Pollution Tax and Social Welfare in Oligopoly
—Asymmetric Taxation on Identical Polluters—

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Abstract We study asymmetric pollution taxation on identical polluting oligopolists engaged in Cournot competition. It has been studied that an identical firms must be treated symmetrically and that the second-best pollution tax is a uniform tax. But, asymmetric treatment of identical firms generates an aggregate cost-saving effect, which is oligopoly-specific property. We consider a manipulation of the uniform tax vector without changing the total emissions which will be emitted under the uniform pollution tax.

We derive a sufficient condition that guarantees that unequal taxation on *ex ante* identical polluters increases welfare. We show that if the sufficient condition is satisfied, unequal emission standards as well as unequal taxation increases welfare.

Keywords: Cournot Duopoly, Firm Asymmetry, Pollution Tax, Environmental Policy

JEL classification: Q5,Q2, L13

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1 Introduction

When firms produce a good with pollution, a pollution tax internalizes the external damages caused by the polluting activities. Under perfect competition, the optimal tax rate equals the marginal environmental damages, regardless of whether polluting firms are identical or not: that is known as a Pigouvian tax. But, this full-internalization result depends upon market structure. In an oligopolistic market, because there is two distortions, environmental externalities and market imperfection, the optimal (second-best) tax rate does not equal the marginal environmental damages. In an early analysis of pollution tax under oligopoly, e.g., Simpson (1995), uniform tax is considered. Recently, Long and Soubeyran (2005) analize firm-specific pollution taxes. They show that the optimal firm-specific pollution taxes require that the more inefficient firm must pay a higher tax rate. They call this tax rule as *selective penalization*. Although imposing firm-specific pollution taxes on heterogeneous oligopolists have studied, there has been little attention to asymmetric taxation on identical oligopolists.

In the context of direct regulation of pollution, however, the following papers deal with asymmetric regulation. Salant and Shaffer (1999) point out that direct regulation with unequal treatment of equals enhance welfare as application of their analysis. Long and Soubeyran(2001a) show that if the social welfare function is convex in output of each firm, non-identical treatment of identical firms is optimal. Amir and Nannerup(2005) show that direct regulation with unequal treatment of identical polluters may increase welfare. They derive a sufficient condition for welfare-improving by asymmetric regulation with keeping total output and total emission at constant levels. This paper is related to and complements work by Amir and Nannerup (2005) [1]. Although their result is interesting but is limited to the case of direct regulation. No studies have ever tried to explore differentiating tax duties among identical polluters. There are further questions to be considered regarding unequal treatment of equals. The first question is whether differentiated taxation imposed on identical polluters increases social welfare. The second question is, if so, whether asymmetric regulation also increases social welfare.

Purpose of this paper is to answer the above questions. The contributions of this paper are as follows. First, we derive a sufficient condition that inducing unequal taxation increases social welfare. Second, under the specific functional form with two firms used by Amir and Nannerup(2005), we also show that if the sufficient condition is satisfied, unequal emission standards as well as unequal taxation increases welfare.

The standard environmental economics theory postulates equal treatment of identical polluters. It asserts that, government imposes firms on the uniform taxation or emission standards. But, when the sufficient conditions given in this paper holds, there is room for a potential welfare improving. Government can improve welfare by setting unequal tax rates.

Asymmetric treatment of firms in our model follows from Salant and

[1] Long and Soubeyran(2005) is different from ours in that their model does not have firm's abatement activity and in that they assume that the marginal cost of public fund is larger than one.

Shaffer (1999) and Long and Soubeyran (2001b). They study two-stage game where firms first invest to manipulate marginal costs and then compete in quantities. They show that manipulating marginal costs of *ex ante* identical firms with keeping the sum of marginal costs at constant increases social welfare. Their results are based on the property pointed out by Bergstrom and Varian(1985a,b): if variance of the marginal costs increase but their sum remains constant, the aggregate production costs will decrease but the industry output will be unchanged.

The structure of the paper is as follows. In section 2, we describe the formal model. In section 3, we derive a condition for asymmetric taxation. In section 4, we compare unequal taxation to unequal standards. Section 5 provides a numerical example. Section 6 concludes the paper.

2 Model

Consider a polluting oligopoly consisting of n identical firms producing a homogenous good. They compete à la Cournot. Each firm i produces output q^i and a negative externality e^i. Let $Q = \sum_{i=1}^{n} q^i$ denote total industry output. The inverse market demand function is given by $P = P(Q)$ where $P'(Q) < 0$. To ensure the existence and uniqueness of Cournot equilibrium, it is assumed that the marginal revenue of each firm is strictly decreasing in output of its rivals, $P'(Q) + P''(Q)q^i < 0$. Each firm has an identical cost function $c^i(q^i, e^i)$. The cost function is increasing in q^i, decreasing in e^i $(c^i_q > 0, c^i_{qq} > 0, c^i_e < 0, c^i_{ee} > 0, c^i_{qe} < 0)$ [2]. Let t_i denote a firm-specific pollution tax for firm i. We allow $t_i \neq t_j$

[2]The subscripts associated with functions denote partial derivatives.

nevertheless assumption of identical firms.

