

with $I_{(T-3)}$ being the $(T-3)$ by $(T-3)$ identity matrix and O being zero matrices, and the weighting matrix

$$W_N^{pr} = \left((1/N)\sum_{i=1}^{N} Z_i^{pr\prime}\epsilon_i^{pr}(\tilde{\theta}_1)\epsilon_i^{pr}(\tilde{\theta}_1)' Z_i^{pr}\right)^{-1}$$

with $\epsilon_i^{pr}(\tilde{\theta}_1)$ being the vector ϵ_i^{pr} realized by incorporating an initial consistent estimate $\tilde{\theta}_1$ for θ. In this paper, the initial estimate $\tilde{\theta}_1$ for the GMM(pr) estimator is obtained by minimizing (4.2.1) with respect to θ after replacing W_N^{pr} with $\left((1/N)\sum_{i=1}^{N} Z_i^{pr\prime} Z_i^{pr}\right)^{-1}$, which is consistent for the case of predetermined explanatory variables.

It is expected that the GMM(pr) estimator improves the poor small sample performance of the GMM(qd) estimator.

4.3 GMM(ex) estimator

For the case of strictly exogenous explanatory variables, in which the model (3.1.1) with (3.1.5) is postulated, the GMM(ex) estimator is constructed using the moment conditions (3.3.24), (3.3.27), and (3.3.32), with the intention of estimating parameters of interest consistently.

After defining the quasi-differenced transformations of a different form

$$e_{it} = u_{it} - (\mu_{it}/\mu_{i,t-1})u_{i,t-1}, \quad \text{for } t = 3, \ldots, T$$

and

$$m_{it} = (u_{i,t-1} - (\mu_{i,t-1}/\mu_{i,t-2})u_{i,t-2})u_{it}, \quad \text{for } t = 4, \ldots, T,$$

the GMM(ex) estimator for the parameter vector $\theta = [\gamma\ \beta]'$ is obtained by minimizing the following criterion function with respect to θ:

$$\left((1/N)\sum_{i=1}^{N}\epsilon_i^{ex\prime}Z_i^{ex}\right)W_N^{ex}\left((1/N)\sum_{i=1}^{N}Z_i^{ex\prime}\epsilon_i^{ex}\right), \tag{4.3.1}$$

where the column vector $\epsilon_i^{ex} = [e_i{}' \ m_i{}']'$ with $e_i = [e_{i3} \ e_{i4} \ \cdots \ e_{iT}]'$ being a $(T-2)$ column vector and $m_i = [m_{i4} \ m_{i5} \ \cdots \ m_{iT}]'$ being a $(T-3)$ column vector, the matrix

$$Z_i^{ex} = \begin{bmatrix} A_i & C_i & O \\ O & O & I_{(T-3)} \end{bmatrix}$$

with

$$C_i = \begin{bmatrix} x_{i1} & x_{i2} & \cdots & x_{iT} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & x_{i1} & x_{i2} & \cdots & x_{iT} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & x_{i1} & x_{i2} & \cdots & x_{iT} \end{bmatrix}$$

being a $(T-2)$ by $((T-2)T)$ matrix, and the weighting matrix

$$W_N^{ex} = \left((1/N)\sum_{i=1}^{N}Z_i^{ex\prime}\epsilon_i^{ex}(\tilde{\theta}_1)\epsilon_i^{ex}(\tilde{\theta}_1)'Z_i^{ex}\right)^{-1}$$

with $\epsilon_i^{ex}(\tilde{\theta}_1)$ being the vector ϵ_i^{ex} realized by incorporating an initial consistent estimate $\tilde{\theta}_1$ for θ. In this paper, the initial estimate $\tilde{\theta}_1$ for the GMM(ex) estimator is obtained by minimizing (4.3.1) with respect to θ after replacing W_N^{ex} with $\left((1/N)\sum_{i=1}^{N}Z_i^{ex\prime}Z_i^{ex}\right)^{-1}$, which is consistent for the case of strictly exogenous explanatory variables but not consistent for the case of predetermined explanatory variables.

The GMM(ex) estimator is not consistent under the assumptions of predetermined explanatory variables.

4.4 GMM(sa) estimator

The estimator constructed using the moment conditions (3.2.25), (3.2.28), (3.4.15), and (3.4.21) is called the GMM(sa) estimator in this paper.

Under the assumptions that x_{it} is predetermined, where the model (3.1.1) with (3.1.4) is postulated, and that y_{it} and x_{it} are stationary in the sense described in subsection 3.4, the GMM(sa) estimator for the parameters of interest is consistent.

After defining the quasi-level transformations

$$q_{it} = (1/\mu_{it})u_{it}, \quad \text{for } t = 2, \ldots, T,$$

the GMM(sa) estimator for the parameter vector $\theta = [\gamma \ \beta]'$ is obtained by minimizing the following criterion function with respect to θ:

$$\left((1/N) \sum_{i=1}^{N} \epsilon_i^{sa\prime} Z_i^{sa} \right) W_N^{sa} \left((1/N) \sum_{i=1}^{N} Z_i^{sa\prime} \epsilon_i^{sa} \right), \tag{4.4.1}$$

where the column vector $\epsilon_i^{sa} = [p_i' \ q_i']'$ with $q_i = [q_{i2} \ q_{i3} \ \cdots \ q_{iT}]'$ being a $(T-1)$ column vector, the matrix

$$Z_i^{sa} = \begin{bmatrix} A_i & B_i & O & O \\ O & O & D_i & E_i \end{bmatrix}$$

with

$$D_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \Delta y_{i2} & 0 & \cdots & 0 \\ 0 & \Delta y_{i3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta y_{i,T-1} \end{bmatrix}$$

being a $(T-1)$ by $(T-2)$ matrix and

$$E_i = \begin{bmatrix} \Delta x_{i2} & 0 & \cdots & 0 \\ 0 & \Delta x_{i3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta x_{iT} \end{bmatrix}$$

being a $(T-1)$ by $(T-1)$ diagonal matrix, and the weighting matrix

$$W_N^{sa} = \left((1/N) \sum_{i=1}^{N} Z_i^{sa\prime} \epsilon_i^{sa}(\tilde{\theta}_1) \epsilon_i^{sa}(\tilde{\theta}_1)' Z_i^{sa} \right)^{-1}$$

with $\epsilon_i^{sa}(\tilde{\theta}_1)$ being the vector ϵ_i^{sa} realized by incorporating an initial consistent estimate $\tilde{\theta}_1$ for θ. In this paper, the initial estimate $\tilde{\theta}_1$ for the GMM(sa) estimator is obtained by minimizing (4.4.1) with respect to θ after replacing W_N^{sa} with $\left((1/N) \sum_{i=1}^{N} Z_i^{sa\prime} Z_i^{sa} \right)^{-1}$, which is consistent under the assumptions that x_{it} is predetermined and that y_{it} and x_{it} are stationary.

Under the assumption that y_{it} and x_{it} are stationary, it is expected that the GMM(sa) estimator improves the poor small sample performance of the GMM(qd) estimator.

4.5 GMM(sb) estimator

Under the assumptions that x_{it} is predetermined, where the model (3.1.1) with (3.1.4) is postulated, and that y_{it} and x_{it} are stationary in the sense described in subsection 3.4, an alternative to the GMM(sa) estimator is constructed using the moment conditions (3.2.25), (3.2.28), (3.4.15), (3.4.21) for $t = 2$, and (3.4.24). The alternative is consistent and called the GMM(sb) estimator in this paper. The difference between both estimators is that the GMM(sb) estimator uses the moment conditions (3.4.24) in place of (3.4.21) for $t = 3, \ldots, T$ used in GMM(sa) estimator.

After defining

$$d_{it} = x_{it}u_{it} - x_{i,t-1}u_{i,t-1}, \quad \text{for } t = 3, \ldots, T,$$

the GMM(sb) estimator for the parameter vector $\theta = [\gamma\ \beta]'$ is obtained by minimizing the following criterion function with respect to θ:

$$\left((1/N)\sum_{i=1}^{N} \epsilon_i^{sb\prime} Z_i^{sb}\right) W_N^{sb} \left((1/N)\sum_{i=1}^{N} Z_i^{sb\prime} \epsilon_i^{sb}\right), \tag{4.5.1}$$

where the column vector $\epsilon_i^{sb} = [p_i{}'\ q_i{}'\ d_i{}']'$ with $d_i = [d_{i3}\ d_{i4}\ \cdots\ d_{iT}]'$ being a $(T-2)$ column vector, the matrix

$$Z_i^{sb} = \begin{bmatrix} A_i & B_i & O & O \\ O & O & F_i & O \\ O & O & O & I_{(T-2)} \end{bmatrix}$$

with

$$F_i = \begin{bmatrix} \Delta x_{i2} & 0 & 0 & \cdots & 0 \\ 0 & \Delta y_{i2} & 0 & \cdots & 0 \\ 0 & 0 & \Delta y_{i3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \Delta y_{i,T-1} \end{bmatrix}$$

being a $(T-1)$ by $(T-1)$ diagonal matrix and $I_{(T-2)}$ being the $(T-2)$ by $(T-2)$

identity matrix, and the weighting matrix

$$W_N^{sb} = \left((1/N) \sum_{i=1}^{N} Z_i^{sb\prime} \epsilon_i^{sb}(\tilde{\theta}_1) \epsilon_i^{sb}(\tilde{\theta}_1)' Z_i^{sb} \right)^{-1}$$

with $\epsilon_i^{sb}(\tilde{\theta}_1)$ being the vector ϵ_i^{sb} realized by incorporating an initial consistent estimate $\tilde{\theta}_1$ for θ. In this paper, the initial estimate $\tilde{\theta}_1$ for the GMM(sb) estimator is obtained by minimizing (4.5.1) with respect to θ after replacing W_N^{sb} with $\left((1/N) \sum_{i=1}^{N} Z_i^{sb\prime} Z_i^{sb}\right)^{-1}$, which is consistent under the assumptions that x_{it} is predetermined and that y_{it} and x_{it} are stationary.

Under the assumption that y_{it} and x_{it} are stationary, it is also expected that the GMM(sa) estimator improves the poor small sample performance of the GMM(qd) estimator.

5 Monte Carlo experiments

In this section, the small sample performances of the GMM estimators exhibited in previous section are investigated with some Monte Carlo experiments. For the sake of the comparison with the GMM estimators, some estimators other than the GMM estimators are also investigated for the LFM. The Monte Carlo experiments are implemented by using an econometric software TSP version 4.5.

5.1 Data generating process

The data used in the Monte Carlo studies is generated from the following data generating process (DGP):

$$y_{it} \sim Poisson(\gamma y_{i,t-1} + \exp(\beta x_{it} + \varpi\eta_i + \varrho\xi_i + \vartheta\varepsilon_{it} - (1/2)\vartheta^2\sigma_\varepsilon^2)), \tag{5.1.1}$$

$$y_{i,-TG+1} \sim Poisson(\pi \exp(\beta x_{i,-TG+1} + \varpi\eta_i + \varrho\xi_i + \vartheta\varepsilon_{i,-TG+1} - (1/2)\vartheta^2\sigma_\varepsilon^2)), \tag{5.1.2}$$

$$x_{it} = \rho x_{i,t-1} + \kappa\eta_i + \iota\zeta_i + \tau w_{it} + \delta\varepsilon_{i,t-1}, \tag{5.1.3}$$

$$x_{i,-TG+1} = (1/(1-\rho))(\kappa\eta_i + \iota\zeta_i) + (1/\sqrt{1-\rho^2})(\tau w_{i,-TG+1} + \delta\nu_i), \tag{5.1.4}$$

$$\eta_i \sim N(0,\sigma_\eta^2); \quad \xi_i \sim N(0,\sigma_\xi^2); \quad \varepsilon_{it} \sim N(0,\sigma_\varepsilon^2);$$

$$\zeta_i \sim N(0,\sigma_\zeta^2); \quad w_{it} \sim N(0,\sigma_w^2); \quad \nu_i \sim N(0,\sigma_\varepsilon^2),$$

where $t = -TG + 1, \ldots, 1, \ldots, T$ with TG being periods of the pre-sample to be generated. In the DGP, the values are set to the parameters γ, β, π, ρ, κ, σ_η^2, ι, σ_ζ^2, τ, σ_w^2, δ, σ_ε^2, ϖ, ϱ, σ_ξ^2, and ϑ. The experiments are carried out with the pre-sample periods $TG = 50$, the cross-sectional sizes $N = 100$, 500, and 1000, the periods used for the estimations $T = 4$ and 8, and the number of replications $NR = 1000$.

