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Dynamic Panel Data Model and Moment Generating Function
by Yoshitsugu Kitazawa

Dynamic Panel Data Model and Moment Generating Function*

Yoshitsugu Kitazawa**

Abstract

This paper proposes new sets of moment restrictions for consistently estimating the dynamic panel data model. These sets are derived from solving the moment generating functions of the error term for the dynamic panel data model and have the relevancy with some well-known sets of moment restrictions proposed up to this point in time. To investigate small sample properties for GMM estimators based on these sets, Monte Carlo experiments were conducted. The Monte Carlo experiments show that the GMM estimators based on some of these sets exhibit good small sample properties for some values of the so-called *adjusting parameter*.

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** Correspondence. Postal: Faculty of Economics, Kyushu Sangyo University, 2-3-1 Matsukadai, Higashi-ku, Fukuoka, 813-8503, Japan. Tel.: +81-92-673-5290; Fax: +81-92-673-5919. E-mail: kitazawa@ip.kyusan-u.ac.jp (Yoshitsugu Kitazawa).

1. Introduction

Many moment restrictions have been developed for the consistent estimation of the dynamic panel data model with additive fixed effects, under the assumption that the number of individuals is large and the number of time series observations is small.[1] We can consistently estimate the dynamic panel data model using GMM (Generalized Method of Moments) estimators (see Hansen, 1982) based on the moment restrictions.

Based on the covariance structure of the error component composed of the fixed effect and the disturbance in the dynamic panel data model, some sets of the moment restrictions are constructed as follows: the *standard* moment restrictions proposed by Holtz-Eakin et al. (1988) and Arellano and Bond (1991); the *non-linear* moment restrictions and the *intertemporal homoscedasticity* moment restrictions proposed by Ahn (1990) and Ahn and Schmidt (1995, 1999); the *mean-stationarity* moment restrictions proposed by Arellano and Bover (1995) and discussed in Ahn and Schmidt (1995) and Blundell and Bond (1998, 2000).[2]

Further, allowing for assumptions generating the covariance structure and assuming the normal disturbance, Kitazawa (2001) proposes the moment restrictions based on the exponential transformations, one of which is one type of the quasi-differencing transformations proposed by Wooldridge (1997) and Chamberlain (1992).

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[1] Anderson and Hsiao (1982) take the initiative in the development.

[2] Other than these sets of the moment restrictions, Crepon et al. (1997) suggest one more set of the moment restrictions assuming that the mean of the individual heterogeneities is zero.

In this paper, by expanding on Kitazawa's (2001) idea, some new types of moment restrictions are proposed to consistently estimate the dynamic panel data model. In this paper, we show that there are numerous sets of the moment restrictions for consistently estimating the dynamic panel data model and they are deeply related with both the *standard* moment restrictions and the *intertemporal homoscedasticity* moment restrictions.

In this paper, we also investigate small sample properties for GMM estimators based on some of the new types of the moment restrictions. GMM estimators based on the *standard* moment restrictions frequently used in empirical studies suffer from small sample biases for persistent autoregressive parameters. To overcome the small sample biases, we often use GMM estimators incorporating the *mean-stationarity* moment restrictions. It is intriguing whether GMM estimators based on the new types of the moment restrictions improve the small sample properties, too.

The rest of the paper is organized as follows: In Section 2, a simple dynamic panel data model is presented, some of the moment restrictions proposed up to now are given, the new types of moment restrictions are shown, and the GMM estimators are defined. In Section 3, Monte Carlo experiments are carried out to investigate the small sample properties for the new types of moment restrictions. Section 4 is the conclusion section of the paper.

2. Model, moment restrictions, and estimators

In this section, we present a simple dynamic panel data model, sets of moment restrictions required for estimating the model consistently, and GMM estimators using the sets of the moment restrictions. Central to this section is that the new types of moment restrictions are proposed and we show the relationship between the new types of the moment restrictions and the sets of the moment restrictions proposed up to now in the literature on the dynamic panel data model.

2.1. A formulation of the simple dynamic panel data model

We consider the case where the number of individuals (N) and the number of time series observations (T) are large and small, respectively. Then we discuss the following AR(1) dynamic panel data model based on the assumption that $N \rightarrow \infty$ and T is fixed:

$$y_{it} = \alpha y_{i,t-1} + u_{it}, \tag{1}$$

$$u_{it} = \eta_i + \sigma_i \varepsilon_{it}, \tag{2}$$

and

$$\varepsilon_{it} \sim iidN(0,1). \tag{3}$$

The dynamic panel data model composed of (1), (2), and (3) is for $i = 1, \ldots, N$ and $t = 2, \ldots, T$. In this model, y_{it} is the dependent variable and α is the parameter of interest (to be estimated). The error term u_{it} is composed of the additive fixed effect η_i and the disturbance $\sigma_i \varepsilon_{it}$, where ε_{it} is independent of η_i, σ_i, and y_{i1}. A (positive) multiplicative fixed effect σ_i implies the *intertemporal homoscedasticity* on the disturbance for the individual i.

2.2. *Standard* and *intertemporal homoscedasticity* moment restrictions

In the model setup above, the *standard* moment restrictions proposed by Holtz-Eakin et al. (1988) and Arellano and Bond (1991) are constructed for estimating α consistently:

$$E[y_{is}\,\Delta u_{it}] = 0\,,\quad \text{for}\ \ s = 1,\ldots,t-2\ \ \text{and}\ \ t = 3,\ldots,T\,, \qquad (4)$$

where Δ is the first-differencing operator and we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ into (4). The number of the *standard* moment restrictions is $(T-2)(T-1)/2$. The moment restrictions (4) are frequently used in empirical econometric analyses, due to the property that they are linear in α. In addition, in the model setup above, the additional *intertemporal homoscedasticity* moment restrictions proposed by Ahn (1990) and Ahn and Schmidt (1995, 1999) are constructed for estimating α consistently:

$$E[u_{it}^2 - u_{i,t-1}^2] = 0\,,\qquad \text{for}\ \ t = 3,\ldots,T\,, \qquad (5)$$

where we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ and $u_{i,t-1} = y_{i,t-1} - \alpha y_{i,t-2}$ into (5). The number of the *intertemporal homoscedasticity* moment restrictions is $T-2$. The two types of the moment restrictions are obtained from the covariance structure on u_{it}.

2.3. Moment restrictions based on the exponential regression

Looking at the model setup above from a different angle, Kitazawa (2001) proposes the following conditional moment restrictions to estimate α consistently:

$$E[\ \exp(\ u_{it}\ ) - \exp(\ u_{i,t-1}\ )\ \mid\ y_i^{t-2}\ ] = 0\,,\qquad \text{for}\ \ t = 3,\ldots,T\,, \quad (6)$$

where y_i^{t-2} is $(y_{i1},\ldots,y_{i,t-2})$. These are the conditional moment restrictions based on a quasi-differencing transformation proposed by Wooldridge (1997) and Chamberlain (1992). Kitazawa's (2001) paper shows that we can obtain the conditional moment restrictions on the quasi-differencing transformation to estimate the parameter of interest consistently for the dynamic panel data model as well as for the count panel data model. From the conditional moment restrictions (6), we can obtain the following $(T-2)(T-1)/2$ unconditional moment restrictions for estimating α consistently:

$$E[\, y_{is}\, \{\exp(\, u_{it}\,) - \exp(\, u_{i,t-1}\,)\,\}\,] = 0\,,$$

$$\text{for } \; s = 1,\ldots,t-2 \; \text{ and } \; t = 3,\ldots,T\,, \qquad (7)$$

where we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ and $u_{i,t-1} = y_{i,t-1} - \alpha y_{i,t-2}$ into (7).