Each firm's profit function can be written as

$$\pi^i = P(Q)q^i - c^i(q^i, e^i) - t_i e^i. \tag{1}$$

The first-order necessary conditions for Cournot-Nash equilibrium choice of output and emission are:

$$P(Q^N) + P'(Q^N)q^{iN} = c^i_q(q^{iN}, e^{iN}), \tag{2}$$

$$-c^i_e(q^{iN}, e^{iN}) = t_i. \tag{3}$$

where the superscript N denotes the Nash equilibrium outcome. The implication of conditions (2) and (3) are as follows. Condition (2) indicates that the marginal revenue of firm equals the marginal production cost. Condition (3) indicates that the marginal abatement cost equals the pollution tax.

Social welfare is given by:

$$W = \int_0^{Q^N} P(u)du - \sum_{i=1}^{n} c^i(q^{iN}, e^{iN}) - D(\sum_{i=1}^{n} e^{iN}) \tag{4}$$

where $D(\cdot)$ is pollution damage, $D' > 0$, and $D'' > 0$. For the benchmark, we derive the *optimal* uniform pollution tax rate t^0 [3]. Differentiating (4) with respect to t and using (2) and (3), we get:

$$t^0 = D' + \frac{\sum_{i=1}^{n} P' q^{iN} \frac{dq^{iN}}{dt}}{\sum_{i=1}^{n} \frac{de^{iN}}{dt}}. \tag{5}$$

When environmental policy is limited to equal treatment of identical firms, the optimal uniform tax is given by (5). The standard result for taxation for polluting oligopolists (Simpson,1995) asserts that the government should impose the tax $(t_1, \cdots, t_n) = (t^0, \cdots, t^0)$ on polluters.

[3] If unequal taxation leads to welfare improving, t^0 is suboptimal.

3 Optimal Condition for Asymmetric Taxation

In this section we consider a sufficient condition that unequal taxation on identical polluters improves welfare. Obviously arbitrary deviation from the uniform taxation effects all variables. It may increase or decrease total emissions. It seems less important that manipulation of tax rates increases total emissions, $E^N(t_1, t_2, \cdots, t_n) = \sum_{i=1}^n e^{iN}(t_1, t_2, \cdots, t_n)$, even when it increases consumers' and producers' surplus more than damages. For this reason, we restrict our attention to manipulation of the tax vector without changing the total emissions under the *optimal* uniform tax, $\bar{E} = E^N(t^0, \cdots, t^0)$.

In this section, we consider the following manipulation Fix the tax rates of firm $3, \cdots, n$ at t^0 but allow the tax rates of firm 1 and firm 2 to vary. We now define a function $\varphi(t_1)$ which satisfies $E^N(t_1, \varphi(t_1), t^0, \cdots, t^0) = \bar{E}$. The existence of $\varphi(t_1)$ within a neighborhood of t^0 is as follows. Obviously E^N is continuously differentiable with respect to t_i. Assuming that $E_i^N \neq 0$ at $(t^0, \cdots, t^0)$, the implicit function theorem guarantees the unique existence of $\varphi(t_1)$ taking some neighborhood of t^0. The pair $(t_1, \varphi(t_1))$ in the neighborhood of $(t_1, t_2) = (t^0, t^0)$ describes the only combination of (t_1, t_2) that remains the aggregate emissions constant given $(t_3, \cdots, t_n) = (t^0, \cdots, t^0)$ [4]. For notational convenience, we define the emission-constrained welfare function $\tilde{W}(t_1) = W(t_1, \varphi(t_1), t^0, \cdots, t^0)$ [5]. It shows that welfare with changing of tax rate of firm 1 and the corresponding changing of tax rate of firm 2 ,without changing of total emissions. Gross consumers' surplus is given by $S = \int_0^{Q^N} P(u)du$ and

[4] We consider only local perturbations in the neighborhood of $(t_1, t_2) = (t^0, t^0)$.

[5] Because we fix $(t_3, \cdots, t_n)$ at t^0, they are dropped.

aggregate costs is given by $C = \sum_{i=1}^{n} c^i(q^i, e^i)$. In a similar fashion, we define the emission-constrained consumers' surplus and cost function $\tilde{S}(t_1)$, and $\tilde{C}(t_1)$, respectively.