5.2 Estimators for comparison

For the model (3.1.1) in subsection 3.1, based on which the DGP composed of (5.1.1), (5.1.2), (5.1.3), and (5.1.4) is designed, three estimators other than the GMM estimators are sketched in this subsection and their properties are described under both sets of the assumptions of (3.1.4) and (3.1.5). These estimators are presented in BGW.

The first is the Level estimator, which is the solution of the following system:

$$\sum_{i=1}^{N}\sum_{t=2}^{T}(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0 + \beta x_{it})) = 0, \tag{5.2.1}$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T} y_{i,t-1}(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0 + \beta x_{it})) = 0, \tag{5.2.2}$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T} x_{it}(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0 + \beta x_{it})) = 0. \tag{5.2.3}$$

The Level estimator for the set of parameters γ and β (and further β_0) is inconsistent for both cases of predetermined explanatory variables and strictly exogenous explanatory variables, because it ignores the presence of the fixed effect. As stated in BGW, it can be expected that the upward bias appears, endemic to the Level estimator.

The second is the within group (WG) estimator, which is the solution of the following system:

$$\sum_{i=1}^{N}\sum_{t=2}^{T} y_{i,t-1}(y_{it} - \gamma y_{i,t-1} - ((\bar{y}_i - \gamma \bar{y}_{i,-1})/\bar{\mu}_i)\mu_{it}) = 0, \tag{5.2.4}$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T} x_{it}(y_{it} - \gamma y_{i,t-1} - ((\bar{y}_i - \gamma \bar{y}_{i,-1})/\bar{\mu}_i)\mu_{it}) = 0, \tag{5.2.5}$$

where $\bar{y}_i = (1/(T-1))\sum_{t=2}^{T} y_{it}$, $\bar{y}_{i,-1} = (1/(T-1))\sum_{t=2}^{T} y_{i,t-1}$, and $\bar{\mu}_i = (1/(T-1))\sum_{t=2}^{T} \mu_{it}$. For both cases of predetermined explanatory variables and strictly exogenous explanatory variables, the WG estimator for the set of parameters γ and

β is inconsistent when T is fixed and $N \to \infty$.[9] The WG estimator is proposed by BGW, and with the setup of $\gamma = 0$ and for the case of strictly exogenous explanatory variables, the WG estimator for the parameter β is consistent when T is fixed and $N \to \infty$ and equivalent to Poisson CMLE proposed by Hausman et al. (1984) and the ordinary Poisson maximum likelihood estimator. As stated in BGW, it can be expected that the downward bias appears, endemic to the WG estimator.

The third is the pre-sample mean (PSM) estimator, which is the solution of the following system:

$$\sum_{i=1}^{N}\sum_{t=2}^{T}(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0^* + \beta x_{it} + \phi \ln(\bar{y}_{ip})) = 0, \qquad (5.2.6)$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T} y_{i,t-1}(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0^* + \beta x_{it} + \phi \ln(\bar{y}_{ip})) = 0, \qquad (5.2.7)$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T} x_{it}(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0^* + \beta x_{it} + \phi \ln(\bar{y}_{ip})) = 0, \qquad (5.2.8)$$

$$\sum_{i=1}^{N}\sum_{t=2}^{T}(\ln(\bar{y}_{ip}))(y_{it} - \gamma y_{i,t-1} - \exp(\beta_0^* + \beta x_{it} + \phi \ln(\bar{y}_{ip})) = 0, \qquad (5.2.9)$$

where $\bar{y}_{ip} = (1/TP)\sum_{r=0}^{TP-1} y_{i,0-r}$ with TP being the number of the pre-sample periods to be used for the estimation. For both cases of predetermined explanatory variables and strictly exogenous explanatory variables, the consistency of the PSM estimator for the set of parameters γ and β (and further β_0^* and ϕ) when T is fixed and $N \to \infty$ relies on the assumptions that $TP \to \infty$, that the fixed effect in the explanatory variable is proportional to the fixed effect in the regression equation, and that the (finite) moment generating function of the disturbance composing the explanatory variable is equal intertemporally and cross-sectionally.[10] The PSM estimator has some attractive small sample properties under the assumptions above, but its defect is that it is felt that the situation where the assumptions above are wholly satisfied does not necessarily come into being in the empirical analysis. In particular, collecting the data of the dependent variable with a certain length of the pre-sample history for each individual is considered to be cumbersome. The PSM estimator is proposed by Blundell et al. (1999) and BGW, based on the idea that the pre-sample mean is able to be used as a proxy for the fixed effect, depending on circumstances.[11]

[9] However, when $T \to \infty$ and $N \to \infty$, the WG estimator is consistent for both cases.

[10] It is premised that the explanatory variable is stationary and composed of the fixed effect, the constant term, and the disturbance with its mean being zero.

[11] This idea originates from Blundell et al. (1995).

5.3 Results for strictly exogenous explanatory variables

Some Monte Carlo experiments are carried out for the case of strictly exogenous explanatory variables. The values of parameters for this case are set in the DGP composed of (5.1.1), (5.1.2), (5.1.3), and (5.1.4). The parameter settings specified in this subsection are the same as those used in Tables 1, 2, and 3 in BGW.

Results for this case are shown in Tables 1, 2, 3, and 4 in terms of bias and root mean squared error (rmse). It is seen from these tables that in almost all cases the GMM estimators cutting down the instruments for the quasi-differenced transformations perform better than the GMM estimators using the full set of instruments for the quasi-differenced transformations, when using the identical estimators. The symbols $\gamma(99)$ and $\beta(99)$ represent the usage of the full set of instruments for estimating γ and β in the GMM estimators, while $\gamma(1)$ and $\beta(1)$ represent the usage of the curtailed set. That is, the figure 99 in the parentheses next to γ and β implies that all the instruments required by the GMM estimators presented in section 4 are used in the estimations, while the figure 1 implies that the past dependent variables (y_{it}) dated $t-3$ and before are not used as the instruments for the quasi-differenced equations dated t and further for the cases of the GMM(qd) and GMM(pr) estimators the past explanatory variables (x_{it}) dated $t-3$ and before are not used as the instruments for the quasi-differenced equations dated t. In addition, since the Level and WG estimators are inconsistent in this situation, bias endemic to these estimators is found, which does not diminish with N increasing, and rmse for the WG estimator does not diminish as well, while the PSM estimators using the long history of the pre-sample perform well, reflecting the property of the consistent estimator. As for the PSM estimator, the figures in the parentheses next to γ and β imply the values of TP.

In Tables 1 and 2, results for moderately persistent y_{it} and x_{it} (which are characterized by $\gamma = 0.5$, $\beta = 0.5$, and $\rho = 0.5$) are shown for the combinations of $N = 100$, 500, and 1000, and $T = 4$ and 8. In this situation, the GMM(qd) estimator is considerably downward biased and sizes of bias and rmse for the GMM(pr) and GMM(ex) estimators are smaller than those for the GMM(qd) estimator for all the combinations of N and T, when the same types of instruments on y_{it} are used for the quasi-differenced transformations. Especially, sizes of bias and rmse for the GMM(ex) estimators are much smaller than those for the GMM(qd) estimator. For example, when $T = 4$ and $N = 500$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are $(-0.031, 0.088)$ and $(-0.053, 0.151)$ respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(ex) estimator are $(-0.025, 0.087)$ and $(-0.027, 0.128)$ respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are $(-0.108, 0.166)$ and $(-0.126, 0.224)$ respectively. Further, when $T = 8$ and $N = 1000$, the performance of the GMM(ex) estimator is slightly superior to that of the PSM estimator with $TP = 50$. That is, when $T = 8$ and $N = 1000$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(ex) estimator are $(-0.017, 0.035)$ and $(-0.025, 0.044)$ respectively, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.040, 0.044)$ and $(0.038, 0.045)$ respectively.

This trend is also true of both situations of the considerably persistent y_{it} and x_{it} and the considerably persistent y_{it} and extremely persistent x_{it}. The former situation is characterized by the settings of parameters $\gamma = 0.7$, $\beta = 1$, and $\rho = 0.9$, whose results are exhibited in Table 3, while the latter situation is characterized by the settings of parameters $\gamma = 0.7$, $\beta = 1$, and $\rho = 0.95$, whose results are exhibited in Table 4. It can be said that in both situations, albeit the GMM(qd) estimator is considerably downward biased, sizes of bias and rmse for the GMM(pr) and GMM(ex) estimators are much smaller than those for the GMM(qd) estimator for $T = 8$ and $N = 100$, 500, and 1000. Especially, sizes of bias and rmse for the GMM(ex) estimator are markedly small for larger sizes of N (i.e. $N = 500$ and 1000), and in the situation of considerably persistent y_{it} and extremely persistent x_{it}, sizes of bias for the GMM(ex) estimator are much smaller than those for the PSM estimator with $TP = 50$ for larger sizes of N (i.e. $N = 500$ and 1000).[12] For example, in the situation of considerably persistent y_{it} and extremely persistent x_{it}, when $T = 8$ and $N = 500$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are $(0.002, 0.082)$ and $(-0.176, 0.546)$ respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(ex) estimator are $(0.006, 0.085)$ and $(-0.041, 0.340)$ respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are $(-0.138, 0.189)$ and $(-0.588, 1.148)$ respectively and further those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.070, 0.074)$ and $(-0.165, 0.190)$ respectively.

It is shown from the Monte Carlo experiments above that for the case of strictly exogenous explanatory variables, the GMM(pr) and GMM(ex) estimators perform fairly better than the GMM(qd) estimator in terms of bias and rmse.

Next, the small sample performances of the GMM estimators proposed in this paper (i.e. the GMM(pr) and GMM(ex) estimators) are compared with that of the GMM(qd) estimator from the viewpoint of efficiency gain and inference. For $N =$ 100, 500, and 1000, results of Monte Carlo standard deviation (mcsd) and Monte Carlo mean of standard error (mcmse) are shown in Table 5 for the situation of

[12] The Monte Carlo experiments by BGW point out that the PSM estimator with the large number of the pre-sample history used is able to be biased in the situation of considerably persistent y_{it} and extremely persistent x_{it}. BGW mentions that this will be due to the multicollinearity between x_{it} and $\ln \bar{y}_{ip}$ originating when x_{it} is extremely persistent.

moderately persistent y_{it} and x_{it} when $T = 4$, Table 6 for the situation of moderately persistent y_{it} and x_{it} when $T = 8$, Table 7 for the situation of considerably persistent y_{it} and x_{it} when $T = 8$, and Table 8 for the situation of considerably persistent y_{it} and extremely persistent x_{it} when $T = 8$.