2.4. New types of the moment restrictions proposed in the paper

For the error term u_{it}, we solve the moment generating functions conditional on the information $(y_i^{t-1}, \eta_i, \sigma_i)$, allowing for the structure of u_{it}:

$$E[\, \exp(\theta u_{it}) \mid y_i^{t-1}, \eta_i, \sigma_i \,] = \exp(\, \theta \eta_i + \theta^2 \sigma_i^2 / 2)\,,$$

$$\text{for } \; t = 2,\ldots,T\,, \qquad (8)$$

where θ is any real number.[3]

Applying the laws of iterated expectations to the first-difference of (8), we can obtain the following conditional moment restrictions to estimate α consistently for any θ:

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[3] The moment generating functions hold for any real number θ when assuming the normal distribution on v_{it} as is given in (3). However, the moment generating functions do not necessarily hold for any real number θ when assuming other distributions.

$$E[\ \exp(\theta u_{it}) - \exp(\theta u_{i,t-1}) \mid y_i^{t-2}\ ] = 0, \quad \text{for } t = 3,\ldots,T. \quad (9)$$

We can recognize that the moment restrictions (6) are a special case of the moment restrictions (9) when $\theta = 1$. We can construct from (9) the following $(T-2)(T-1)/2$ unconditional moment restrictions:

$$E[\ y_{is}\{\exp(\theta u_{it}) - \exp(\theta u_{i,t-1})\}] = 0,$$

$$\text{for } s = 1,\ldots,t-2 \text{ and } t = 3,\ldots,T, \quad (10)$$

where we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ and $u_{i,t-1} = y_{i,t-1} - \alpha y_{i,t-2}$ into (10).

After first-differentiating the conditional moment restrictions (9) with respect to θ, we obtain the relationship that the first derivative of (9) equals zero:

$$E[\ u_{it}\exp(\theta u_{it}) - u_{i,t-1}\exp(\theta u_{i,t-1}) \mid y_i^{t-2}\ ] = 0,$$

$$\text{for } t = 3,\ldots,T.^{4} \quad (11)$$

We can construct from (11) the following $(T-2)(T-1)/2$ unconditional moment restrictions for estimating α consistently:

$$E[\ y_{is}\{u_{it}\exp(\theta u_{it}) - u_{i,t-1}\exp(\theta u_{i,t-1})\}] = 0,$$

$$\text{for } s = 1,\ldots,t-2 \text{ and } t = 3,\ldots,T, \quad (12)$$

where we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ and $u_{i,t-1} = y_{i,t-1} - \alpha y_{i,t-2}$ into (12). When we set $\theta = 0$, the unconditional moment restrictions (12) are equivalent to the *standard* moment restrictions (4). It can be clearly seen that the *standard* moment restrictions (4) are a special case of the moment restrictions (12).

After first-differentiating the conditional moment restrictions (11) with respect to θ (that is to say, after second-differentiating the conditional moment restrictions (9)

[4] See page 460 in Davidson (2000) and page 154 in Davidson (1994).

with respect to θ), we obtain the relationship that the second derivative of (9) equals zero:

$$E[\, u_{it}^2 \exp(\theta u_{it}) - u_{i,t-1}^2 \exp(\theta u_{i,t-1}) \mid y_i^{t-2} \,] = 0,$$

$$\text{for } t = 3, \ldots, T. \qquad (13)$$

From (13), we can obtain the following $(T-2)(T-1)/2$ unconditional moment restrictions for estimating α consistently:

$$E[\, y_{is} \{u_{it}^2 \exp(\theta u_{it}) - u_{i,t-1}^2 \exp(\theta u_{i,t-1})\}] = 0,$$

$$\text{for } s = 1, \ldots, t-2 \text{ and } t = 3, \ldots, T, \qquad (14)$$

where we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ and $u_{i,t-1} = y_{i,t-1} - \alpha y_{i,t-2}$ into (14). In addition, applying the laws of iterated expectations to (13) and then setting $\theta = 0$, we can generate the *intertemporal homoscedasticity* moment restrictions (5).

Further, after n th-differentiating the conditional moment restrictions (9) with respect to θ where n is any integer with $n > 0$, we obtain the relationship that the n th derivative of (9) equals zero:

$$E[\, u_{it}^n \exp(\theta u_{it}) - u_{i,t-1}^n \exp(\theta u_{i,t-1}) \mid y_i^{t-2} \,] = 0,$$

$$\text{for } t = 3, \ldots, T. \qquad (15)$$

When n equals zero, the conditional moment restrictions (15) are equivalent to (9). Accordingly, (15) is generalized for any integer value n with $n \geq 0$.[5] From (15), we can obtain the following $(T-2)(T-1)/2$ unconditional moment restrictions for estimating α consistently:

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[5] We take a standpoint that $0^0 = 1$, allowing for the case that $u_{it} = 0$ for some i and t. If we do not take this standpoint, the generalization of (9) into (15) fails.

$$E[\, y_{is}\, \{u_{it}^{n} \exp(\theta u_{it}) - u_{i,t-1}^{n} \exp(\theta u_{i,t-1})\}\,] = 0\,,$$

$$\text{for } s = 1, \ldots, t-2 \text{ and } t = 3, \ldots, T\,, \qquad (16)$$

where we substitute $u_{it} = y_{it} - \alpha y_{i,t-1}$ and $u_{i,t-1} = y_{i,t-1} - \alpha y_{i,t-2}$ into (16). The unconditional moment restrictions (16) are non-linear for α, except for two cases: the first is where $n = 1$ and $\theta = 0$, and the second is where $n = 0$ and $\theta = 0$.[6]

Considering the moment generating functions for the dynamic panel data model, we find that there are numerous sets of the conditional moment restrictions to estimate the parameter of interest consistently, such as (15).

We will call θ the *adjusting parameter*. The next section reveals the reason for this name. It seems to be considered that by changing the values of θ, we can adjust the fitness of the moment restrictions for the data in the small sample.

2.5. GMM estimators

From now on, in this study, the GMM estimators for α will be defined using the unconditional moment restrictions (16) for n (zero or any positive integer value) and θ (any real number except for the case that $\theta = 0$ when $n = 0$).

To begin with, the unconditional moment restrictions (16) are rewritten in the following $[(T-2)(T-1)/2] \times 1$ vector form:

$$E[\phi_i(n, \theta, \alpha)] = E[Z_i' \,\xi_i(n, \theta, \alpha)] = 0\,, \qquad (17)$$

where defined are $(T-2) \times 1$ error term vector

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[6] The former exceptional case corresponds to the case that the unconditional moment restrictions (16) are linear for α and coincide with the *standard* moment restrictions (4). The latter exceptional case is the case that the unconditional moment restrictions (16) do not hold to consistently estimate α.

$$\xi_i(n, \theta, \alpha) = \begin{bmatrix} u_{i3}^n \exp(\theta u_{i3}) - u_{i2}^n \exp(\theta u_{i2}) \\ u_{i4}^n \exp(\theta u_{i4}) - u_{i3}^n \exp(\theta u_{i3}) \\ \vdots \\ u_{iT}^n \exp(\theta u_{iT}) - u_{i,T-1}^n \exp(\theta u_{i,T-1}) \end{bmatrix}, \tag{18}$$

into which $u_{it} = y_{it} - \alpha y_{i,t-1}$ for $t = 2,\ldots,T$ are substituted, and $(T-2)\times[(T-2)(T-1)/2]$ instrumental matrix

$$Z_i = \begin{bmatrix} y_{i1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & 0 & \cdots & 0 \\ . & . & . & \cdots & . & . & \cdots & . \\ 0 & 0 & 0 & \cdots & y_{i1} & y_{i2} & \cdots & y_{i,T-2} \end{bmatrix}. \tag{19}$$

The empirical counterpart for (17) is written as follows:

$$\Phi(n, \theta, \alpha) = (1/N)\sum_{i=1}^{N} \phi_i(n, \theta, \alpha). \tag{20}$$

The following quadratic form for solving the GMM estimator of α is defined based on the empirical counterpart (20) and any $[(T-2)(T-1)/2]\times[(T-2)(T-1)/2]$ positive definite weighting matrix A_N (which is symmetric):

$$N\,\Phi(n, \theta, \alpha)' A_N\, \Phi(n, \theta, \alpha). \tag{21}$$

We minimize the quadratic form (21) with respect to α so that we can obtain any GMM estimator of α for both any real number θ and any integer value n with $n \geq 0$ ruling out the exceptional case.