Now social welfare can be rewritten as:

$$\tilde{W}(t_1) = \tilde{S}(t_1) - \tilde{C}(t_1) - D(\bar{E}). \tag{6}$$

Differentiating (6) with respect to t_1 and evaluating at $t_1 = t^0$, we get:

$$\tilde{W}'(t^0) = \tilde{S}'(t^0) - \tilde{C}'(t^0) = 0. \tag{7}$$

From (5), it is obvious that (7) is zero. Differentiating (6) with respect to t_1 once more and evaluating at $t_1 = t^0$, we get:

$$\tilde{W}''(t^0) = \tilde{S}''(t^0) - \tilde{C}''(t^0). \tag{8}$$

If (8) is positive, $\tilde{W}(t_1)$ is convex in the neighborhood of t^0. If so, the convexity of $\tilde{W}(t_1)$ and (7) implies that $\tilde{W}(t_1)$ achieves a local minimum at t_0; we can improve welfare inducing asymmetric taxation along the path of constant total emissions. The following proposition characterizes the sufficient condition for welfare-improving by deviating from the symmetric pollution tax.

Proposition 1. *Asymmetric taxation on identical polluters improves welfare if*

$$\tilde{S}''(t^0) - \tilde{C}''(t^0) \quad = \quad 2\Bigg(\sum_{i=1}^{n}(P(Q^N) - c_q^i)(q_{11}^{iN} - q_{12}^{iN}) - \sum_{i=1}^{n} c_e^i(e_{11}^{iN} - e_{12}^{iN})$$

$$+\sum_{i=1}^{n}(c_q^i q_2^{iN} + c_e^i e_2^{iN} - q_2^{iN})\frac{\sum_{j=1}^{n}(e_{11}^{jN} - e_{12}^{jN})}{\sum_{h=1}^{n} e_2^{hN}}\Bigg)$$
$$> 0. \tag{9}$$

Proof See the Appendix A.

Unequal taxation implies manipulation of marginal costs [6]. Hence, it induces changes of the equilibrium output of each firm, and then the consumers' surplus and aggregate production costs without change of total emissions. If cost reduction dominates changes of the consumers' surplus (may be positive or negative), then asymmetry leads to welfare-improving.

4 Equivalence between the Conditions for Asymmetric Taxation and Asymmetric Emission Standards

Now we compare unequal taxation on emissions to unequal emission standards. Whether the conditions that asymmetric treatment of identical polluters leads to welfare-improving are the same between pollution tax and emission standards? To this purpose we will introduce a specific functional form used by Amir and Nannerup (2005) which deals with direct regulation. In this section we set $n = 2$. For simplicity, we assume that direct production costs are zero. Let a^i be the abatement

[6] Long and Soubeyran(2001) study the cost manipulation game in the context of cost-saving R&D where a firm invests to reduce its marginal cost. Their model is manipulating marginal cost with cost, whereas our situation is without cost.

units of firm i, r the price of abatement unit. Pollution emissions of firm i is given by $e^i = q^i/a^i$. Notice that $a^i = q^i/e^i$. Then firm i's cost is $c^i(q^i, e^i) = ra^i = r(q^i/e^i)$. Denote $\bar{e}^i$ firm i's maximum allowable emissions. With standards, firm i maximizes the profit given q^j and $\bar{e}^i$:

$$\pi^i = P(Q)q^i - r\frac{q^i}{\bar{e}^i}. \tag{10}$$

The first order condition with respect to q^i is:

$$P(Q^N) + P'(Q^N)q^{iN} = \frac{r}{\bar{e}^i}. \tag{11}$$

In this section, we introduce a lump sum transfer to firm i , T_i, which assures non-negative profits and is financed from pollution tax revenue. A lump sum transfer does not affect the firm's marginal decisions. With taxation, firm i maximizes the profit given q^j, t_i and T_i:

$$\pi^i = P(Q) - r\frac{q^i}{e^i} - t_i e^i + T_i. \tag{12}$$

Here it is more convenient to use e^i rather than a^i as the choice variable of firm. The first order conditions with respect to q^i and e^i are:

$$P(Q^N) + P'(Q^N)q^{iN} = \frac{r}{e^{iN}}, \tag{13}$$

$$r\frac{q^{iN}}{(e^{iN})^2} = t_i. \tag{14}$$

Let us define

$$\theta^i = \frac{r}{e^{iN}}. \tag{15}$$

where θ^i denotes the reciprocal of the equilibrium emissions times abatement unit cost. From (11) or (13), θ^i is interpreted as marginal cost of firm i.