For the situation of moderately persistent y_{it} and x_{it} when $T = 4$, it can be said that values of mcsd of γ and β for the GMM(pr) and GMM(ex) estimators are, at some level, smaller than those for the GMM(qd) estimator for $N = 100$, 500, and 1000, while when $T = 8$, it can be said with difficulty that values of mcsd of γ and β for the GMM(pr) and GMM(ex) estimators are smaller than those for the GMM(qd) estimator only for $N = 1000$. For example, when $T = 4$ and $N = 1000$, values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are 0.065 and 0.125 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(ex) estimator are 0.060 and 0.093 respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are 0.096 and 0.157 respectively.

For the situation of considerably persistent y_{it} and x_{it} when $T = 8$, it cannot be said at all that for $N = 100$ values of mcsd of γ and β for the GMM(pr) and GMM(ex) estimators are smaller than those for the GMM(qd) estimators, while it can be said that for $N = 500$ and 1000 values of mcsd of β for the GMM(pr) and GMM(ex) estimators are smaller than those for the GMM(qd) estimator when using the curtailed set of instruments and values of mcsd of γ are about the same among the three GMM estimators. For example, when $N = 1000$, values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are 0.045 and 0.230 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(ex) estimator are 0.046 and 0.135 respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are 0.058 and 0.301 respectively.

For the situation of considerably persistent y_{it} and extremely persistent x_{it}, when $T = 8$, it can be said without doubt that for $N = 500$ and 1000 values of mcsd of β for the GMM(pr) and GMM(ex) estimators are clearly smaller than those for the GMM(qd) estimator and values of mcsd of γ are about the same among the three GMM estimators. For example, when $N = 1000$, values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are 0.055 and 0.443 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(ex) estimator are 0.056 and 0.224 respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are 0.082 and 0.738 respectively.

For the case of strictly exogenous explanatory variables, it seems that it is not until the cross-sectional size is large that the efficiency gain is recognized when the GMM(pr) and GMM(ex) estimators are used as opposed to the GMM(qd) estimator, judging from the results of Monte Carlo experiments as mentioned above.

As seen from the comparison of mcsd and mcmse in Tables 5, 6, 7, and 8, it is conceivable that the estimated standard errors for the GMM(pr) and GMM(ex) estimators as well as those for the GMM(qd) estimator are fairly biased downward in almost all cases when $N = 100$. This implies that the inferences using the the GMM(pr) and GMM(ex) estimators as well as the GMM(qd) estimator are problematic when the cross-sectional size is small.[13]

5.4 Results for predetermined explanatory variables

In the experiments carried out in this subsection, the settings of parameters in the DGP are arranged in order that the explanatory variable x_{it} is able to be predetermined, as characterized by $\delta = 1$ and $\vartheta = 1$. In addition, the settings of parameters are arranged in order that the dependent variable y_{it} is mean-stationary starting from the mean-stationary initial value, as characterized by $\pi = 1/(1-\gamma)$.

Results for this case are shown in Tables 9, 10, 11, and 12 in terms of bias and rmse. The results are about the same as those for the case of strictly exogenous explanatory variables in previous subsection, except for those for the GMM(ex) estimator.

The Level and WG estimators are upward and downward biased respectively, and values of bias for the Level and WG estimators (which are also inconsistent in this case) level out with N increasing from 100, 500, to 1000 and values of rmse for the WG estimators level out as well. Since the GMM(ex) estimator is inconsistent for the case of predetermined x_{it}, bulky sizes of (downward) bias and rmse for the GMM(ex) estimator with respect to β do not decrease sharply with N increasing. It can be said that the consistent GMM estimators cutting down the instruments for the quasi-differenced transformations roughly perform better than the consistent GMM estimators using the full set of instruments for the quasi-differenced transformations in terms of bias and rmse. The PSM estimator using the long history of the pre-sample performs well, reflecting the property of the consistent estimator.

For the situation of moderately persistent y_{it} and x_{it} where the typical settings of parameters are $\gamma = 0.5$, $\beta = 0.5$, and $\rho = 0.5$, results on bias and rmse are shown in Table 9 for $T = 4$ and Table 10 for $T = 8$. The GMM(qd) estimator is considerably downward biased for the smaller cross-sectional sizes (i.e. $N = 100$ and 500). Sizes of bias and rmse for the GMM(qd), GMM(pr), GMM(sa), and GMM(sb) estimators decrease with N increasing, reflecting the property of the consistent estimators, while it can be reckoned that those for the inconsistent GMM(ex) estimator do not

[13] One possibility of solving this problem is to use the finite sample corrected variance proposed by Windmeijer (2005, 2006) as the estimated variance for the purpose of conducting the inferences.

decrease so much with N increasing. It can be seen that sizes of bias and rmse for the GMM(pr), GMM(sa), and GMM(sb) estimators are considerably smaller than those for the GMM(qd) estimators for each N and each T, when the same types of instruments composed of lagged y_{it} and x_{it} are used for the quasi-differenced transformations and moreover it can be said that the small sample performance of the GMM(sa) estimators is best among the three estimators. For example, when $T = 4$ and $N = 100$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are $(-0.318, 0.474)$ and $(-0.242, 0.374)$ respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are $(-0.079, 0.278)$ and $(-0.140, 0.318)$ respectively, those of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are $(-0.010, 0.182)$ and $(-0.072, 0.243)$ respectively, and those of $\gamma(1)$ and $\beta(1)$ for the GMM(sb) estimator are $(-0.007, 0.192)$ and $(-0.110, 0.291)$ respectively. In addition, it is no exaggeration to say that when $N = 500$ and 1000, the performances of the GMM(pr), GMM(sa), and GMM(sb) estimators bear comparison to or are superior to that for the PSM estimator with $TP = 50$ whose small sample property is very preferable. For example, when $T = 8$ and $N = 500$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are $(-0.001, 0.082)$ and $(-0.042, 0.098)$ respectively, those of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are $(-0.012, 0.056)$ and $(-0.033, 0.082)$ respectively, and those of $\gamma(1)$ and $\beta(1)$ for the GMM(sb) estimator are $(-0.008, 0.057)$ and $(-0.041, 0.090)$ respectively, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.042, 0.056)$ and $(0.018, 0.038)$ respectively.

For the situation of the considerably persistent y_{it} and x_{it} where the typical settings of parameters are $\gamma = 0.7$, $\beta = 1$, and $\rho = 0.9$, results on bias and rmse are shown in Table 11 for $T = 8$, and for the situation of the considerably persistent y_{it} and extremely persistent x_{it} where the typical settings of parameters are $\gamma = 0.7$, $\beta = 1$, and $\rho = 0.95$, results on bias and rmse are shown in Table 12 for $T = 8$. In both situations, the same tendency as in the situation of the moderately persistent y_{it} and x_{it} is found as a whole. In both situations, the GMM(qd) estimator is considerably downward biased and sizes of rmse for the GMM(qd) estimator are also large even when $N = 1000$. However, sizes of bias and rmse for the GMM(pr), GMM(sa), and GMM(sb) estimators are much smaller than those for the GMM(qd) estimator. Especially, sizes of bias and rmse for the GMM(sa) estimators are, to a considerable degree, small for the larger sizes of N (i.e. $N = 500$ and 1000), and in the situation of considerably persistent y_{it} and extremely x_{it}, sizes of bias of $\gamma(1)$ and $\beta(1)$ of the GMM(sa) estimators are considerably smaller than those for the

PSM estimator with $TP = 50$ for the larger sizes of N (i.e. $N = 500$ and 1000).[14] For example, in the situation of considerably persistent y_{it} and extremely persistent x_{it} when $T = 8$ and $N = 500$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are $(-0.001, 0.088)$ and $(-0.159, 0.550)$ respectively, those of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are $(0.008, 0.049)$ and $(-0.143, 0.367)$ respectively, and those of $\gamma(1)$ and $\beta(1)$ for the GMM(sb) estimator are $(0.008, 0.053)$ and $(-0.230, 0.479)$ respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are $(-0.167, 0.229)$ and $(-0.637, 1.167)$ respectively, and further those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.075, 0.080)$ and $(-0.211, 0.230)$ respectively.

It is shown from the Monte Carlo experiments above that for the case of predetermined explanatory variables where further the stationary dependent and explanatory variables are assumed, the GMM(pr), GMM(sa), and GMM(sb) estimators perform fairly better than the GMM(qd) estimator in terms of bias and rmse.

Next, the consistent GMM estimators proposed in this paper (i.e. the GMM(pr), GMM(sa), and GMM(sb) estimators) are compared with the GMM(qd) estimator from the viewpoint of efficiency gain and inference. Results of mcsd and mcmse are shown in Table 13 for the case of moderately persistent y_{it} and x_{it} when $T = 4$, Table 14 for the case of moderately persistent y_{it} and x_{it} when $T = 8$, Table 15 for the case of considerably persistent y_{it} and x_{it} when $T = 8$, and Table 16 for the case of considerably persistent y_{it} and extremely persistent x_{it} when $T = 8$. The comparison is conducted, confined to the cases where the same types of instruments are used for the quasi-differenced equations.

For the situation of moderately persistent y_{it} and x_{it} when $T = 4$, values of mcsd of γ and β for the GMM(sa) estimator are, to some degree, smaller than those for the GMM(qd) estimator for $N = 100$, 500, and 1000 and those for the GMM(sb) estimator are, to some degree, smaller than those for the GMM(qd) estimator for $N = 500$ and 1000, while it would be safe to say that only values of mcsd of γ for the GMM(pr) estimator are, to some degree, smaller than those for the GMM(qd) estimator for $N = 100$, 500, and 1000. For example, when $T = 4$ and $N = 500$, values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are 0.100 and 0.135 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(sb) estimator are 0.103 and 0.144 respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are 0.167 and 0.190 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are 0.130 and 0.188 respectively.

[14]As pointed out in previous subsection, the multicollinearity between x_{it} and $\ln \bar{y}_{ip}$ is a potential source of the fact that the PSM estimator with the large number of the pre-sample history used is biased in the situation of considerably persistent y_{it} and extremely persistent x_{it}.

On the contrary, for the situation of moderately persistent y_{it} and x_{it}, when $T = 8$, it can be said that values of mcsd of γ for the GMM(sa) and GMM(sb) estimators are, to some degree, smaller than those for the GMM(qd) estimator for $N = 100$, 500, and 1000, while values of the mcsd of β for the GMM(sa) and GMM(sb) estimators are little less than those for the GMM(qd) estimator for $N = 500$ and are smaller than those for the GMM(qd) estimator only for $N = 1000$ where values of mcsd for the three estimators are considerably small. Values of mcsd of γ and β for the GMM(pr) estimator are little less than those for the GMM(qd) estimator only for 1000. For example, when $T = 8$ and $N = 500$, values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are 0.054 and 0.076 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(sb) estimator are 0.056 and 0.080 respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are 0.083 and 0.081 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are 0.082 and 0.088 respectively.

For both cases of considerably persistent y_{it} and x_{it} and considerably persistent y_{it} and extremely persistent x_{it} when $T = 8$, it can be said that values of mcsd of γ and β for the GMM(sa) and GMM(sb) estimators are much smaller than those of γ and β for the GMM(qd) estimator for $N = 100$, 500 and 1000, except for those of $\beta(99)$ for the GMM(sa) and GMM(sb) estimators for the case of considerably persistent y_{it} and x_{it} when $N = 100$. On the contrary, it can be said that values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are comparatively mildly smaller than those for the GMM(qd) estimator for $N = 100$, 500, and 1000. For example, for the case of considerably persistent y_{it} and extremely persistent x_{it} when $T = 8$ and $N = 500$, values of mcsd of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are 0.049 and 0.338 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(sb) estimator are 0.052 and 0.420 respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are 0.157 and 0.977 respectively and those of $\gamma(1)$ and $\beta(1)$ for the GMM(pr) estimator are 0.088 and 0.526 respectively.