For the number of individuals N, an optimal selection of the weighting matrix A_N for both any real number θ and any integer value n satisfying $n \geq 0$ is

$$A_N^* = \left[(1/N)\sum_{i=1}^{N} E[\phi_i(n, \theta, \alpha^*)\phi_i(n, \theta, \alpha^*)']\right]^{-1}, \tag{22}$$

where α^* is the true value of the parameter α. The optimal selection generates the efficient GMM estimator $\hat{\alpha}$, in the sense of the asymptotic. However, since we cannot know A_N^*, we use the consistent estimate for A_N^* instead of A_N^*. The weighting matrix to be consistently estimated for A_N^* is

$$\widetilde{A}_N = \left[(1/N)\sum_{i=1}^{N}\phi_i(n,\,\theta,\,\widetilde{\alpha})\phi_i(n,\,\theta,\,\widetilde{\alpha})'\right]^{-1}, \tag{23}$$

where $\widetilde{\alpha}$ is an arbitrary consistent estimate for α.

That is to say, for both any real number θ and any integer value n satisfying $n \geq 0$, we can obtain the asymptotically efficient GMM estimator $\hat{\alpha}$ by minimizing (21) with respect to α when $A_N = \widetilde{A}_N$

In large samples, $\hat{\alpha}$ is approximately distributed as normal with

$$E[\hat{\alpha}] = \alpha^* \tag{24}$$

and

$$Var[\hat{\alpha}] = \left[\ (\nabla\Phi(n,\,\theta,\,\alpha^*))'\,A_N^*\,(\nabla\Phi(n,\,\theta,\,\alpha^*))\ \right]^{-1}, \tag{25}$$

where $\nabla\Phi(n,\,\theta,\,\alpha^*) = \{\partial\Phi(n,\,\theta,\,\alpha)/\partial\alpha\}\,|_{\alpha=\alpha^*} = E[\{\partial\phi_i(n,\,\theta,\,\alpha)/\partial\alpha\}\,|_{\alpha=\alpha^*}]$. The estimated variance for the estimated $\hat{\alpha}$ is calculated, both replacing α^* with the estimated $\hat{\alpha}$ and replacing A_N^* with $\widetilde{A}_N$ in (25).

The *Sargan* test statistic (the test of over-identifying restrictions) for the estimated $\hat{\alpha}$ with the aim of testing the validity of the moment restrictions is calculated, both replacing α with the estimated $\hat{\alpha}$ and replacing A_N with $\widetilde{A}_N$ in (21).

3. Monte Carlo experiments

It is a great interest to investigate the small sample properties on the GMM estimators for α based on the unconditional moment restrictions (16) derived from the moment generating functions (8). We conduct Monte Carlo experiments for some of the GMM estimators using a data generating process (DGP), under some settings of parameters in the Monte Carlo experiments. The experiments are implemented with econometric software TSP 4.5.

3.1. DGP

The DGP for the experiments is as follows:

$$y_{i1} = \frac{\eta_i}{1-\alpha} + \frac{v_{i1}}{\sqrt{1-\alpha^2}},$$

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it}, \qquad \text{for } t = 2, \ldots, T,$$

$$\eta_i \sim iidN(0, \sigma_\eta^2),$$

and

$$v_{it} \sim iidN(0, \sigma_v^2), \qquad \text{for } t = 1, \ldots, T,$$

where η_i and v_{it} are independent of each other and $i = 1, \ldots, N$. In the DGP, T is considerably small and N is moderately large. This type of DGP is used in Arellano and Bover (1995) and Blundell and Bond (1998). It is known that when applying the GMM estimators for α using the *standard* moment restrictions (4) to the data set from this type of DGP, serious downward biases and imprecision for the GMM estimators can appear especially when persistent values of $\alpha*$ and (or) high values of $\sigma_\eta^2 / \sigma_v^2$ are set.[7]

3.2. GMM estimators to be investigated

For some values of the *adjusting parameter* θ, we investigate the GMM estimators using the unconditional moment restrictions (16) in the previous section for the three cases: $n = 0, 1, 2$. Hereafter, we call the GMM estimator for $n = 0$ as **GMM(MGF[0]-FD)**, the GMM estimator for $n = 1$ as **GMM(MGF[1]-FD)**, and the GMM estimator for $n = 2$ as **GMM(MGF[2]-FD)**.[8]

The error term used in **GMM(MGF[0]-FD)** is

$$\xi_i(0, \theta, \alpha) = \begin{bmatrix} \exp(\theta u_{i3}) - \exp(\theta u_{i2}) \\ \exp(\theta u_{i4}) - \exp(\theta u_{i3}) \\ \vdots \\ \exp(\theta u_{iT}) - \exp(\theta u_{i,T-1}) \end{bmatrix},$$

the error term vector used in GMM(MGF[1]-FD) is

$$\xi_i(1, \theta, \alpha) = \begin{bmatrix} u_{i3}\exp(\theta u_{i3}) - u_{i2}\exp(\theta u_{i2}) \\ u_{i4}\exp(\theta u_{i4}) - u_{i3}\exp(\theta u_{i3}) \\ \vdots \\ u_{iT}\exp(\theta u_{iT}) - u_{i,T-1}\exp(\theta u_{i,T-1}) \end{bmatrix},$$

and the error term vector used in GMM(MGF[2]-FD) is

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[7] With the aim of overcoming the downward biases and imprecision when using the *standard* moment restrictions, the *mean-stationarity* moment restrictions are used.

[8] **GMM(MGF[0]-FD)**, **GMM(MGF[1]-FD)**, and **GMM(MGF[2]-FD)** are named after the GMM estimators using the moment restrictions (16) originating from the moment generating functions when $n = 0, 1, 2$, which are the first-differenced forms. According to the expression with equation number, the unconditional moment restrictions used for **GMM(MGF[0]-FD)**, **GMM(MGF[1]-FD)**, and **GMM(MGF[2]-FD)** are (10), (12), and (14), respectively.

$$\xi_i(2,\theta,\alpha)=\begin{bmatrix} u_{i3}^2\exp(\theta u_{i3})-u_{i2}^2\exp(\theta u_{i2}) \\ u_{i4}^2\exp(\theta u_{i4})-u_{i3}^2\exp(\theta u_{i3}) \\ \vdots \\ u_{iT}^2\exp(\theta u_{iT})-u_{i,T-1}^2\exp(\theta u_{i,T-1}) \end{bmatrix},$$

where $u_{it}=y_{it}-\alpha y_{i,t-1}$ for $t=2,\ldots,T$ are substituted into each of the error terms.

The initial consistent estimate $\tilde{\alpha}$ must be obtained to construct $\tilde{A}_N$ in (23). In the experiments, $\tilde{\alpha}$ is obtained by minimizing the quadratic form (21) with respect to α after selecting a positive definite weighting matrix $\left[\sum_{i=1}^{N} Z_i' H Z_i\right]^{-1}$ for A_N, where H is a $(T-2)$ symmetric matrix with 2 on the main diagonal, -1 on the first sub-diagonal, and 0 elsewhere.[9] The starting value of α in the minimization to obtain $\tilde{\alpha}$ is the two-step estimate in Arellano and Bond (1991) using the moment restrictions (4).

After substituting $A_N=\tilde{A}_N$ into the quadratic form (21), we perform a non-linear minimization of (21) with respect to α by setting the starting value to $\tilde{\alpha}$, so that we obtain the efficient GMM estimate for $\hat{\alpha}$.