Assuming that $P(Q) = A - Q$, by (13) and (15), the firm i's equilibrium output is

$$q^{iN} = \frac{A - 2\theta^i + \theta^j}{3}. \tag{16}$$

From (14)-(16), the relationship between t_i and θ^i, θ^j are given by:

$$t_i = \frac{rq^{iN}}{(e^{iN})^2} = \frac{(\theta^i)^2(A - 2\theta^i + \theta^j)}{3r}. \tag{17}$$

For given (t_1, t_2), the equilibrium output and emissions (q^{iN}, e^{iN}) are determined as functions of (t_1, t_2), and then θ^i is determined as a function of e^{iN}. Then we can consider θ^i as a function of (t_1, t_2) [7]. In this section, environmental damages are given by $D(e^1 + e^2) = (d/2)(e^1 + e^2)^2$ for a given positive parameter d . Now social welfare is:

$$W = \int_0^{Q^N} P(u)du - \frac{rq^{1N}}{e^{1N}} - \frac{rq^{2N}}{e^{2N}} - \frac{d}{2}(e^{1N} + e^{2N})^2. \tag{18}$$

By (15) and (16), we can express social welfare as a function of θ^1, θ^2:

$$\begin{aligned} \hat{W}(\theta^1, \theta^2) \quad &= \quad \int_0^{\frac{2A-\theta^1-\theta^2}{3}} P(u)du - r\frac{A - 2\theta^1 + \theta^2}{3} - r\frac{A + \theta^1 - 2\theta^2}{3} \\ &\quad -\frac{dr^2}{2}\left(\frac{1}{\theta^1} + \frac{1}{\theta^2}\right)^2. \end{aligned} \tag{19}$$

First, we consider the pollution tax policy. Now the *optimal* uniform pollution tax rate t^0 is given by (5). The *optimal* transfer associated with it T_i is set to an arbitrary value which assures a non-negative profit. We define a function $t_2(t_1)$ satisfying

$$e^{1N}(t_1, t_2(t_1)) + e^{2N}(t_1, t_2(t_1)) \equiv e^{1N}(t^0, t^0) + e^{2N}(t^0, t^0). \tag{20}$$

The above definition assures that the government can change the symmetric tax vector (t^0, t^0) into an asymmetric tax vector $(t_1, t_2(t_1)), t_1 \neq$

[7] Unfortunately we cannot explicitly solve (17) for θ^i but this does not affect our results.

$t_2(t_1)$ without changing the total emissions. With a slight abuse of notations, we also use $\tilde{W}(t_1) = W(t_1, t_2(t_1))$, $\tilde{S}(t_1) = S(t_1, t_2(t_1))$, and $\tilde{C}(t_1) = C(t_1, t_2(t_1))$.

Lemma 1 *Under the uniform pollution tax rate, t^0, the following conditions are equivalent.*

$$\tilde{W}''(t^0) > 0 \iff \hat{W}_{\theta^1\theta^1}(\theta^1, \theta^2) - \hat{W}_{\theta^1\theta^2}(\theta^1, \theta^2) > 0$$

where the derivatives of $\hat{W}$ are evaluated at $(\theta^1, \theta^2) = (\theta^1(t^0, t^0), \theta^2(t^0, t^0))$.

Proof See Appendix B.

If $\tilde{W}''$ is positive, $\tilde{W}$ is convex in the neighborhood of t^0, and $\hat{W}$ is also convex in the neighborhood of its corresponding point $(\theta^1(t^0, t^0), \theta^2(t^0, t^0))$. It implies that deviation from the equal taxation improves welfare. By lemma1, if so, the following proposition characterizes the sufficient condition for welfare-improving by unequal taxation in the duopoly case; it is equivalent to the condition (9) because $\tilde{W}'' = \tilde{S}'' - \tilde{C}''$.

Proposition 2 *Asymmetric taxation on identical polluting duopolists improves welfare if*

$$\hat{W}_{\theta^1\theta^1}(\theta^1, \theta^2) - \hat{W}_{\theta^1\theta^2}(\theta^1, \theta^2) > 0. \tag{21}$$

where the left-hand side is evaluated at $(\theta^1, \theta^2) = (\theta^1(t^0, t^0), \theta^2(t^0, t^0))$ [8].

[8] Under asymmetric taxation, a set of transfer (T_1, T_2) has to be set to an arbitrary value which assures a non-negative profit.

Next, we consider a sufficient condition for welfare-improving by asymmetric standards. Denote *optimal* uniform standards by $\bar{e}^0$ and its corresponding θ by $\theta^0 = r/\bar{e}^0$. Following Amir and Nannerup (2005), we do not use (e^1, e^2) as the choice variables of the government but (θ^1, θ^2). It is innocuous because θ^i is a monotonic transformation of e^i. The first-order condition for *optimal* uniform standards and (15) give us $d\hat{W}/d\theta(\theta, \theta) = 0$ when (θ, θ) is evaluated at (θ^0, θ^0).