For the situation of predetermined explanatory variables where further the stationary dependent and explanatory variables are assumed, it seems that it is not until N is large that the efficiency gain is recognized when the GMM(pr) estimator is used as opposed to the GMM(qd) estimator as well as for the situation of strictly exogenous explanatory variables, while it can be said that the significant efficiency gain by using the GMM(sa) and GMM(sb) estimators as opposed to the GMM(qd) estimator is recognized for the smaller N as well as for the larger N.

For the case of predetermined explanatory variables where further the stationary dependent and explanatory variables are assumed, the comparison of mcsd and mcmse in Tables 13, 14, 15, and 16 says that the estimated standard errors for the

GMM(pr), GMM(sa), and GMM(sb) estimators as well as those for the GMM(qd) estimators are fairly biased downwards when the cross-sectional size is small, and therefore the inferences using these estimators should be implemented with caution when the cross-sectional size is small.[15]

5.5 Results for different types of fixed effect

Using three different types of the fixed effect in the LFM, the experiments are carried out in this subsection. The first type corresponds to the case where the fixed effect composing the explanatory variable is proportional to the fixed effect in the count regression, the second type corresponds to the case where the fixed effect composing the explanatory variable is correlated with the fixed effect in the count regression but not proportional to the fixed effect in the count regression, and the third type corresponds to the case where the fixed effect composing the explanatory variable is uncorrelated with the fixed effect in the count regression. The first type is characterized by $\varpi = 1$ and $\varrho = 0$, the second type characterized by $\varpi = 0.6$ and $\varrho = 0.8$, and the third type characterized by $\varpi = 0$ and $\varrho = 1$. From now on, the cases for the first, second, and third types are called the case of the proportional fixed effect, the case of the correlated fixed effect, and the case of the uncorrelated fixed effect, respectively. Results of the experiments for these three cases are shown in Tables 17, 18, and 19 in terms of bias and rmse. The sizes of the variances of the fixed effect composing the explanatory variable are equal among the three cases, and the sizes of the variances of the fixed effect in the count regression are equal among the three cases as well. For the three cases, the values of parameters are set in such a way that the explanatory variable x_{it} is predetermined and stationary and the dependent variable y_{it} is mean-stationary.

For the case of the proportional fixed effect, it is verified from Table 17 that the GMM(pr), GMM(sa), and GMM(sb) estimators behave spectacularly better than the GMM(qd) estimator. For example, when $N = 100$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are $(-0.109, 0.261)$ and $(0.065, 0.209)$ respectively, while those of $\gamma(1)$ and $\beta(1)$ for the GMM(qd) estimator are $(-0.460, 0.576)$ and $(-0.546, 0.776)$ respectively. Further, the PSM estimator with the pre-sample history used being large (i.e. $TP = 50$) behaves better than the GMM(pr), GMM(sa), and GMM(sb) estimators when $N = 100$, and sizes of bias of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) and GMM(sb) estimators are not superior

[15]It is conceivable that the finite sample variance correction proposed by Windmeijer (2005, 2006) make it possible to conduct the acceptable inferences, because it may improve the downward bias.

to or not on a par with those of the PSM estimator with $TP = 50$ until the cross-sectional size of $N = 500$ or 1000 is reached, reflecting the consistency of the PSM estimator with TP being large for the case of the proportional fixed effect. That is, when $N = 100$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ are $(0.094, 0.221)$ and $(0.294, 0.397)$ for the GMM(pr) estimator, $(-0.109, 0.261)$ and $(0.065, 0.209)$ for the GMM(sa) estimator, and $(-0.161, 0.280)$ and $(0.081, 0.238)$ for the GMM(sb) estimator, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.004, 0.177)$ and $(-0.003, 0.084)$, and when $N = 1000$, values of bias of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are -0.027 and -0.001 respectively, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are 0.068 and -0.005 respectively.

However, for both cases of the correlated fixed effect and the uncorrelated fixed effect, the PSM estimator is inconsistent under the assumptions of $N \to \infty$ and $TP \to \infty$. Although the GMM(pr), GMM(sa), and GMM(sb) estimators behave spectacularly better than the GMM(qd) estimators for the smaller size of cross-section (i.e. $N = 100$) as well as for the larger sizes of cross-section (i.e. $N = 500$ and $N = 1000$), the performance of the inconsistent PSM estimator is not preferable even with N and TP large. In other words, the accuracy of the PSM estimator for both cases of the correlated fixed effect and the uncorrelated fixed effect remains in low degree when TP is large (i.e. $TP = 50$) and N is larger (i.e. $N = 500$ or $N = 1000$), while the accuracies of the GMM(sa) and GMM(sb) estimators largely surpass that of PSM estimator with $TP = 50$ for the larger sizes of cross-section (i.e. $N = 500$ and $N = 1000$), reflecting the property of the consistent estimator, and further it can be said that for the case of the uncorrelated fixed effect, the accuracies of the GMM(sa) and GMM(sb) estimators are better than that of PSM estimator with $TP = 50$ even for the smaller sizes of cross-section (i.e. $N = 100$), judging from the comprehensive standpoint. In addition, it can be said that the GMM(pr) estimator performs better than the PSM estimator with $TP = 50$ for the case of the uncorrelated fixed effect even for the smaller sizes of cross-section (i.e. $N = 100$), judging from the comprehensive standpoint. It is confirmed in the Monte Carlo experiments that even when the usage of the PSM estimator is not valid, the GMM(pr), GMM(sa), and GMM(sb) estimators are able to perform considerably well. These results are different from those for the case of the proportional fixed effect.

For the case of the correlated fixed effect, to illustrate, when $N = 100$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ are $(0.075, 0.221)$ and $(0.177, 0.338)$ for the GMM(pr) estimator, $(-0.056, 0.200)$ and $(0.008, 0.189)$ for the GMM(sa) estimator, and $(-0.115, 0.226)$ and $(0.033, 0.233)$ for the GMM(sb) estimator, while those of

$\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.082, 0.134)$ and $(-0.123, 0.152)$, and when $N = 1000$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are $(-0.008, 0.086)$ and $(-0.011, 0.109)$, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.135, 0.147)$ and $(-0.127, 0.130)$.

For the case of the uncorrelated fixed effect, to illustrate, when $N = 100$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ are $(0.050, 0.177)$ and $(-0.020, 0.296)$ for the GMM(pr) estimator, $(-0.003, 0.126)$ and $(-0.098, 0.215)$ for the GMM(sa) estimator, and $(-0.049, 0.152)$ and $(-0.073, 0.251)$ for the GMM(sb) estimator, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.114, 0.135)$ and $(-0.218, 0.235)$, and when $N = 1000$, the sets of bias and rmse of $\gamma(1)$ and $\beta(1)$ for the GMM(sa) estimator are $(0.004, 0.044)$ and $(-0.044, 0.102)$, while those of $\gamma(50)$ and $\beta(50)$ for the PSM estimator are $(0.145, 0.150)$ and $(-0.221, 0.222)$.

In addition, casting a spotlight on sizes of bias and rmse of $\gamma(25)$ and $\beta(25)$ and $\gamma(50)$ and $\beta(50)$ in Tables 17, 18, and 19, it can be recognized that the accuracy of the PSM estimator with TP being large decreases with the decrease of magnitude of the correlation between the fixed effect composing the explanatory variable and the fixed effect in the count regression.

Due to the above observation, it is felt that taking into consideration the magnitude of the correlation between the fixed effect composing the explanatory variable and the fixed effect in the count regression is important in the empirical applications using the PSM estimator. It is recommended that even if the long pre-sample history for the dependent variable is available, the GMM(pr), GMM(sa), and GMM(sb) estimators are used to the empirical applications where the number of time periods is very small and the number of cross-sectional sizes is moderately large and then the results using these estimators are compared with those using the other estimators (i.e. the Level, WG, and PSM estimators).

6 Conclusion

In this paper, some moment conditions for consistently estimating the LFM for count panel data were derived on the basis of the structure of variance and covariance of the disturbance in the LFM. The moment conditions were derived by using the implicit operation under the four assumptions: the assumption of predetermined explanatory variables, the assumption of strictly exogenous explanatory variables, the assumption of mean-stationary dependent variables, and the assumption of equidispersion. Under the assumption of predetermined explanatory variables, the set of the moment conditions based on the quasi-differenced transformation proposed by Chamberlain

(1992) and Wooldridge (1997) and a variant of the set of the moment conditions proposed by Windmeijer (2000) were obtained, while under the assumption of strictly exogenous explanatory variables, a different set of the moment conditions based on the quasi-differenced transformation and a variant of the set of the moment conditions proposed by Crépon and Duguet (1997) were obtained, both of which are not valid under the assumption of predetermined explanatory variables. Further, imposing the assumption of mean-stationary dependent variables has given birth to the stationarity moment conditions compatible with count panel data. In addition, imposing the assumption of equidispersion has generated further moment conditions remaining unspotted until now.

Using these sets of the moment conditions, the following five GMM estimators were constructed: the GMM(qd) estimator using the set based on the quasi-differenced transformation only, the GMM(pr) estimator using the set based on the quasi-differenced transformation and the variant valid under the assumption of predetermined explanatory variables, the GMM(ex) estimator using the set based on the quasi-differenced transformation and the variant valid under the assumption of strictly exogenous explanatory variables, and the GMM(sa) and GMM(sb) estimators incorporating the stationarity moment conditions compatible with count panel data.

Monte Carlo experiments were carried out in order to investigate the small sample performances of these GMM estimators and compare them with those of the estimators developed until now (i.e. the Level, WG, and PSM estimators). The results of the experiments indicated that the GMM(pr) and GMM(ex) estimators perform considerably better than the GMM(qd) estimator under the assumption of strictly exogenous explanatory variables, and the GMM(pr), GMM(sa), and GMM(sb) estimators perform considerably better than the GMM(qd) estimator under the assumption of predetermined and stationary explanatory variables and stationary dependent variables. In addition, it was shown that the small sample performances of the GMM(pr), GMM(ex), GMM(sa), and GMM(sb) estimators are occasionally by no means inferior to that of the PSM estimator using the large size of pre-sample history for situations where each of these estimators are valid, and further the GMM(pr), GMM(ex), GMM(sa), and GMM(sb) estimators perform well in situations where the PSM estimator is not valid.