3.3. Parameter settings

In the experiments, the combination of sample sizes is $N=100$ and $T=7$. The number of replications (NR) is 500. The true values of α (i.e. α^*) to be examined are 0.2, 0.5, and 0.8. The value combinations of (σ_η^2, σ_v^2) to be examined are (0.25, 1), (1, 1),

[9] This weighting matrix is asymptotically efficient for the case that $n=1$ and $\theta=0$. That is to say, this weighting matrix is the weighting matrix used for the one-step estimator in Arellano and Bond (1991). For other n and θ, this weighting matrix is not asymptotically efficient.

and (4,1), which in order correspond to the cases of $\sigma_\eta^2/\sigma_v^2 = 0.25$, $\sigma_\eta^2/\sigma_v^2 = 1$, and $\sigma_\eta^2/\sigma_v^2 = 4$. The above three types of GMM estimators are investigated, in which the values of the *adjusting parameter* θ vary from –1.10 to 1.10 at the intervals of 0.01.

3.4. Results

Monte Carlo results are reported in **[Exhibit 0-0.25-0.2]** to **[Exhibit 0-4-0.8]** for **GMM(MGF[0]-FD)**, **[Exhibit 1-0.25-0.2]** to **[Exhibit 1-4-0.8]** for **GMM(MGF[1]-FD)**, and **[Exhibit 2-0.25-0.2]** to **[Exhibit 2-4-0.8]** for **GMM(MGF[2]-FD)**, starting from the explanations on the exhibits in **Exhibits of the Monte Carlo results**. Each of the exhibits consists of four line graphs and one table.

In each exhibit, the graph located on the left upper part shows the change of the Monte Carlo mean of the estimated $\hat{\alpha}$ associated with the change of θ, denoted with **MEAN**. The graph located on the right upper part shows the changes of the Monte Carlo standard deviation of the estimated $\hat{\alpha}$ and the Monte Carlo mean of the estimated standard errors for $\hat{\alpha}$ associated with the change of θ, denoted with **SD** and **SE** respectively. The graph located on the left lower part shows the change of the Monte Carlo mean of the *Sargan* test statistics associated with the change of θ, denoted with **MEAN(SARGAN)**. The graph located on the right lower part shows the change of the Monte Carlo standard deviation of the *Sargan* test statistics associated with the change of θ, denoted with **SD(SARGAN)**.[10] In each of the graphs, the horizontal axis represents the value of θ and the vertical axis represents the value of

[10] In this paper, we depict the graph for **SD(SARGAN)** in each exhibit, without examining the movement of **SD(SARGAN)**. This graph is a reference data.

the Monte Carlo statistics explained above.

In addition, the table located on the bottom part shows the Monte Carlo result using Arellano and Bond's (1991) one-step estimator, the Monte Carlo result using Arellano and Bond's (1991) two-step estimator, the Monte Carlo result in which **MEAN** is nearest to the true value $\alpha*$ for negative θ, and the Monte Carlo result in which **MEAN(SARGAN)** is the local minimum for negative θ. The results are indexed with **A-B(ONE-STEP)**, **A-B(TWO-STEP)**, **NEREST**, and **MINIMUM**, respectively.

(a) Results for **GMM(MGF[0]-FD)**

[Exhibit 0-0.25-0.2] to **[Exhibit 0-4-0.8]** represent the Monte Carlo results using **GMM(MGF[0]-FD)** estimators. We find some interesting evidences.

When θ varies from –1.10 to 1.10, the Monte Carlo statistics change. Note that since we cannot define **GMM(MGF[0]-FD)** estimator at $\theta = 0$, instead we use the statistics for A-B(TWO-STEP) at $\theta = 0$ in the graphs.

Looking at the graphs for **MEAN**, it is found that the Monte Carlo means of the estimated $\hat{\alpha}$ (**MEAN**) increase as the absolute value of θ increases. The graphs for **MEAN** are almost symmetric to the vertical axis for $\theta = 0$. In each $\alpha*$, the values of **MEAN** for some values of θ are located in the vicinity of $\alpha*$ in both cases of negative and positive θ. The larger $\sigma_\eta^2 / \sigma_v^2$ and (or) $\alpha*$, the smaller the absolute values of θ for which the values of **MEAN** are located in the vicinity of $\alpha*$ would be.

For the high value of $\alpha*$ (viz. $\alpha* = 0.8$), the values of **SD** for the absolute values of θ for which the values of **MEAN** are located in the vicinity of $\alpha*$ are considerably smaller than the values of **SD** for **A-B(ONE-STEP)**, **A-B(TWO-STEP)** and differences between the values of **SE** and the values of **SD** for these absolute values of

θ are much smaller than the differences for **A-B(ONE-STEP)**, **A-B(TWO-STEP)**, as is seen from the graphs and tables (i.e. compare **A-B(ONE-STEP)** and **A-B(TWO-STEP)** with **NEREST**) in the exhibits for the case of $\alpha^* = 0.8$.

The graphs of **MEAN(SARGAN)** have two local minimum values for both cases of negative and positive θ, as is read off the graphs for **MEAN(SARGAN)**.[11] In addition, roughly speaking, the values of θ for which the values of **MEAN** are located in the vicinity of α^* almost coincide with the values of θ for which the graphs of **MEAN(SARGAN)** have the minimum values, as is seen from the graphs and tables (i.e. compare **NEREST** and **MINIMUM**) in the exhibits.[12]

It is recognized from the Monte Carlo experiments that a considerable improvement of the small sample properties for **GMM(MGF[0]-FD)** estimators at some values of θ are conducted compared to the small sample properties for **A-B(ONE-STEP)**, and **A-B(TWO-STEP)** estimators. The improvement is dramatic in the case of large $\sigma_\eta^2 / \sigma_v^2$ and (or) persistent α^*.

(b) Results for **GMM(MGF[1]-FD)**

[Exhibit 1-0.25-0.2] to **[Exhibit 1-4-0.8]** represent the Monte Carlo results using the **GMM(MGF[1]-FD)** estimators. We find some interesting evidences, which resemble the evidences for **GMM(MGF[0]-FD)**.

When θ varies from –1.10 to 1.10, the Monte Carlo statistics change. Note

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[11] However, the local minimum values for positive θ are nebulous when α^* is small (i.e. $\alpha^* = 0.2$).

[12] However, the gap between the former values of θ and the latter values of θ seems to diminish as $\sigma_\eta^2 / \sigma_v^2$ and (or) α^* increase(s).

that **GMM(MGF[1]-FD)** estimator at $\theta = 0$ coincides with **A-B(TWO-STEP)** estimator.

The movements of values of **MEAN**, **SD**, **SE**, and **MEAN(SARGAN)** for **GMM(MGF[1]-FD)** estimators are considerably similar to the movements for **GMM(MGF[0]-FD)**, as is read off the graphs and the tables in the exhibits. However, for the identical combinations of $(\sigma_\eta^2/\sigma_v^2, \alpha^*)$, the absolute values of θ for which the values of **MEAN** are located in the vicinity of α^* for **GMM(MGF[1]-FD)** are small compared to the absolute values of θ for which the values of **MEAN** are located in the vicinity of α^* for **GMM(MGF[0]-FD)**.

As the same as the case of **GMM(MGF[0]-FD)** estimators, it is recognized from the Monte Carlo experiments that a considerable improvement of the small sample properties for **GMM(MGF[1]-FD)** estimators at some values of θ are conducted compared to the small sample properties for **A-B(ONE-STEP)**, and **A-B(TWO-STEP)** estimators. The improvement is also dramatic in the case of large σ_η^2/σ_v^2 and (or) persistent α^*.

(c) Results for **GMM(MGF[2]-FD)**

[Exhibit 2-0.25-0.2] to **[Exhibit 2-4-0.8]** represent the Monte Carlo results using the **GMM(MGF[2]-FD)** estimators.