We define a function $\theta_2(\theta_1)$ satisfying

$$\frac{r}{\theta^1} + \frac{r}{\theta^2} \equiv 2\bar{e}^0. \tag{22}$$

The above definition assures that the government can change the symmetric standards (θ^0, θ^0) into an asymmetric standards $(\theta^1, \theta^2), \theta^1 \neq \theta^2$ without changing the total emissions. The following proposition characterizes the sufficient condition for welfare-improving by asymmetric standards in the duopoly case, which is essentially identical to the result stated in Amir and Nannerup(2005) [9].

Proposition 3 *Asymmetric standards on identical polluting duopolists improve welfare if*

$$\hat{W}_{\theta^1\theta^1}(\theta^0, \theta^0) - \hat{W}_{\theta^1\theta^2}(\theta^0, \theta^0) > 0. \tag{23}$$

Proof Applying the implicit function theorem to (22) evaluated at $\theta^1 = \theta^0$, we easily obtain $\theta^{2'}(\theta^0) = -1$. Differentiating $\hat{W}(\theta^1, \theta^2(\theta^1))$ with respect to θ^1, we obtain:

$$\frac{d\hat{W}(\theta^1, \theta^2(\theta^1))}{d\theta^1} = \hat{W}_{\theta^1} + \hat{W}_{\theta^2}\theta^{2'}. \tag{24}$$

[9] The condition in Proposition 3 is identical to Eq.(8) in Amir and Nannerup(2005) although there are notational difference between the expressions of here and theirs.

Differentiating $\hat{W}(\theta^1, \theta^2(\theta^1))$ twice with respect to θ^1, we obtain:

$$\frac{d^2\hat{W}(\theta^1, \theta^2(\theta^1))}{d(\theta^1)^2} = \hat{W}_{\theta^1\theta^1} + \hat{W}_{\theta^1\theta^2}\theta^{2\prime} + \hat{W}_{\theta^2\theta^1}\theta^{2\prime} + \hat{W}_{\theta^2\theta^2}(\theta^{2\prime})^2 + \hat{W}_{\theta^2}\theta^{2\prime\prime}. \tag{25}$$

By the optimality condition for the *optimal* uniform standards, $\hat{W}_{\theta^1} = \hat{W}_{\theta^2} = 0$. Note that $\hat{W}_{\theta^1\theta^1} = \hat{W}_{\theta^2\theta^2}$ at the symmetric point and $\hat{W}_{\theta^1\theta^2} = \hat{W}_{\theta^2\theta^1}$ by the Young's theorem. Then, we get:

$$\frac{d^2\hat{W}(\theta^1, \theta^2(\theta^1))}{d(\theta^1)^2} = 2(\hat{W}_{\theta^1\theta^1} - \hat{W}_{\theta^1\theta^2}). \tag{26}$$

The conditions of Proposition 2 and 3 are identical. This immediately leads to the following central result of this section.

Proposition 4 *The sufficient condition for welfare-improving by unequal taxation is equivalent to the sufficient condition for welfare-improving by asymmetric standards.*

5 Numerical Result

In this section, we provide numerical examples to illustrate the result obtained in the preceding section. We adopt a set of parameters as follows: $A = r = 100, d = 50$. Under these parameters, $\hat{W}_{\theta^1\theta^1} - \hat{W}_{\theta^1\theta^2} = 0.122$ and $\tilde{W}''(t^0) = 0.014$. Because the condition of Proposition 2 is satisfied, the uniform pollution tax is suboptimal. Here the suboptimal uniform pollution tax rate is 233.475, and the corresponding social welfare is 1078.585. Table 1 shows deviation from the suboptimal uniform pollution tax rate. In the table, c^i denotes firm i's cost (without tax). From (13) and (15), θ^i in the table denotes the marginal cost of firm i.

Here (t_1, t_2) is chosen to keep the total emission level at 6.226, which is emitted under the suboptimal uniform pollution tax [10].

For example, consider deviation from the symmetric taxation, $t_1 = t_2 = 233.475$. Let us that government alter firm 1's tax rate equals 238.475 (symmetric tax rate plus 5) and adjust firm 2'a tax rate equals 228.259 to keep total emission at the initial level under the symmetric case. Here social welfare is 1078.771, which is larger than the initial level. The reason is as follows. An increase in firm's tax involves an increase in its marginal cost and a decrease in its output, and *vice versa*. The low-cost firm gets a high market share. Then the aggregate costs decrease. Here, firm 1 (respectively, 2) becomes high-cost (respectively, low-cost) firm. The pair of output levels is $(21.389, 23.831)$. Because the low-cost firm produces more, aggregate costs may decrease. Remember that the total emissions are unchanged. When this cost-saving effect dominates reduction of the consumers' surplus, then asymmetry leads to welfare-improving even if total ouput decreases.