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Table 1: Monte Carlo results for strictly exogenous x_{it}, $T = 4$, bias and rmse (Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1; \rho = 0.5; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 1; \sigma_w^2 = 0.5; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	*0.259*	*0.267*	0.275	0.277	0.277	0.278
	β	*0.543*	*0.642*	0.570	0.633	0.555	0.567
WG	γ	-0.454	0.464	-0.446	0.448	-0.447	0.448
	β	-0.261	0.272	-0.261	0.264	-0.261	0.262
GMM(qd)	$\gamma(99)$	-0.272	0.372	-0.107	0.153	-0.068	0.109
	$\beta(99)$	-0.256	0.347	-0.136	0.206	-0.091	0.160
	$\gamma(1)$	-0.281	0.415	-0.108	0.166	-0.063	0.114
	$\beta(1)$	-0.246	0.377	-0.126	0.224	-0.075	0.175
GMM(pr)	$\gamma(99)$	-0.100	0.203	-0.039	0.087	-0.025	0.065
	$\beta(99)$	-0.170	0.277	-0.072	0.144	-0.043	0.118
	$\gamma(1)$	-0.093	0.199	-0.031	0.088	-0.019	0.068
	$\beta(1)$	-0.148	0.278	-0.053	0.151	-0.026	0.128
GMM(ex)	$\gamma(99)$	-0.093	0.221	-0.028	0.087	-0.019	0.063
	$\beta(99)$	-0.109	0.256	-0.033	0.126	-0.019	0.095
	$\gamma(1)$	-0.090	0.212	-0.025	0.087	-0.017	0.063
	$\beta(1)$	-0.099	0.253	-0.027	0.128	-0.015	0.094
PSM	$\gamma(4)$	*0.136*	*0.158*	0.160	0.166	0.162	0.165
	$\beta(4)$	*0.198*	*0.316*	0.214	0.243	0.211	0.221
	$\gamma(8)$	0.108	0.132	0.128	0.135	0.130	0.134
	$\beta(8)$	0.141	0.227	0.154	0.177	0.153	0.162
	$\gamma(25)$	0.048	0.092	0.063	0.075	0.066	0.072
	$\beta(25)$	0.062	0.152	0.065	0.088	0.066	0.076
	$\gamma(50)$	0.023	0.085	0.036	0.053	0.038	0.047
	$\beta(50)$	0.036	0.135	0.035	0.064	0.036	0.050

Notes: (1) The number of replications is 1000. (2) The instrument sets for GMM estimators include no time dummies. (3) The replications where no convergence of the estimations is achieved are eliminated when calculating the values of the Monte Carlo statistics. Their rates are below about one percent. (4) The individuals where the pre-sample means are zero are eliminated in each replication when estimating the parameters of interest using the PSM estimator. The number of these individuals is fairly small for each replication. (5) Although there may be a few replications where the Level and PSM estimators generate the estimates of γ and β with their absolute values exceeding 10, these replications are eliminated when calculating the values of the Monte Carlo statistics. The values of the statistics obtained by conducting the eliminations are written in an italic type for the Level and PSM estimators. (6) The values of the Monte Carlo statistics are exhibited in the table, which are obtained using the true values of γ and β as the starting values in the optimization for each replication. The values of the statistics obtained using the true values are not much different from those obtained using two different types of the starting values. The differences are below about 0.01 in terms of the absolute value in almost all cases.

Table 2: Monte Carlo results for strictly exogenous x_{it}, $T = 8$, bias and rmse (Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1; \rho = 0.5; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$
$\tau = 1; \sigma_w^2 = 0.5; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$

		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.263	0.268	0.275	0.276	0.277	0.277
	β	0.538	0.592	0.552	0.565	0.554	0.560
WG	γ	−0.189	0.197	−0.183	0.185	−0.184	0.185
	β	−0.128	0.142	−0.126	0.129	−0.127	0.128
GMM(qd)	$\gamma(99)$	−0.315	0.332	−0.100	0.109	−0.060	0.069
	$\beta(99)$	−0.285	0.298	−0.141	0.151	−0.090	0.099
	$\gamma(1)$	−0.237	0.273	−0.075	0.094	−0.043	0.061
	$\beta(1)$	−0.238	0.270	−0.104	0.131	−0.059	0.088
GMM(pr)	$\gamma(99)$	−0.017	0.138	−0.029	0.049	−0.025	0.036
	$\beta(99)$	−0.170	0.230	−0.093	0.112	−0.061	0.074
	$\gamma(1)$	−0.002	0.133	−0.026	0.052	−0.022	0.041
	$\beta(1)$	−0.133	0.210	−0.060	0.094	−0.039	0.070
GMM(ex)	$\gamma(99)$	0.005	0.173	−0.011	0.057	−0.015	0.032
	$\beta(99)$	−0.136	0.217	−0.046	0.079	−0.028	0.046
	$\gamma(1)$	0.015	0.170	−0.014	0.057	−0.017	0.035
	$\beta(1)$	−0.124	0.211	−0.039	0.073	−0.025	0.044
PSM	$\gamma(4)$	0.145	0.155	0.163	0.166	0.164	0.166
	$\beta(4)$	0.192	0.229	0.212	0.221	0.213	0.219
	$\gamma(8)$	0.116	0.127	0.132	0.135	0.133	0.135
	$\beta(8)$	0.140	0.174	0.155	0.164	0.157	0.162
	$\gamma(25)$	0.058	0.077	0.068	0.073	0.069	0.072
	$\beta(25)$	0.061	0.098	0.068	0.078	0.069	0.075
	$\gamma(50)$	0.029	0.059	0.039	0.047	0.040	0.044
	$\beta(50)$	0.030	0.076	0.037	0.049	0.038	0.045

Notes: See Table 1.

Table 3: Monte Carlo results for strictly exogenous x_{it}, $T = 8$, bias and rmse (Situation of considerably persistent y_{it} and x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1; \rho = 0.9; \kappa = 0; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 1; \sigma_w^2 = 0.05; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.170	0.174	0.180	0.181	0.183	0.183
	β	0.421	0.669	0.425	0.471	0.428	0.455
WG	γ	-0.251	0.258	-0.245	0.246	-0.244	0.245
	β	-0.369	0.403	-0.367	0.373	-0.367	0.371
GMM(qd)	$\gamma(99)$	-0.496	0.515	-0.145	0.160	-0.078	0.090
	$\beta(99)$	-0.706	0.772	-0.465	0.549	-0.347	0.409
	$\gamma(1)$	-0.361	0.415	-0.109	0.143	-0.060	0.084
	$\beta(1)$	-0.696	0.880	-0.412	0.595	-0.272	0.406
GMM(pr)	$\gamma(99)$	0.034	0.163	-0.004	0.068	-0.013	0.042
	$\beta(99)$	-0.370	0.610	-0.314	0.416	-0.253	0.318
	$\gamma(1)$	0.035	0.155	-0.017	0.069	-0.019	0.049
	$\beta(1)$	-0.347	0.631	-0.215	0.371	-0.175	0.289
GMM(ex)	$\gamma(99)$	0.066	0.201	*0.024*	*0.094*	*0.005*	*0.050*
	$\beta(99)$	-0.476	0.845	*−0.134*	*0.342*	*−0.052*	*0.160*
	$\gamma(1)$	0.073	0.196	*−0.001*	*0.072*	*−0.008*	*0.046*
	$\beta(1)$	-0.482	0.863	*−0.090*	*0.247*	*−0.051*	*0.144*
PSM	$\gamma(4)$	0.115	0.125	0.133	0.136	0.137	0.139
	$\beta(4)$	0.046	0.460	0.070	0.289	0.066	0.167
	$\gamma(8)$	0.105	0.115	0.122	0.125	0.126	0.127
	$\beta(8)$	0.011	0.360	0.025	0.165	0.024	0.119
	$\gamma(25)$	0.076	0.089	0.091	0.094	0.094	0.096
	$\beta(25)$	-0.017	0.206	-0.004	0.100	-0.004	0.074
	$\gamma(50)$	0.056	0.073	0.069	0.073	0.072	0.074
	$\beta(50)$	-0.009	0.182	-0.001	0.081	-0.001	0.060

Notes: See Table 1. Further, (7) The values of the Monte Carlo statistics written in an italic type for the GMM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are slightly different from those obtained using the two different values.

Table 4: Monte Carlo results for strictly exogenous x_{it}, $T = 8$, bias and rmse (Situation of considerably persistent y_{it} and extremely persistent x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1; \rho = 0.95; \kappa = 0; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 1; \sigma_w^2 = 0.015; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.175	0.178	0.183	0.184	0.184	0.184
	β	0.244	0.524	0.250	0.322	0.234	0.272
WG	γ	-0.274	0.280	-0.272	0.273	-0.271	0.272
	β	-0.367	0.469	-0.360	0.380	-0.363	0.373
GMM(qd)	$\gamma(99)$	-0.540	0.561	-0.156	0.175	-0.075	0.091
	$\beta(99)$	-0.682	0.927	-0.513	0.820	-0.401	0.661
	$\gamma(1)$	-0.449	0.511	-0.138	0.189	-0.070	0.108
	$\beta(1)$	-0.746	1.378	-0.588	1.148	-0.379	0.829
GMM(pr)	$\gamma(99)$	0.034	0.165	0.011	0.081	0.002	0.049
	$\beta(99)$	-0.237	0.797	-0.304	0.564	-0.280	0.474
	$\gamma(1)$	0.042	0.167	0.002	0.082	-0.005	0.056
	$\beta(1)$	-0.236	0.810	-0.176	0.546	-0.150	0.467
GMM(ex)	$\gamma(99)$	0.061	0.193	0.038	0.106	*0.014*	*0.059*
	$\beta(99)$	-0.421	1.240	-0.116	0.537	*−0.027*	*0.275*
	$\gamma(1)$	0.069	0.187	*0.006*	*0.085*	*−0.004*	*0.056*
	$\beta(1)$	-0.423	1.249	*−0.041*	*0.340*	*−0.027*	*0.225*
PSM	$\gamma(4)$	0.112	0.122	0.128	0.131	0.131	0.132
	$\beta(4)$	-0.205	0.406	-0.185	0.250	-0.196	0.223
	$\gamma(8)$	0.101	0.111	0.116	0.119	0.118	0.119
	$\beta(8)$	-0.248	0.385	-0.231	0.270	-0.240	0.258
	$\gamma(25)$	0.074	0.087	0.087	0.091	0.089	0.091
	$\beta(25)$	-0.236	0.332	-0.224	0.248	-0.233	0.245
	$\gamma(50)$	0.058	0.073	0.070	0.074	0.071	0.073
	$\beta(50)$	-0.173	0.277	-0.165	0.190	-0.173	0.184

Notes: See Table 1. Further, (7) The values of the Monte Carlo statistics written in an italic type for the GMM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are slightly different from those obtained using the two different values.

Table 5: Monte Carlo results for strictly exogenous x_{it}, $T = 4$,
mcsd and mcmse for GMM estimators
(Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1; \rho = 0.5; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$
$\tau = 1; \sigma_w^2 = 0.5; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$

		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.254	0.202	0.110	0.130	0.085	0.103
	$\beta(99)$	0.233	0.205	0.155	0.185	0.132	0.166
	$\gamma(1)$	0.305	0.246	0.126	0.152	0.096	0.121
	$\beta(1)$	0.285	0.259	0.185	0.224	0.157	0.205
GMM(pr)	$\gamma(99)$	0.177	0.143	0.078	0.095	0.060	0.071
	$\beta(99)$	0.219	0.215	0.124	0.170	0.110	0.137
	$\gamma(1)$	0.176	0.164	0.083	0.104	0.065	0.078
	$\beta(1)$	0.236	0.257	0.141	0.196	0.125	0.155
GMM(ex)	$\gamma(99)$	0.200	0.124	0.082	0.079	0.060	0.058
	$\beta(99)$	0.231	0.179	0.122	0.123	0.093	0.092
	$\gamma(1)$	0.192	0.130	0.084	0.081	0.060	0.059
	$\beta(1)$	0.233	0.189	0.125	0.129	0.093	0.095

Notes: See Table 1 except as described in (4) and (5).

Table 6: Monte Carlo results for strictly exogenous x_{it}, $T = 8$,
mcsd and mcmse for GMM estimators
(Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1; \rho = 0.5; \kappa = 0.1; \sigma^2_\eta = 0.5; \iota = 0; \sigma^2_\zeta = 0.5;$ $\tau = 1; \sigma^2_w = 0.5; \delta = 0; \sigma^2_\varepsilon = 0.5; \varpi = 1; \varrho = 0; \sigma^2_\xi = 0.5; \vartheta = 0;$							
		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.105	0.035	0.044	0.037	0.033	0.031
	$\beta(99)$	0.088	0.027	0.054	0.043	0.043	0.042
	$\gamma(1)$	0.135	0.081	0.057	0.056	0.044	0.045
	$\beta(1)$	0.126	0.075	0.079	0.074	0.066	0.068
GMM(pr)	$\gamma(99)$	0.137	0.020	0.039	0.026	0.026	0.022
	$\beta(99)$	0.156	0.031	0.063	0.043	0.042	0.038
	$\gamma(1)$	0.133	0.050	0.046	0.041	0.035	0.033
	$\beta(1)$	0.163	0.082	0.073	0.069	0.058	0.057
GMM(ex)	$\gamma(99)$	0.173	0.014	0.055	0.023	0.029	0.020
	$\beta(99)$	0.169	0.021	0.064	0.033	0.036	0.028
	$\gamma(1)$	0.170	0.026	0.055	0.029	0.031	0.024
	$\beta(1)$	0.171	0.038	0.061	0.039	0.036	0.031

Notes: See Table 1 except as described in (4) and (5).