When θ varies from –1.10 to 1.10, the Monte Carlo statistics change.

As is seen from the graphs for **MEAN** in the exhibits, the values of **MEAN** are markedly upward-biased toward unity for the all values of θ. The upward-biases (i.e. the gap between MEAN and α^*) expand as σ_η^2/σ_v^2 is large. In addition, the graphs for MEAN have two local minimum values, which are however considerably apart from

$\alpha*$.

The values of **SD** for **GMM(MGF[2]-FD)** are considerably small in the case of persistent $\alpha*$ (and large σ_η^2/σ_v^2) compared to **A-B(ONE-STEP)** and **A-B(TWO-STEP)** estimators. In addition, for some values of θ, the gap between the values of **SD** and **SE** is minimized. However, we cannot enjoy these merits due to the upward biases of the values of **MEAN**.

The graphs of **MEAN(SARGAN)** have one local minimum value when the value of θ is extremely close to zero, as is read off the graphs for **MEAN(SARGAN)**. This shape is different from the shapes for **GMM(MGF[0]-FD)** and **GMM(MGF[1]-FD)** estimators.

Different from both **GMM(MGF[0]-FD)** and **GMM(MGF[1]-FD)** estimators, we never say that **GMM(MGF[2]-FD)** estimators improve the small sample properties compared to **A-B(ONE-STEP)**, and **A-B(TWO-STEP)** estimators.

Finally, the Monte Carlo experiments provide an indication that **GMM(MGF[0]-FD)** and **GMM(MGF[1]-FD)** estimators for some values of the *adjusting parameter* θ would behave much better in the small sample than **A-B(ONE-STEP)** and **A-B(TWO-STEP)** estimators, but **GMM(MGF[2]-FD)** estimators would behave much worse. The GMM estimators using the moment restrictions (16) have good small sample properties for some appropriate values of the *adjusting parameter* θ when $n = 0$ and $n = 1$.

4. Conclusion

In this paper, new types of conditional moment restrictions (15) (and new types of unconditional moment restrictions (16)) were presented for consistently estimating the dynamic panel data model. The moment restrictions (15) are derived from the moment generating functions conditional on some of information sets and are found to generate the *standard* moment restrictions and the *intertemporal homoscedasticity* moment restrictions. It was announced that numerous sets of the moment restrictions are defined to consistently estimate the dynamic panel data model. In addition, some Monte Carlo experiments were conducted to investigate the small sample properties in the framework of GMM estimators using the unconditional moment restrictions (16). The Monte Carlo results indicate that the GMM estimators have the considerably good small sample properties for some values of the *adjusting parameter* θ, by using the unconditional moment restrictions (16) for the case that $n = 0$ and $n = 1$.

The future works to be conducted on the unconditional moment restrictions (16) are to elucidate the reason why the good small sample properties are attained and to implement the empirical study using them.

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Exhibits of the Monte Carlo results

We present the Monte Carlo results for some of the GMM estimators in the paper. The explanations on the exhibits for the Monte Carlo results are conducted and then the exhibits are displayed.

Explanation 1. The exhibits corresponding to each of the GMM estimators are as follows:

[Exhibit 0-0.25-0.2] to [Exhibit 0-4-0.8]: for GMM(MGF[0]-FD).

[Exhibit 1-0.25-0.2] to [Exhibit 1-4-0.8]: for GMM(MGF[1]-FD).

[Exhibit 2-0.25-0.2] to [Exhibit 2-4-0.8]: for GMM(MGF[2]-FD).

Explanation 2. The Monte Carlo settings common in all the exhibits are as follows:

Number of individuals: $N = 100$.

Number of time series observations: $T = 7$.

Number of replications: $NR = 500$.

Explanation 3. The Monte Carlo statistics shown in the exhibits are as follows:

MEAN: Monte Carlo mean of the estimated $\hat{\alpha}$.

SD: Monte Carlo standard deviation of the estimated $\hat{\alpha}$.

SE: Monte Carlo mean of the estimated standard errors for $\hat{\alpha}$.

MEAN(SARGAN): Monte Carlo mean of the *Sargan* test statistics.

SD(SARGAN): Monte Carlo standard deviation of the *Sargan* test statistics.

DF: Degree of freedom.

Explanation 4. The Monte Carlo results for some of the GMM estimators shown in the tables in the exhibits are as follows:

A-B(ONE-STEP): Monte Carlo result using the one-step GMM estimator in Arellano and Bond (1991). In this case, both $\widetilde{A}_N$ for the estimated standard error for $\hat{\alpha}$ and the *Sargan* test statistic are calculated with the estimated $\hat{\alpha}$.

A-B(TWO-STEP): Monte Carlo result using the two-step GMM estimator in Arellano and Bond (1991).

NEAREST: Monte Carlo result in which MEAN is nearest to the true value for the negative θ in the graph of MEAN.

MINIMUM: Monte Carlo result in which MEAN(SARGAN) is a local minimum value for the negative θ in the graph of MEAN(SARGAN).

Explanation 5. Some of the replications in each Monte Carlo experiment are not accomplished because the estimates of $\hat{\alpha}$ do not converge. Minimums in rates of number of the convergences (NC) to number of replications (NR) are $NC/NR = 483/500$ for GMM(MGF[0]-FD), $NC/NR = 471/500$ for GMM(MGF[1]-FD), and $NC/NR = 469/500$ for GMM(MGF[2]-FD), respectively.

[Exhibit 0-0.25-0.2]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators $(n = 0)$

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (0.25, 0.2)$ when the value of θ changes.