[10] As far as profit of firm i is non-negative, a lump sum transfer T_i is arbitrary. For example, $T_i = t_i e^{iN}/2$.

Table 1

The Effects on Deviations from the Uniform Pollution Tax

t_1	t_2	θ^1	θ^2	c^1	c^2	$c^1 + c^2$	W
218.475	246.368	28.964	36.057	754.304	683.279	1437.583	1080.464
223.475	242.637	29.952	34.634	746.103	700.581	1446.684	1079.319
228.475	238.277	30.995	33.337	737.131	714.759	1451.890	1078.756
233.475	233.475	32.123	32.123	726.808	726.808	1453.616	1078.585
238.475	228.259	33.391	30.949	714.197	737.541	1451.738	1078.771
243.475	222.434	34.918	29.743	697.275	747.858	1445.133	1079.500
248.475	214.831	37.202	28.265	667.914	760.061	1427.975	1081.881

From the table, aggregate production costs, $c^1 + c^2$, decreases with an increase in the variance of marginal costs across firms. Asymmetric taxation changes two *ex ante* identical firms into *ex post* high-cost firm and low-cost firm. Hence, it leads to increase the Herfindahl index of concentration. This is the conflict between efficiency and equity.

6 Conclusions

This paper has examined unequal treatment of identical polluters engaged in Cournot competition. We have derived a sufficient condition that guarantees that unequal taxation on equals increases welfare. We have shown that if the sufficient condition is satisfied, unequal standards as well as unequal taxation improve welfare.

A policy implication of this paper is equal treatment of identical polluters is not always optimal. When the condition in the paper is satisfied, the regulator can improve social welfare by unequal treatment of equals. We would like to emphasize the regulator however faces the trade-off between efficiency and equity.

The model can be extended to allow for international oligopolistic competition [11]. In the context of international environmental agreement, such as the Kyoto global warming treaty, unequal treatment of international oligopolistic firms may increase global welfare levels. Consider the following polluting international oligopolies. There are two industries i and j, each industry with two identical firms each located in a different country, A and B. For each industry, output produced by the two firms is consumed in these two countries. Environmental damages caused by pollution are not local or regional , but global. Let us suppose that two governments implement the following environmental agreement: government in country A (respectively, B) imposes a higher pollution tax on firms in industry i (respectively, j) in country A (respectively, B) and imposes a lower pollution tax on firms in industry j (respectively, i) in country A (respectively, B). The above asymmetric taxation may increase the sum of social welfare of two countries than symmetric taxation on two firms in each industry without explicit monetary transfer.

Finally, we present the limitations of our model, along with further extensions. First, our model deals with unequal treatments of pollution taxes and emission standards. What remains to be done is to examine whether unequal treatment of other economic instruments, such as

[11] There has been a large literature on strategic environmental and trade policy ,e.g., Barrett (1994), Kennedy (1994), Ulph and Ulph (1996), Nannerup(2001).

tradable permits and subsidies, increase social welfare. Second, we only examine asymmetric manipulation with keeping total emission constant and , in section 3, with keeping tax rates of firm $3, \cdots, n$ at constant. The future direction of this study will be to get a global optimal solution by means of different class of asymmetric manipulation which is not subject to such constraints.

Appendix A

Proof of Proposition 1. We use some technique following the Appendix of Salant and Shaffer(1999) throughout our Appendices, which yields a simplification of and equivalence of cross derivatives of an objective function.

Applying the implicit function theorem to $\sum_{i=1}^n e^{iN}(t_1, \varphi(t_1), t^0, \cdots, t^0) = \bar{E}$ at $t_1 = t^0$, we get:

$$\varphi'(t^0) = -\frac{\sum_{i=1}^n e_1^{iN}}{\sum_{i=1}^n e_2^{iN}}. \tag{A.1}$$

Notice that $e_1^{1N} = e_2^{2N}, e_2^{1N} = e_1^{2N}, e_1^{iN} = e_2^{iN}, i = 3, \cdots, n$ when $t_1 = t^0$. Then, $\varphi'(t^0) = -1$. Differentiating φ twice with respect to t_1 and evaluating at t^0, we get:

$$\varphi''(t^0) = -\frac{\sum_{j=1}^n (e_{11}^{jN} + e_{12}^{jN}\varphi') \sum_{i=1}^n e_2^{iN} - \sum_{i=1}^n e_1^{iN} \sum_{j=1}^n (e_{21}^{jN} - e_{22}^{jN}\varphi')}{(\sum_{i=1}^n e_2^{iN})^2}. \tag{A.2}$$