Table 7: Monte Carlo results for strictly exogenous x_{it}, $T = 8$,
mcsd and mcmse for GMM estimators
(Situation of considerably persistent y_{it} and x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1; \rho = 0.9; \kappa = 0; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$
$\tau = 1; \sigma_w^2 = 0.05; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$

		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.139	0.044	0.066	0.047	0.045	0.038
	$\beta(99)$	0.312	0.083	0.291	0.172	0.216	0.173
	$\gamma(1)$	0.206	0.106	0.093	0.075	0.058	0.058
	$\beta(1)$	0.538	0.263	0.429	0.316	0.301	0.284
GMM(pr)	$\gamma(99)$	0.160	0.022	0.068	0.031	0.040	0.027
	$\beta(99)$	0.485	0.110	0.274	0.180	0.192	0.165
	$\gamma(1)$	0.151	0.055	0.067	0.051	0.045	0.043
	$\beta(1)$	0.527	0.307	0.302	0.298	0.230	0.255
GMM(ex)	$\gamma(99)$	0.190	0.013	*0.091*	*0.025*	*0.050*	*0.022*
	$\beta(99)$	0.698	0.086	*0.315*	*0.128*	*0.152*	*0.099*
	$\gamma(1)$	0.181	0.028	*0.072*	*0.037*	*0.046*	*0.031*
	$\beta(1)$	0.717	0.167	*0.230*	*0.146*	*0.135*	*0.113*

Notes: See Table 1 except as described in (4) and (5). Further, (7) The values of the Monte Carlo statistics written in an italic type for the GMM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are slightly different from those obtained using the two different values.

Table 8: Monte Carlo results for strictly exogenous x_{it}, $T = 8$,
mcsd and mcmse for GMM estimators
(Situation of considerably persistent y_{it} and extremely persistent x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1; \rho = 0.95; \kappa = 0; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 1; \sigma_w^2 = 0.015; \delta = 0; \sigma_\varepsilon^2 = 0.5; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 0;$							
		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.149	0.048	0.079	0.054	0.052	0.044
	$\beta(99)$	0.628	0.166	0.640	0.364	0.525	0.373
	$\gamma(1)$	0.244	0.122	0.128	0.091	0.082	0.068
	$\beta(1)$	1.158	0.489	0.985	0.600	0.738	0.536
GMM(pr)	$\gamma(99)$	0.162	0.023	0.080	0.034	0.049	0.029
	$\beta(99)$	0.761	0.189	0.475	0.324	0.383	0.315
	$\gamma(1)$	0.162	0.060	0.082	0.056	0.055	0.046
	$\beta(1)$	0.775	0.530	0.517	0.538	0.443	0.454
GMM(ex)	$\gamma(99)$	0.183	0.014	0.099	0.026	*0.058*	*0.023*
	$\beta(99)$	1.166	0.160	0.525	0.233	*0.273*	*0.170*
	$\gamma(1)$	0.174	0.030	*0.085*	*0.043*	*0.056*	*0.038*
	$\beta(1)$	1.175	0.304	*0.338*	*0.249*	*0.224*	*0.182*

Notes: See Table 1 except as described in (4) and (5). Further, (7) The values of the Monte Carlo statistics written in an italic type for the GMM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are slightly different from those obtained using the two different values.

Table 9: Monte Carlo results for predetermined x_{it}, $T = 4$, bias and rmse (Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.5; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 1; \sigma_w^2 = 0.25; \delta = 1; \sigma_\varepsilon^2 = 0.25; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 1;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	*0.242*	*0.254*	0.259	0.263	0.263	0.265
	β	*0.232*	*0.382*	0.243	0.287	0.238	0.258
WG	γ	−0.523	0.540	−0.520	0.525	−0.518	0.520
	β	−0.289	0.298	−0.289	0.290	−0.290	0.291
GMM(qd)	$\gamma(99)$	−0.322	0.443	−0.138	0.205	−0.087	0.138
	$\beta(99)$	−0.259	0.358	−0.140	0.213	−0.089	0.163
	$\gamma(1)$	−0.318	0.474	−0.133	0.213	−0.081	0.146
	$\beta(1)$	−0.242	0.374	−0.120	0.224	−0.070	0.184
GMM(pr)	$\gamma(99)$	−0.090	0.272	−0.043	0.140	−0.028	0.093
	$\beta(99)$	−0.172	0.306	−0.072	0.189	−0.037	0.140
	$\gamma(1)$	−0.079	0.278	−0.039	0.135	−0.022	0.097
	$\beta(1)$	−0.140	0.318	−0.052	0.195	−0.019	0.153
GMM(ex)	$\gamma(99)$	−0.097	0.300	−0.112	0.177	−0.132	0.167
	$\beta(99)$	−0.343	0.425	−0.313	0.329	−0.312	0.319
	$\gamma(1)$	−0.102	0.301	−0.123	0.182	−0.139	0.171
	$\beta(1)$	−0.342	0.424	−0.307	0.323	−0.310	0.318
GMM(sa)	$\gamma(99)$	−0.023	0.183	−0.025	0.101	−0.019	0.077
	$\beta(99)$	−0.101	0.245	−0.035	0.133	−0.023	0.104
	$\gamma(1)$	−0.010	0.182	−0.021	0.102	−0.015	0.079
	$\beta(1)$	−0.072	0.243	−0.022	0.137	−0.014	0.110
GMM(sb)	$\gamma(99)$	−0.019	0.193	−0.025	0.102	−0.019	0.077
	$\beta(99)$	−0.137	0.284	−0.043	0.139	−0.027	0.108
	$\gamma(1)$	−0.007	0.192	−0.020	0.104	−0.015	0.079
	$\beta(1)$	−0.110	0.291	−0.028	0.147	−0.017	0.116
PSM	$\gamma(4)$	0.132	0.166	0.161	0.170	0.164	0.171
	$\beta(4)$	0.075	0.231	0.093	0.141	0.091	0.163
	$\gamma(8)$	0.108	0.146	0.132	0.143	0.136	0.142
	$\beta(8)$	0.050	0.174	0.069	0.108	0.066	0.114
	$\gamma(25)$	0.046	0.107	0.069	0.089	0.071	0.083
	$\beta(25)$	0.019	0.178	0.030	0.067	0.027	0.054
	$\gamma(50)$	0.017	0.098	0.039	0.068	0.040	0.058
	$\beta(50)$	0.003	0.123	0.017	0.057	0.014	0.042

Notes: See Table 1.

Table 10: Monte Carlo results for predetermined x_{it}, $T = 8$, bias and rmse (Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.5; \kappa = 0.1; \sigma^2_\eta = 0.5; \iota = 0; \sigma^2_\zeta = 0.5;$ $\tau = 1; \sigma^2_w = 0.25; \delta = 1; \sigma^2_\varepsilon = 0.25; \varpi = 1; \varrho = 0; \sigma^2_\xi = 0.5; \vartheta = 1;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.246	0.253	0.262	0.264	0.263	0.264
	β	0.237	0.286	0.239	0.254	0.237	0.243
WG	γ	−0.242	0.253	−0.240	0.243	−0.239	0.241
	β	−0.153	0.163	−0.157	0.159	−0.157	0.158
GMM(qd)	$\gamma(99)$	−0.394	0.422	−0.149	0.167	−0.087	0.100
	$\beta(99)$	−0.290	0.308	−0.154	0.167	−0.102	0.115
	$\gamma(1)$	−0.295	0.348	−0.106	0.135	−0.060	0.084
	$\beta(1)$	−0.230	0.268	−0.110	0.136	−0.065	0.095
GMM(pr)	$\gamma(99)$	0.046	0.197	0.007	0.089	−0.010	0.051
	$\beta(99)$	−0.175	0.259	−0.095	0.133	−0.062	0.085
	$\gamma(1)$	0.058	0.187	−0.001	0.082	−0.010	0.055
	$\beta(1)$	−0.118	0.221	−0.042	0.098	−0.028	0.075
GMM(ex)	$\gamma(99)$	0.118	0.236	0.100	0.162	0.063	0.117
	$\beta(99)$	−0.426	0.490	−0.340	0.368	−0.302	0.316
	$\gamma(1)$	0.133	0.242	0.068	0.143	0.020	0.092
	$\beta(1)$	−0.415	0.483	−0.313	0.341	−0.276	0.288
GMM(sa)	$\gamma(99)$	−0.078	0.131	−0.040	0.063	−0.027	0.045
	$\beta(99)$	−0.175	0.214	−0.080	0.103	−0.051	0.069
	$\gamma(1)$	−0.003	0.100	−0.012	0.056	−0.009	0.042
	$\beta(1)$	−0.098	0.171	−0.033	0.082	−0.019	0.059
GMM(sb)	$\gamma(99)$	−0.071	0.134	−0.035	0.064	−0.024	0.045
	$\beta(99)$	−0.192	0.236	−0.088	0.114	−0.055	0.075
	$\gamma(1)$	0.008	0.109	−0.008	0.057	−0.007	0.043
	$\beta(1)$	−0.125	0.204	−0.041	0.090	−0.024	0.062
PSM	$\gamma(4)$	0.143	0.156	0.165	0.170	0.168	0.171
	$\beta(4)$	0.093	0.147	0.096	0.112	0.096	0.104
	$\gamma(8)$	0.117	0.133	0.138	0.144	0.141	0.144
	$\beta(8)$	0.072	0.124	0.073	0.092	0.073	0.081
	$\gamma(25)$	0.057	0.085	0.075	0.084	0.078	0.083
	$\beta(25)$	0.032	0.088	0.033	0.050	0.034	0.044
	$\gamma(50)$	0.026	0.068	0.042	0.056	0.046	0.054
	$\beta(50)$	0.017	0.077	0.018	0.038	0.018	0.031

Notes: See Table 1.