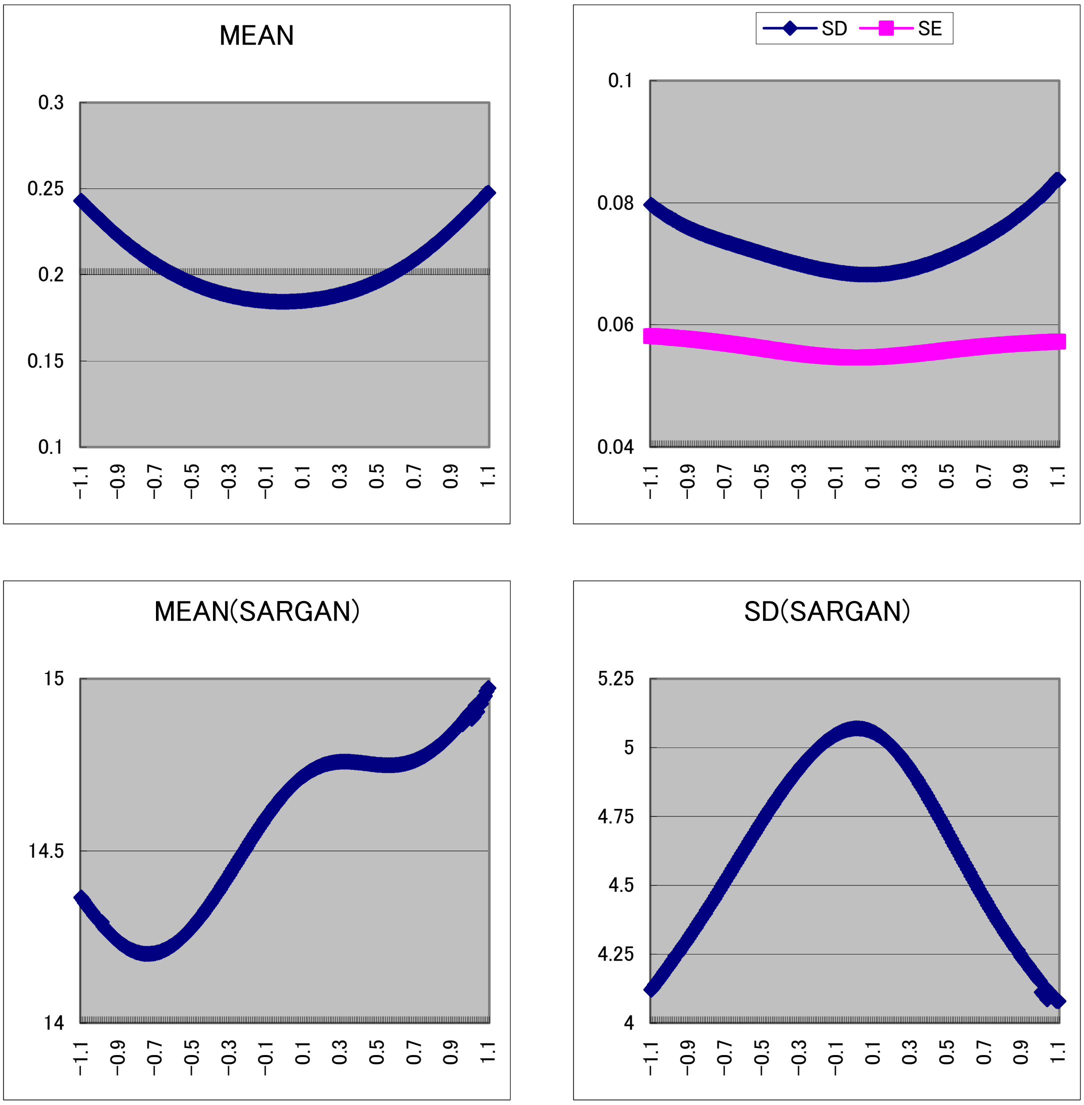


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.60	-0.74
MEAN	0.184	0.184	0.200	0.209
[SD, SE]	[0.064, 0.055]	[0.068, 0.055]	[0.073, 0.057]	[0.074, 0.057]
MEAN(SARGAN)	14.904	14.667	14.228	14.201
SD(SARGAN)	5.147	5.069	4.626	4.473

[Exhibit 0-0.25-0.5]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n = 0$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (0.25, 0.5)$ when the value of θ changes.

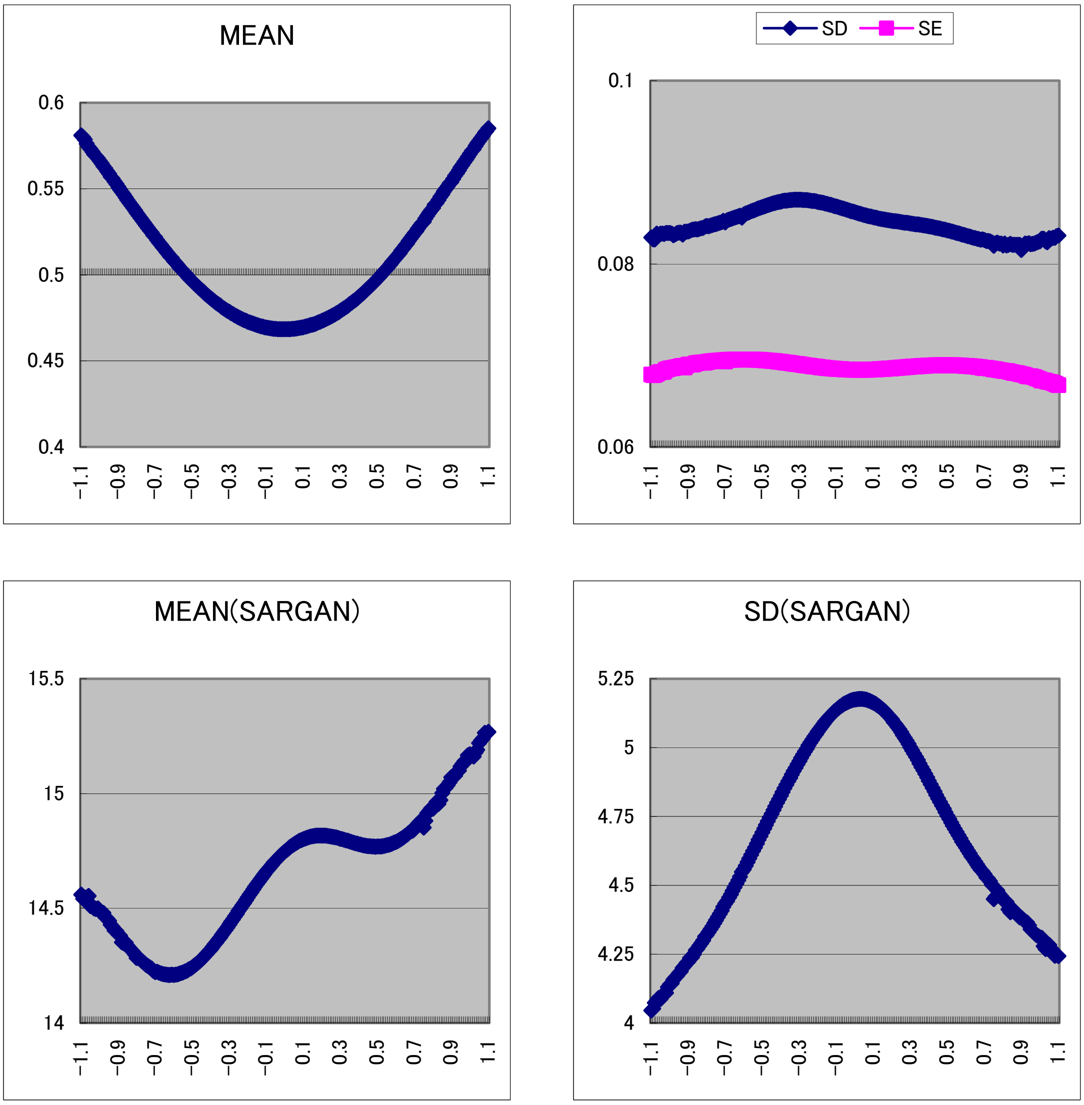


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.53	-0.62
MEAN	0.467	0.468	0.500	0.511
[SD, SE]	[0.080, 0.068]	[0.086, 0.068]	[0.086, 0.070]	[0.085, 0.070]
MEAN(SARGAN)	14.982	14.748	14.228	14.209
SD(SARGAN)	5.252	5.175	4.652	4.531

[Exhibit 0-0.25-0.8]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n=0$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (0.25, 0.8)$ when the value of θ changes.