Notice that $e_{11}^{1N} = e_{22}^{2N}$, $e_{22}^{1N} = e_{11}^{2N}$, $e_{12}^{1N} = e_{21}^{2N}$, $e_{21}^{1N} = e_{12}^{2N}$, $e_{11}^{jN} = e_{22}^{jN}$, and $e_{12}^{jN} = e_{21}^{jN}$, $j = 3, \cdots, n$ when $t_1 = t^0$. Then,

$$\varphi''(t^0) = -\frac{2\sum_{i=1}^n (e_{11}^{iN} - e_{12}^{iN})}{\sum_{i=1}^n e_2^i}. \tag{A.3}$$

Next, we derive $\tilde{S}''$. Differentiating $\tilde{S}$ with respect to t_1 and evaluating at t^0, we get:

$$\tilde{S}'(t^0) = P(Q^N)\sum_{i=1}^{n}(q_1^{iN} + q_2^{iN}\varphi'). \tag{A.4}$$

Notice that $\varphi' = -1$, $q_1^{1N} = q_2^{2N}$,$q_2^{1N} = q_1^{2N}$, and $q_1^{jN} = q_2^{jN}, j = 3, \cdots, n$ when $t_1 = t^0$. Then $\tilde{S}' = 0$. Differentiating $\tilde{S}$ twice with respect to t_1, we get:

$$\tilde{S}''(t^0) = P'(Q^N)(\sum_{i=1}^{n}(q_1^{iN} + q_2^{iN}\varphi'))^2$$

$$+P(Q^N)\sum_{i=1}^{n}(q_{11}^{iN} + q_{21}^{2N}\varphi' + q_2^{iN}\varphi'' + q_{12}^{iN}\varphi' + q_{22}^{iN}(\varphi')^2). \tag{A.5}$$

Notice that $\varphi' = -1$, $\sum_{i=1}^{n}(q_1^{iN} + q_2^{iN}\varphi') = 0$, $q_{11}^{1N} = q_{22}^{2N}$, $q_{22}^{1N} = q_{11}^{2N}$, $q_{12}^{1N} = q_{21}^{2N}$, $q_{21}^{1N} = q_{12}^{2N}$, $q_{11}^{jN} = q_{22}^{jN}$, and $q_{12}^{jN} = q_{21}^{jN}$, $j = 3, \cdots, n$ when $t_1 = t^0$. Hence,

$$\tilde{S}''(t^0) = 2P(Q^N)\sum_{i=1}^{n}(q_{11}^{iN} - q_{12}^{iN} - q_2^{iN}\frac{\sum_{j=1}^{n}(e_{11}^{jN} - e_{12}^{jN})}{\sum_{j=1}^{n} e_2^{jN}}) \tag{A.6}$$

Next, we derive $\tilde{C}''$. Differentiating $\tilde{C}$ with respect to t_1 and evaluating at t^0, we get:

$$\tilde{C}'(t^0) = \sum_{i=1}^{n}(c_q^i q_1^{iN} + c_q^i q_2^{iN}\varphi' + c_e^i e_1^{iN} + c_e^i e_2^{iN}\varphi'). \tag{A.7}$$

Notice that $\varphi' = -1$, $q_1^{1N} = q_2^{2N}$,$q_2^{1N} = q_1^{2N}$, $q_1^{jN} = q_2^{jN}, j = 3, \cdots, n$ and $c_q^1 = c_q^2 = \cdots = c_q^n$, $c_e^1 = c_e^2 = \cdots = c_e^n$, when $t_1 = t^0$. Then $\tilde{C}'(t^0) = 0$. Note that $\varphi' = -1$, and $q_1^{iN} + q_2^{iN}\varphi' = e_1^{iN} + e_2^{iN}\varphi' = 0$. Differentiating $\tilde{C}$ twice with respect to t_1 and evaluating at t^0, we get:

$$\begin{aligned}
\tilde{C}''(t^0) &= \sum_{i=1}^{n}\Bigg(c_q^i(q_{11}^{iN} - q_{12}^{iN}) - c_q^i(q_{21}^{iN} - q_{22}^{iN}) + c_q^i q_2^{iN}\varphi'' \\
&\quad + c_e^i(e_{11}^{iN} - e_{12}^{iN}) - c_e^i(e_{21}^{iN} - e_{22}^{iN}) + c_e^i e_2^{iN}\varphi''\Bigg) \\
&= 2\sum_{i=1}^{n}\Bigg(c_q^i(q_{11}^{iN} - q_{12}^{iN}) + c_e^i(e_{11}^{iN} - e_{12}^{iN}) \\
&\quad -(c_q^i q_2^{iN} + c_e^i e_2^{iN})\frac{\sum_{h=1}^{n}(e_{11}^h - e_{12}^h)}{\sum_{j=1}^{n} e_2^i}\Bigg).
\end{aligned} \tag{A.8}$$

Subtracting (A.6) from (A.9), we obtain (9).