Table 11: Monte Carlo results for predetermined x_{it}, $T = 8$, bias and rmse (Situation of considerably persistent y_{it} and x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1/(1-\gamma); \rho = 0.9; \kappa = 0; \sigma^2_\eta = 0.5; \iota = 0; \sigma^2_\zeta = 0.5;$ $\tau = 0; \sigma^2_w = 0.5; \delta = 1; \sigma^2_\varepsilon = 0.05; \varpi = 1; \varrho = 0; \sigma^2_\xi = 0.5; \vartheta = 1;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	*0.176*	*0.180*	0.187	0.188	0.188	0.188
	β	*0.182*	*0.438*	0.210	0.310	0.199	0.240
WG	γ	−0.274	0.281	−0.270	0.272	−0.266	0.267
	β	−0.443	0.468	−0.450	0.454	−0.444	0.446
GMM(qd)	$\gamma(99)$	−0.559	0.583	−0.182	0.203	−0.096	0.111
	$\beta(99)$	−0.769	0.834	−0.578	0.656	−0.415	0.484
	$\gamma(1)$	−0.447	0.511	−0.151	0.194	−0.078	0.113
	$\beta(1)$	−0.791	0.998	−0.488	0.650	−0.308	0.452
GMM(pr)	$\gamma(99)$	0.056	0.174	0.014	0.091	0.002	0.060
	$\beta(99)$	−0.352	0.617	−0.346	0.449	−0.251	0.335
	$\gamma(1)$	0.059	0.172	−0.002	0.088	−0.007	0.064
	$\beta(1)$	−0.301	0.596	−0.210	0.376	−0.138	0.281
GMM(ex)	$\gamma(99)$	0.101	0.215	0.072	0.136	0.044	0.100
	$\beta(99)$	−0.864	1.106	−0.716	0.793	−0.656	0.698
	$\gamma(1)$	0.109	0.209	0.035	0.111	0.010	0.077
	$\beta(1)$	−0.822	1.091	−0.644	0.710	−0.587	0.620
GMM(sa)	$\gamma(99)$	−0.060	0.113	−0.026	0.056	−0.017	0.037
	$\beta(99)$	−0.433	0.540	−0.265	0.322	−0.169	0.222
	$\gamma(1)$	0.017	0.091	−0.007	0.050	−0.006	0.036
	$\beta(1)$	−0.338	0.499	−0.160	0.264	−0.090	0.189
GMM(sb)	$\gamma(99)$	−0.094	0.148	−0.026	0.063	−0.013	0.040
	$\beta(99)$	−0.437	0.574	−0.310	0.389	−0.211	0.274
	$\gamma(1)$	0.004	0.106	−0.003	0.056	−0.002	0.039
	$\beta(1)$	−0.406	0.619	−0.219	0.342	−0.129	0.235
PSM	$\gamma(4)$	0.126	0.136	0.145	0.148	0.148	0.149
	$\beta(4)$	−0.093	0.394	−0.071	0.193	−0.064	0.222
	$\gamma(8)$	0.116	0.127	*0.135*	*0.138*	0.137	0.139
	$\beta(8)$	−0.116	0.320	*−0.090*	*0.275*	−0.091	0.172
	$\gamma(25)$	0.088	0.102	0.106	0.110	0.108	0.110
	$\beta(25)$	−0.095	0.251	−0.080	0.178	−0.078	0.114
	$\gamma(50)$	0.064	0.083	0.081	0.087	0.083	0.086
	$\beta(50)$	−0.065	0.197	−0.052	0.113	−0.048	0.080

Notes: See Table 1.

Table 12: Monte Carlo results for predetermined x_{it}, $T = 8$, bias and rmse (Situation of considerably persistent y_{it} and extremely persistent x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1/(1-\gamma); \rho = 0.95; \kappa = 0; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$
$\tau = 0; \sigma_w^2 = 0.5; \delta = 1; \sigma_\varepsilon^2 = 0.015; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 1;$

		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.176	0.180	0.186	0.187	0.187	0.187
	β	0.116	0.436	0.107	0.254	0.104	0.168
WG	γ	−0.285	0.290	−0.279	0.281	−0.280	0.280
	β	−0.445	0.517	−0.434	0.450	−0.442	0.450
GMM(qd)	$\gamma(99)$	−0.558	0.581	−0.168	0.188	−0.082	0.099
	$\beta(99)$	−0.777	0.985	−0.636	0.943	−0.516	0.757
	$\gamma(1)$	−0.483	0.551	−0.167	0.229	−0.082	0.127
	$\beta(1)$	−0.836	1.426	−0.637	1.167	−0.438	0.909
GMM(pr)	$\gamma(99)$	0.037	0.169	0.012	0.083	0.006	0.057
	$\beta(99)$	−0.203	0.781	−0.282	0.536	−0.269	0.478
	$\gamma(1)$	0.047	0.169	−0.001	0.088	−0.001	0.063
	$\beta(1)$	−0.192	0.847	−0.159	0.550	−0.110	0.455
GMM(ex)	$\gamma(99)$	0.073	0.192	0.049	0.118	0.029	0.084
	$\beta(99)$	−0.839	1.422	−0.603	0.772	−0.541	0.632
	$\gamma(1)$	0.081	0.190	0.016	0.105	0.005	0.074
	$\beta(1)$	−0.805	1.406	−0.545	0.703	−0.496	0.572
GMM(sa)	$\gamma(99)$	−0.064	0.118	−0.010	0.045	−0.006	0.032
	$\beta(99)$	−0.301	0.553	−0.241	0.385	−0.188	0.307
	$\gamma(1)$	0.019	0.095	0.008	0.049	0.003	0.035
	$\beta(1)$	−0.248	0.579	−0.143	0.367	−0.107	0.288
GMM(sb)	$\gamma(99)$	−0.105	0.156	−0.014	0.049	−0.006	0.033
	$\beta(99)$	−0.263	0.613	−0.291	0.474	−0.243	0.376
	$\gamma(1)$	−0.007	0.105	0.008	0.053	0.003	0.036
	$\beta(1)$	−0.278	0.704	−0.230	0.479	−0.170	0.364
PSM	$\gamma(4)$	0.117	0.127	0.133	0.136	0.135	0.137
	$\beta(4)$	−0.282	0.451	−0.293	0.326	−0.291	0.309
	$\gamma(8)$	*0.106*	*0.116*	0.121	0.124	0.123	0.125
	$\beta(8)$	*−0.319*	*0.438*	−0.330	0.353	−0.327	0.339
	$\gamma(25)$	0.079	0.092	0.094	0.097	0.096	0.098
	$\beta(25)$	−0.286	0.372	−0.295	0.314	−0.296	0.304
	$\gamma(50)$	0.061	0.076	0.075	0.080	0.077	0.079
	$\beta(50)$	−0.206	0.293	−0.211	0.230	−0.212	0.221

Notes: See Table 1 except for (5). Instead of (5), (8) The values of the Monte Carlo statistics written in an italic type for the PSM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are slightly different from those obtained using the two different values.

Table 13: Monte Carlo results for predetermined x_{it}, $T = 4$,
mcsd and mcmse for GMM estimators
(Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.5; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$
$\tau = 1; \sigma_w^2 = 0.25; \delta = 1; \sigma_\varepsilon^2 = 0.25; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 1;$

		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.305	0.213	0.152	0.149	0.107	0.121
	$\beta(99)$	0.246	0.184	0.160	0.166	0.137	0.151
	$\gamma(1)$	0.352	0.264	0.167	0.176	0.121	0.141
	$\beta(1)$	0.285	0.235	0.190	0.214	0.170	0.190
GMM(pr)	$\gamma(99)$	0.257	0.158	0.133	0.113	0.089	0.093
	$\beta(99)$	0.254	0.211	0.175	0.164	0.135	0.141
	$\gamma(1)$	0.266	0.181	0.130	0.127	0.094	0.106
	$\beta(1)$	0.286	0.254	0.188	0.193	0.152	0.168
GMM(sa)	$\gamma(99)$	0.182	0.094	0.097	0.075	0.074	0.063
	$\beta(99)$	0.224	0.147	0.129	0.106	0.101	0.087
	$\gamma(1)$	0.182	0.102	0.100	0.079	0.078	0.065
	$\beta(1)$	0.232	0.169	0.135	0.113	0.109	0.092
GMM(sb)	$\gamma(99)$	0.192	0.093	0.099	0.075	0.074	0.063
	$\beta(99)$	0.249	0.151	0.132	0.108	0.104	0.088
	$\gamma(1)$	0.192	0.101	0.103	0.078	0.078	0.065
	$\beta(1)$	0.269	0.176	0.144	0.116	0.115	0.094

Notes: See Table 1 except as described in (4) and (5).

Table 14: Monte Carlo results for predetermined x_{it}, $T = 8$,
mcsd and mcmse for GMM estimators
(Situation of moderately persistent y_{it} and x_{it})

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.5; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 1; \sigma_w^2 = 0.25; \delta = 1; \sigma_\varepsilon^2 = 0.25; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 1;$							
		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.150	0.034	0.076	0.042	0.050	0.036
	$\beta(99)$	0.104	0.022	0.065	0.035	0.054	0.035
	$\gamma(1)$	0.185	0.085	0.083	0.065	0.059	0.054
	$\beta(1)$	0.137	0.064	0.081	0.063	0.069	0.061
GMM(pr)	$\gamma(99)$	0.192	0.019	0.089	0.029	0.050	0.026
	$\beta(99)$	0.191	0.029	0.093	0.042	0.058	0.037
	$\gamma(1)$	0.178	0.053	0.082	0.050	0.054	0.042
	$\beta(1)$	0.186	0.085	0.088	0.071	0.069	0.059
GMM(sa)	$\gamma(99)$	0.105	0.014	0.049	0.025	0.036	0.023
	$\beta(99)$	0.123	0.019	0.064	0.032	0.047	0.030
	$\gamma(1)$	0.100	0.033	0.054	0.035	0.041	0.030
	$\beta(1)$	0.140	0.054	0.076	0.049	0.056	0.042
GMM(sb)	$\gamma(99)$	0.113	0.014	0.054	0.025	0.038	0.023
	$\beta(99)$	0.138	0.019	0.072	0.032	0.050	0.030
	$\gamma(1)$	0.109	0.032	0.056	0.034	0.042	0.030
	$\beta(1)$	0.162	0.054	0.080	0.050	0.057	0.042

Notes: See Table 1 except as described in (4) and (5).

Table 15: Monte Carlo results for predetermined x_{it}, $T = 8$,
mcsd and mcmse for GMM estimators
(Situation of considerably persistent y_{it} and x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1/(1-\gamma); \rho = 0.9; \kappa = 0; \sigma^2_\eta = 0.5; \iota = 0; \sigma^2_\zeta = 0.5;$
$\tau = 0; \sigma^2_w = 0.5; \delta = 1; \sigma^2_\varepsilon = 0.05; \varpi = 1; \varrho = 0; \sigma^2_\xi = 0.5; \vartheta = 1;$

		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.164	0.046	0.089	0.054	0.056	0.044
	$\beta(99)$	0.321	0.074	0.311	0.157	0.248	0.158
	$\gamma(1)$	0.248	0.115	0.121	0.087	0.082	0.067
	$\beta(1)$	0.609	0.230	0.429	0.271	0.330	0.254
GMM(pr)	$\gamma(99)$	0.165	0.022	0.090	0.034	0.060	0.030
	$\beta(99)$	0.507	0.111	0.287	0.177	0.221	0.164
	$\gamma(1)$	0.161	0.057	0.088	0.056	0.063	0.048
	$\beta(1)$	0.514	0.315	0.312	0.296	0.245	0.255
GMM(sa)	$\gamma(99)$	0.095	0.016	0.049	0.026	0.033	0.023
	$\beta(99)$	0.323	0.068	0.184	0.121	0.144	0.113
	$\gamma(1)$	0.090	0.034	0.050	0.034	0.036	0.029
	$\beta(1)$	0.366	0.184	0.210	0.178	0.166	0.154
GMM(sb)	$\gamma(99)$	0.114	0.015	0.057	0.026	0.038	0.024
	$\beta(99)$	0.372	0.062	0.235	0.124	0.176	0.116
	$\gamma(1)$	0.106	0.033	0.056	0.035	0.039	0.030
	$\beta(1)$	0.467	0.180	0.262	0.187	0.197	0.161

Notes: See Table 1 except as described in (4) and (5).