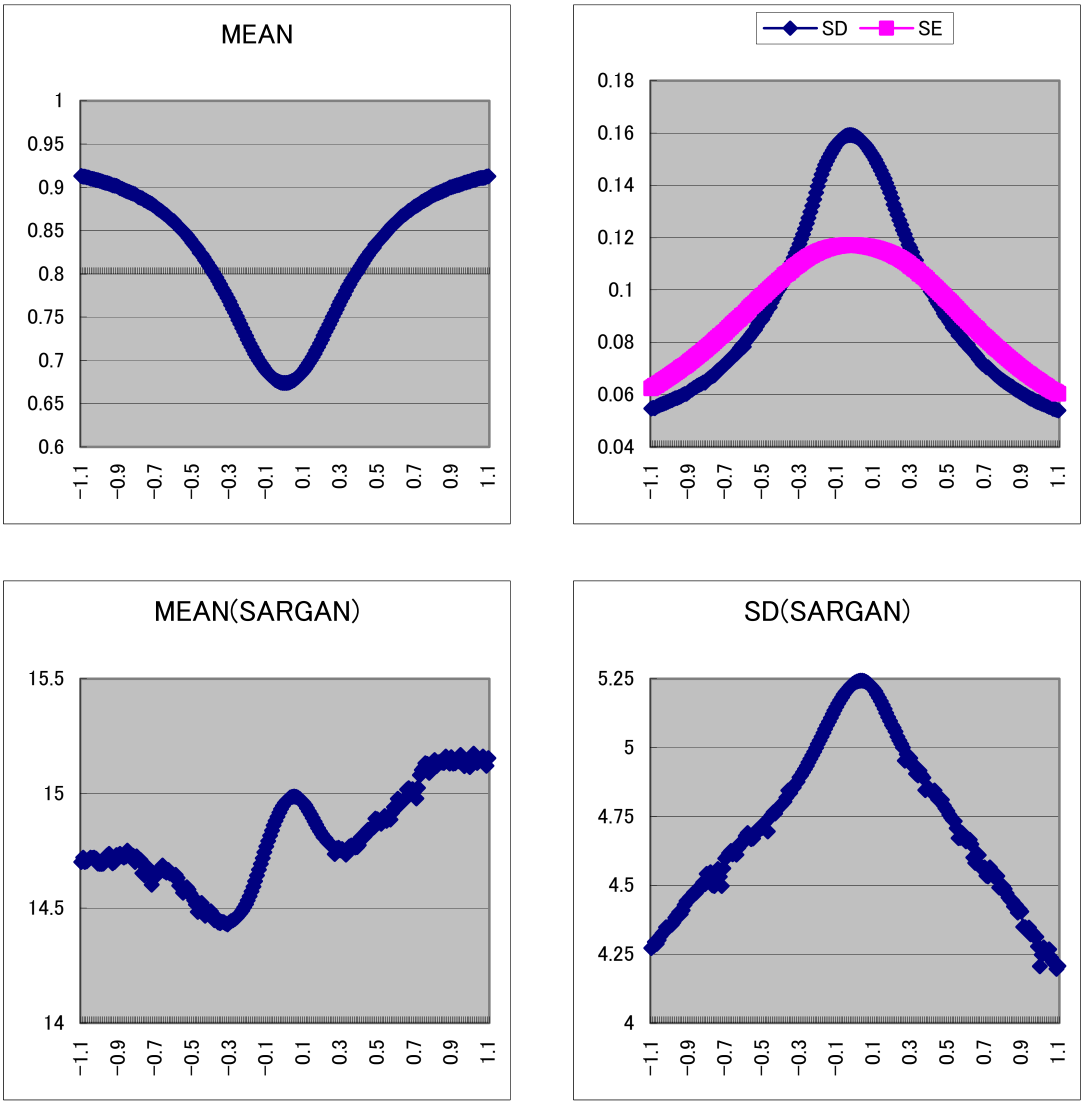


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.38	-0.31
MEAN	0.681	0.674	0.799	0.772
[SD, SE]	[0.140, 0.117]	[0.159, 0.117]	[0.105, 0.106]	[0.116, 0.110]
MEAN(SARGAN)	15.215	14.958	14.447	14.429
SD(SARGAN)	5.305	5.236	4.804	4.876

[Exhibit 0-1-0.2]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators $(n=0)$

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.2)$ when the value of θ changes.

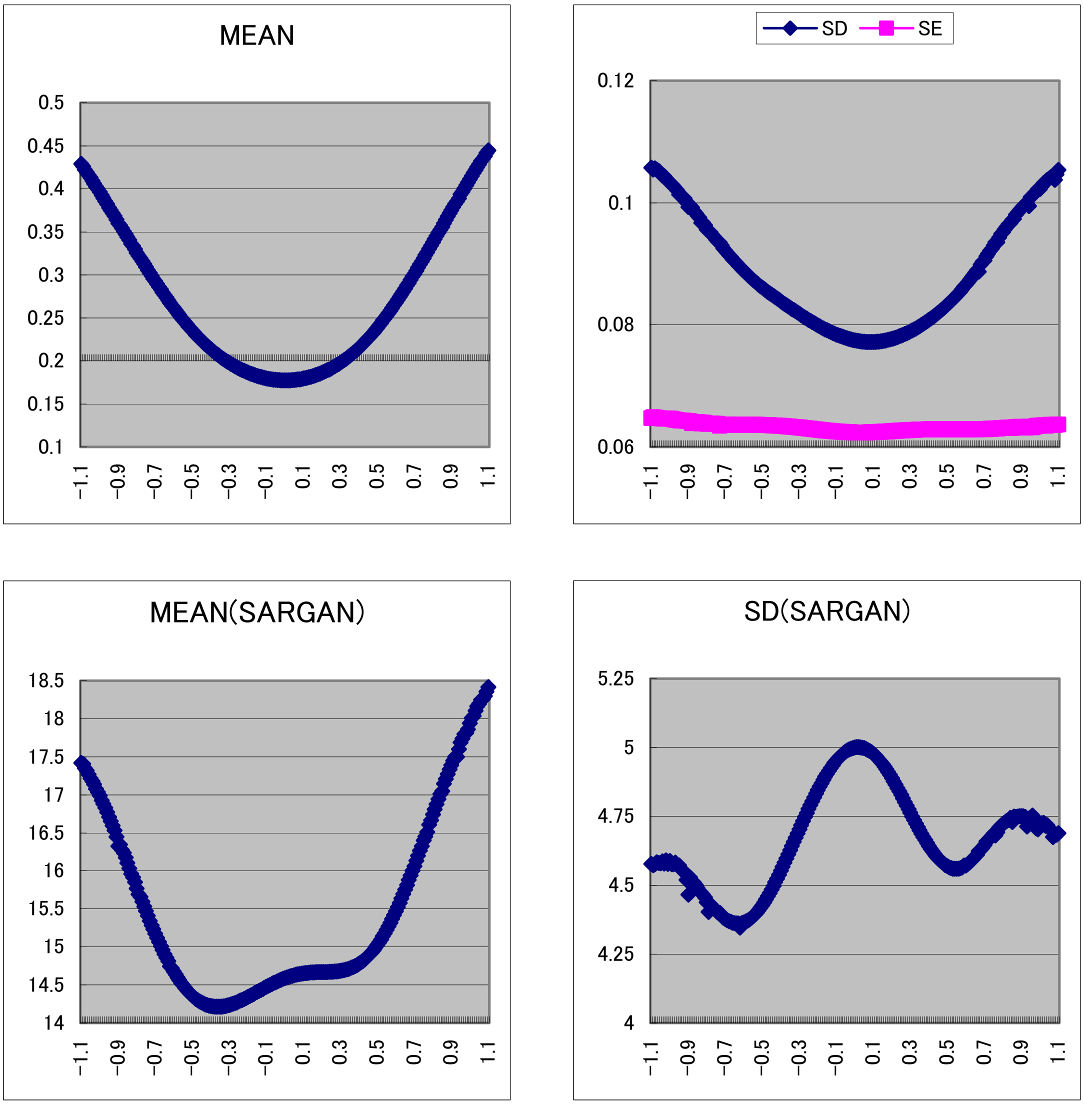


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.32	-0.36
MEAN	0.179	0.177	0.200	0.207
[SD, SE]	[0.071, 0.062]	[0.077, 0.062]	[0.082, 0.063]	[0.083, 0.063]
MEAN(SARGAN)	14.825	14.588	14.224	14.213
SD(SARGAN)	5.077	5.000	4.673	4.612

[Exhibit 0-1-0.5]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n=0$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.5)$ when the value of θ changes.