Appendix B

Proof of lemma 1. Here we do not directly differentiate W with respect to t^1, but use the chain rule to derive the derivatives of $\hat{W} = \hat{W}(\theta^1(t_1, t_2(t_1)), \theta^2(t_1, t_2(t_1)))$. Differentiating $\hat{W}$ with respect to t_1, by the chain rule, we get:

$$\frac{d\hat{W}}{dt_1}(\theta^1(t_1, t_2(t_1)), \theta^2(t_1, t_2(t_1))) = \hat{W}_{\theta^1}\frac{d\theta^1}{dt_1} + \hat{W}_{\theta^2}\frac{d\theta^2}{dt_1} \tag{B.1}$$

where $d\theta^i/dt_1 = \partial\theta^i/\partial t_1 + (\partial\theta^i/\partial t_2)(dt_2/dt_1)$. Under symmetric constraint, when t^0 is optimal $e^0 = r/\theta^i(t^0, t^0)$ is optimal emission. Hence, $d\hat{W}(r/e^0, r/e^0)/de^0 = (\hat{W}_{\theta^1} + \hat{W}_{\theta^2})(-r/(e^0)^2)$ must be zero. Because of $-r/(e^0)^2$ is not zero and $\hat{W}_{\theta^1} = \hat{W}_{\theta^2}$ at the symmetric point, we obtain $\hat{W}_{\theta^1} = \hat{W}_{\theta^2} = 0$. Hence (B.1) is zero.

Differentiating $\hat{W}$ twice with respect to t_1, again by using the chain rule, we get:

$$\begin{aligned} &\frac{d^2\hat{W}}{dt_1^2}(\theta^1(t_1, t_2(t_1)), \theta^2(t_1, t_2(t_1))) \\ &= \hat{W}_{\theta^1\theta^1}\left(\frac{d\theta^1}{dt_1}\right)^2 + 2\hat{W}_{\theta^1\theta^2}\frac{d\theta^1}{dt_1}\frac{d\theta^2}{dt_1} \\ &\quad + \hat{W}_{\theta^2\theta^2}\left(\frac{d\theta^2}{dt_1}\right)^2 + \hat{W}_{\theta^1}\frac{d^2\theta^1}{dt_1^2} + \hat{W}_{\theta^2}\frac{d^2\theta^2}{dt_1^2} \end{aligned} \tag{B.2}$$

where the fourth and fifth terms of the right-hand side equal zero respectively by $\hat{W}_{\theta^1} = \hat{W}_{\theta^2} = 0$.

With some manipulation, we obtain:

$$\frac{d^2\hat{W}}{dt_1^2} = \left(\frac{d\theta^1}{dt_1}\right)^2\left(\hat{W}_{\theta^1\theta^1} + 2\frac{\frac{d\theta^2}{dt_1}}{\frac{d\theta^1}{dt_1}}\hat{W}_{\theta^1\theta^2} + \left(\frac{\frac{d\theta^2}{dt_1}}{\frac{d\theta^1}{dt_1}}\right)^2\hat{W}_{\theta^2\theta^2}\right). \tag{B.3}$$

Substituting $e^{iN} = r/\theta^i$ into (20) and applying the implicit function theorem to it evaluated at $t_1 = t^0$, we easily obtain $t_2'(t_0) = -1$. Note that $\theta_1^1 = \theta_2^2$ and $\theta_2^1 = \theta_1^2$ when $t_1 = t^0$. Then $d\theta^i/dt_1 = \theta_1^i - \theta_2^i$. Hence, we obtain:

$$\frac{d\theta^1}{dt_1}\Big|_{t_1=t_2=t^0} = -\frac{d\theta^2}{dt_1}\Big|_{t_1=t_2=t^0} \tag{B.4}$$

Recall that $\hat{W}_{\theta^1\theta^1} = \hat{W}_{\theta^2\theta^2}$ when (θ_1, θ_2) is evaluated at the symmetric point. By (B.3) and (B.4), we obtain:

$$sign\frac{d^2\hat{W}}{dt_1^2}\Big|_{t_1=t_2=t^0} = sign\Big(\hat{W}_{\theta^1\theta^1} - W_{\theta^1\theta^2}\Big)\Big|_{t_1=t_2=t^0}. \tag{B.5}$$

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