Table 16: Monte Carlo results for predetermined x_{it}, $T = 8$,
mcsd and mcmse for GMM estimators
(Situation of considerably persistent y_{it} and extremely persistent x_{it})

$\gamma = 0.7; \beta = 1; \pi = 1/(1-\gamma); \rho = 0.95; \kappa = 0; \sigma^2_\eta = 0.5; \iota = 0; \sigma^2_\zeta = 0.5;$
$\tau = 0; \sigma^2_w = 0.5; \delta = 1; \sigma^2_\varepsilon = 0.015; \varpi = 1; \varrho = 0; \sigma^2_\xi = 0.5; \vartheta = 1;$

		$N = 100$		$N = 500$		$N = 1000$	
		mcsd	mcmse	mcsd	mcmse	mcsd	mcmse
GMM(qd)	$\gamma(99)$	0.159	0.048	0.085	0.058	0.055	0.046
	$\beta(99)$	0.605	0.159	0.697	0.360	0.554	0.367
	$\gamma(1)$	0.264	0.128	0.157	0.100	0.097	0.074
	$\beta(1)$	1.155	0.469	0.977	0.572	0.796	0.517
GMM(pr)	$\gamma(99)$	0.164	0.023	0.082	0.036	0.057	0.031
	$\beta(99)$	0.754	0.186	0.455	0.327	0.395	0.313
	$\gamma(1)$	0.162	0.062	0.088	0.059	0.063	0.048
	$\beta(1)$	0.825	0.523	0.526	0.518	0.442	0.452
GMM(sa)	$\gamma(99)$	0.100	0.017	0.043	0.026	0.031	0.023
	$\beta(99)$	0.464	0.118	0.301	0.225	0.242	0.204
	$\gamma(1)$	0.093	0.036	0.049	0.036	0.035	0.029
	$\beta(1)$	0.523	0.304	0.338	0.314	0.268	0.260
GMM(sb)	$\gamma(99)$	0.115	0.017	0.047	0.027	0.032	0.023
	$\beta(99)$	0.554	0.111	0.374	0.235	0.286	0.218
	$\gamma(1)$	0.105	0.036	0.052	0.036	0.036	0.030
	$\beta(1)$	0.647	0.298	0.420	0.339	0.321	0.285

Notes: See Table 1 except as described in (4) and (5).

Table 17: Monte Carlo results for proportional fixed effect, $T = 8$, bias and rmse

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.9; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$
$\tau = 0.5; \sigma_w^2 = 0.25; \delta = 0.5; \sigma_\varepsilon^2 = 0.25; \varpi = 1; \varrho = 0; \sigma_\xi^2 = 0.5; \vartheta = 1;$

		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	*0.236*	*0.254*	0.283	0.289	0.292	0.297
	β	*0.359*	*0.381*	0.356	0.369	0.357	0.363
WG	γ	−0.271	0.301	−0.257	0.270	−0.255	0.265
	β	−0.229	0.254	−0.225	0.231	−0.225	0.228
GMM(qd)	$\gamma(99)$	−0.490	0.548	−0.256	0.316	−0.181	0.257
	$\beta(99)$	−0.509	0.638	−0.544	0.657	−0.499	0.639
	$\gamma(1)$	−0.460	0.576	−0.289	0.383	−0.236	0.345
	$\beta(1)$	−0.546	0.776	−0.592	0.811	−0.584	0.825
GMM(pr)	$\gamma(99)$	0.028	0.202	0.036	0.127	0.031	0.111
	$\beta(99)$	0.231	0.337	0.136	0.229	0.106	0.200
	$\gamma(1)$	0.094	0.221	0.072	0.157	0.055	0.135
	$\beta(1)$	0.294	0.397	0.213	0.298	0.173	0.249
GMM(ex)	$\gamma(99)$	0.229	0.327	0.225	0.279	0.218	0.268
	$\beta(99)$	−0.974	1.280	−0.911	1.089	−0.916	1.072
	$\gamma(1)$	0.241	0.331	0.229	0.277	0.228	0.270
	$\beta(1)$	−0.950	1.294	−0.898	1.087	−0.934	1.112
GMM(sa)	$\gamma(99)$	−0.232	0.321	−0.106	0.184	−0.067	0.164
	$\beta(99)$	−0.017	0.163	−0.064	0.143	−0.069	0.141
	$\gamma(1)$	−0.109	0.261	−0.046	0.148	−0.027	0.141
	$\beta(1)$	0.065	0.209	0.008	0.137	−0.001	0.130
GMM(sb)	$\gamma(99)$	−0.260	0.333	−0.117	0.195	−0.069	0.164
	$\beta(99)$	−0.014	0.171	−0.069	0.163	−0.081	0.169
	$\gamma(1)$	−0.161	0.280	−0.062	0.160	−0.032	0.143
	$\beta(1)$	0.081	0.238	0.003	0.167	−0.009	0.163
PSM	$\gamma(4)$	*0.117*	*0.191*	0.180	0.197	0.192	0.207
	$\beta(4)$	*0.012*	*0.136*	0.013	0.115	0.018	0.281
	$\gamma(8)$	0.094	0.190	0.160	0.181	*0.172*	*0.190*
	$\beta(8)$	−0.011	0.117	−0.013	0.137	*−0.018*	*0.129*
	$\gamma(25)$	0.036	0.241	0.101	0.138	0.112	0.148
	$\beta(25)$	−0.008	0.094	−0.015	0.076	−0.013	0.111
	$\gamma(50)$	0.004	0.177	0.056	0.141	0.068	0.128
	$\beta(50)$	−0.003	0.084	−0.006	0.058	−0.005	0.038

Notes: See Table 1.

Table 18: Monte Carlo results for correlated fixed effect, $T = 8$, bias and rmse

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.9; \kappa = 0.1; \sigma^2_\eta = 0.5; \iota = 0; \sigma^2_\zeta = 0.5;$ $\tau = 0.5; \sigma^2_w = 0.25; \delta = 0.5; \sigma^2_\varepsilon = 0.25; \varpi = 0.6; \varrho = 0.8; \sigma^2_\xi = 0.5; \vartheta = 1;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.272	0.282	0.309	0.312	0.313	0.315
	β	0.173	0.225	0.171	0.196	0.175	0.185
WG	γ	−0.263	0.285	−0.252	0.260	−0.249	0.254
	β	−0.227	0.254	−0.224	0.230	−0.223	0.225
GMM(qd)	$\gamma(99)$	−0.459	0.507	−0.201	0.252	−0.131	0.188
	$\beta(99)$	−0.500	0.615	−0.544	0.661	−0.430	0.553
	$\gamma(1)$	−0.411	0.508	−0.234	0.337	−0.183	0.270
	$\beta(1)$	−0.581	0.808	−0.549	0.784	−0.483	0.728
GMM(pr)	$\gamma(99)$	0.021	0.208	0.029	0.103	0.018	0.073
	$\beta(99)$	0.143	0.310	0.075	0.198	0.062	0.167
	$\gamma(1)$	0.075	0.221	0.053	0.125	0.034	0.093
	$\beta(1)$	0.177	0.338	0.130	0.226	0.116	0.196
GMM(ex)	$\gamma(99)$	0.176	0.283	0.179	0.237	0.171	0.218
	$\beta(99)$	−0.879	1.106	−0.905	1.024	−0.906	1.004
	$\gamma(1)$	0.192	0.289	0.189	0.241	0.186	0.226
	$\beta(1)$	−0.856	1.112	−0.886	1.019	−0.916	1.027
GMM(sa)	$\gamma(99)$	−0.182	0.264	−0.061	0.128	−0.036	0.105
	$\beta(99)$	−0.048	0.179	−0.083	0.158	−0.072	0.131
	$\gamma(1)$	−0.056	0.200	−0.012	0.107	−0.008	0.086
	$\beta(1)$	0.008	0.189	−0.013	0.136	−0.011	0.109
GMM(sb)	$\gamma(99)$	−0.219	0.288	−0.072	0.138	−0.039	0.112
	$\beta(99)$	−0.040	0.196	−0.087	0.182	−0.084	0.159
	$\gamma(1)$	−0.115	0.226	−0.025	0.117	−0.013	0.096
	$\beta(1)$	0.033	0.233	−0.016	0.176	−0.019	0.137
PSM	$\gamma(4)$	0.164	0.189	0.212	0.221	0.219	0.225
	$\beta(4)$	−0.126	0.176	−0.133	0.172	−0.121	0.224
	$\gamma(8)$	0.148	0.176	0.194	0.204	0.202	0.209
	$\beta(8)$	−0.143	0.182	−0.149	0.164	−0.145	0.152
	$\gamma(25)$	0.108	0.146	0.152	0.167	0.161	0.171
	$\beta(25)$	−0.136	0.168	−0.140	0.150	−0.138	0.142
	$\gamma(50)$	0.082	0.134	0.126	0.145	*0.135*	*0.147*
	$\beta(50)$	−0.123	0.152	−0.128	0.137	*−0.127*	*0.130*

Notes: See Table 1.

Table 19: Monte Carlo results for uncorrelated fixed effect, $T = 8$, bias and rmse

$\gamma = 0.5; \beta = 0.5; \pi = 1/(1-\gamma); \rho = 0.9; \kappa = 0.1; \sigma_\eta^2 = 0.5; \iota = 0; \sigma_\zeta^2 = 0.5;$ $\tau = 0.5; \sigma_w^2 = 0.25; \delta = 0.5; \sigma_\varepsilon^2 = 0.25; \varpi = 0; \varrho = 1; \sigma_\xi^2 = 0.5; \vartheta = 1;$							
		$N = 100$		$N = 500$		$N = 1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.269	0.274	0.291	0.293	0.295	0.296
	β	−0.067	0.134	−0.067	0.085	−0.067	0.078
WG	γ	−0.252	0.266	−0.251	0.255	−0.247	0.249
	β	−0.223	0.249	−0.223	0.229	−0.226	0.228
GMM(qd)	$\gamma(99)$	−0.402	0.438	−0.128	0.164	−0.069	0.096
	$\beta(99)$	−0.478	0.591	−0.446	0.564	−0.325	0.435
	$\gamma(1)$	−0.343	0.424	−0.145	0.216	−0.086	0.154
	$\beta(1)$	−0.473	0.752	−0.356	0.607	−0.252	0.481
GMM(pr)	$\gamma(99)$	0.011	0.179	0.014	0.080	0.005	0.048
	$\beta(99)$	−0.044	0.301	−0.070	0.199	−0.057	0.157
	$\gamma(1)$	0.050	0.177	0.016	0.089	0.008	0.060
	$\beta(1)$	−0.020	0.296	−0.015	0.182	−0.007	0.141
GMM(ex)	$\gamma(99)$	0.095	0.231	0.099	0.152	0.084	0.117
	$\beta(99)$	−0.848	0.980	−0.820	0.879	−0.809	0.838
	$\gamma(1)$	0.116	0.237	0.112	0.160	0.107	0.135
	$\beta(1)$	−0.836	0.981	−0.814	0.880	−0.841	0.876
GMM(sa)	$\gamma(99)$	−0.114	0.176	−0.024	0.065	−0.009	0.041
	$\beta(99)$	−0.134	0.225	−0.124	0.172	−0.089	0.127
	$\gamma(1)$	−0.003	0.126	0.001	0.061	0.004	0.044
	$\beta(1)$	−0.098	0.215	−0.066	0.139	−0.044	0.102
GMM(sb)	$\gamma(99)$	−0.152	0.210	−0.027	0.072	−0.006	0.043
	$\beta(99)$	−0.117	0.242	−0.146	0.211	−0.117	0.165
	$\gamma(1)$	−0.049	0.152	−0.001	0.067	0.005	0.045
	$\beta(1)$	−0.073	0.251	−0.086	0.179	−0.063	0.136
PSM	$\gamma(4)$	0.177	0.189	0.206	0.210	0.211	0.214
	$\beta(4)$	−0.242	0.262	−0.246	0.250	−0.247	0.249
	$\gamma(8)$	0.161	0.175	0.191	0.195	0.196	0.199
	$\beta(8)$	−0.251	0.267	−0.254	0.257	−0.254	0.256
	$\gamma(25)$	0.131	0.148	0.160	0.165	0.165	0.169
	$\beta(25)$	−0.233	0.247	−0.236	0.239	−0.237	0.238
	$\gamma(50)$	0.114	0.135	0.140	0.147	0.145	0.150
	$\beta(50)$	−0.218	0.235	−0.220	0.223	−0.221	0.222

Notes: See Table 1.