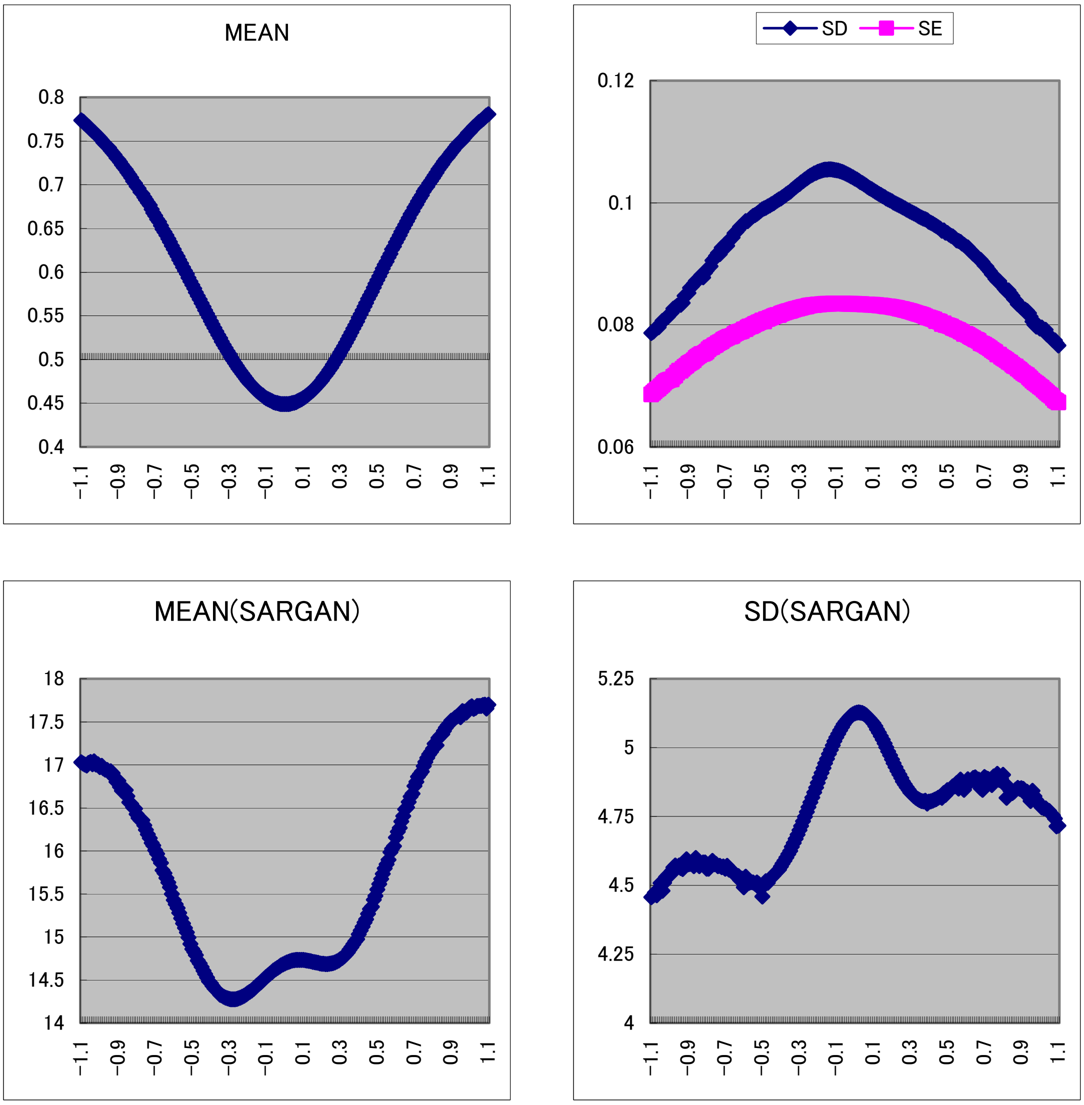


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.28	-0.28
MEAN	0.450	0.449	0.499	0.499
[SD, SE]	[0.095, 0.083]	[0.104, 0.083]	[0.104, 0.083]	[0.104, 0.083]
MEAN(SARGAN)	14.930	14.693	14.275	14.275
SD(SARGAN)	5.192	5.125	4.733	4.733

[Exhibit 0-1-0.8]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n = 0$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (1, 0.8)$ when the value of θ changes.

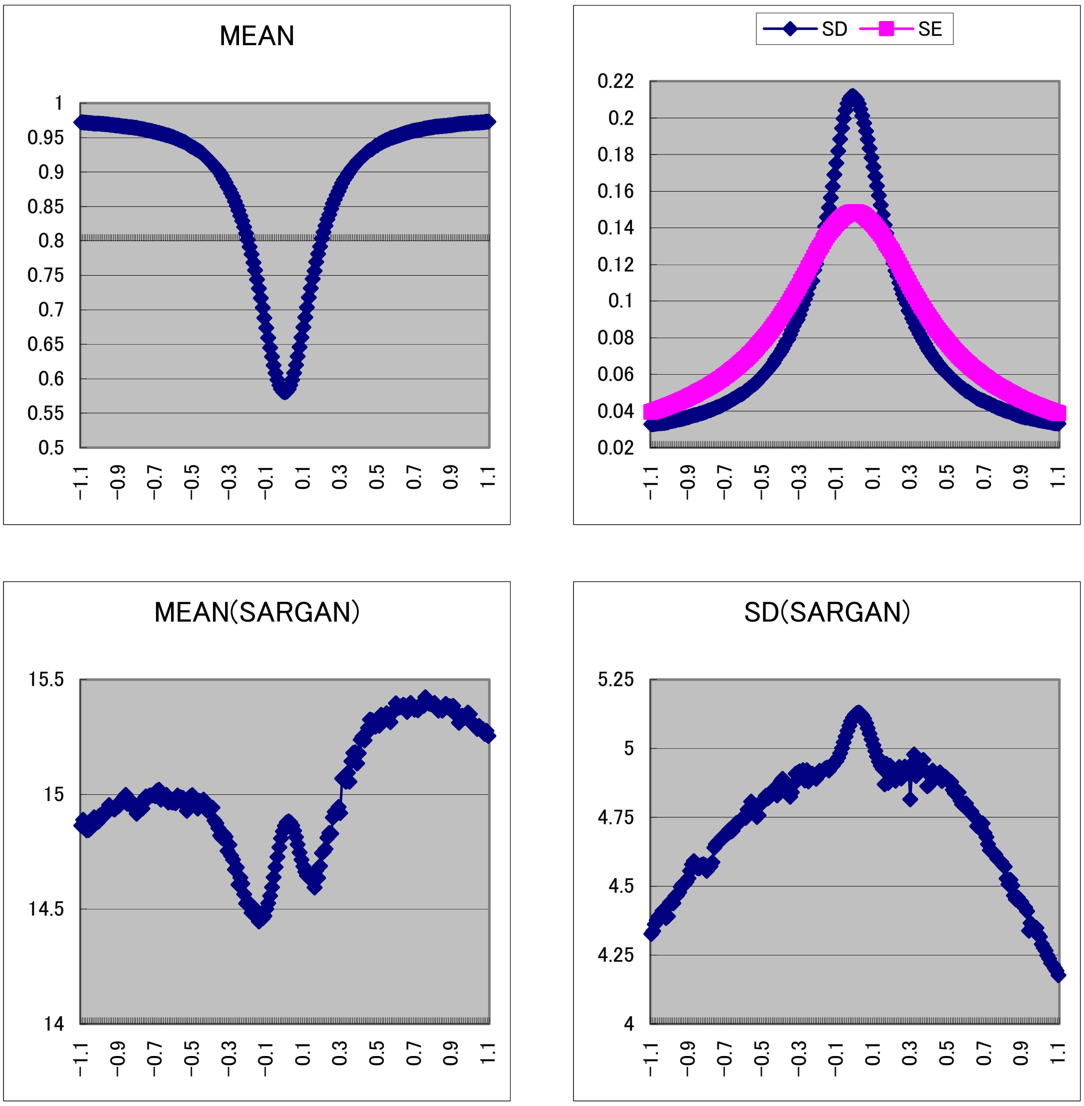


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.20	-0.14
MEAN	0.604	0.580	0.801	0.731
[SD, SE]	[0.177, 0.149]	[0.211, 0.149]	[0.124, 0.127]	[0.151, 0.137]
MEAN(SARGAN)	15.174	14.863	14.533	14.447
SD(SARGAN)	5.228	5.122	4.909	4.921

[Exhibit 0-4-0.2]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n = 0$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (4, 0.2)$ when the value of θ changes.

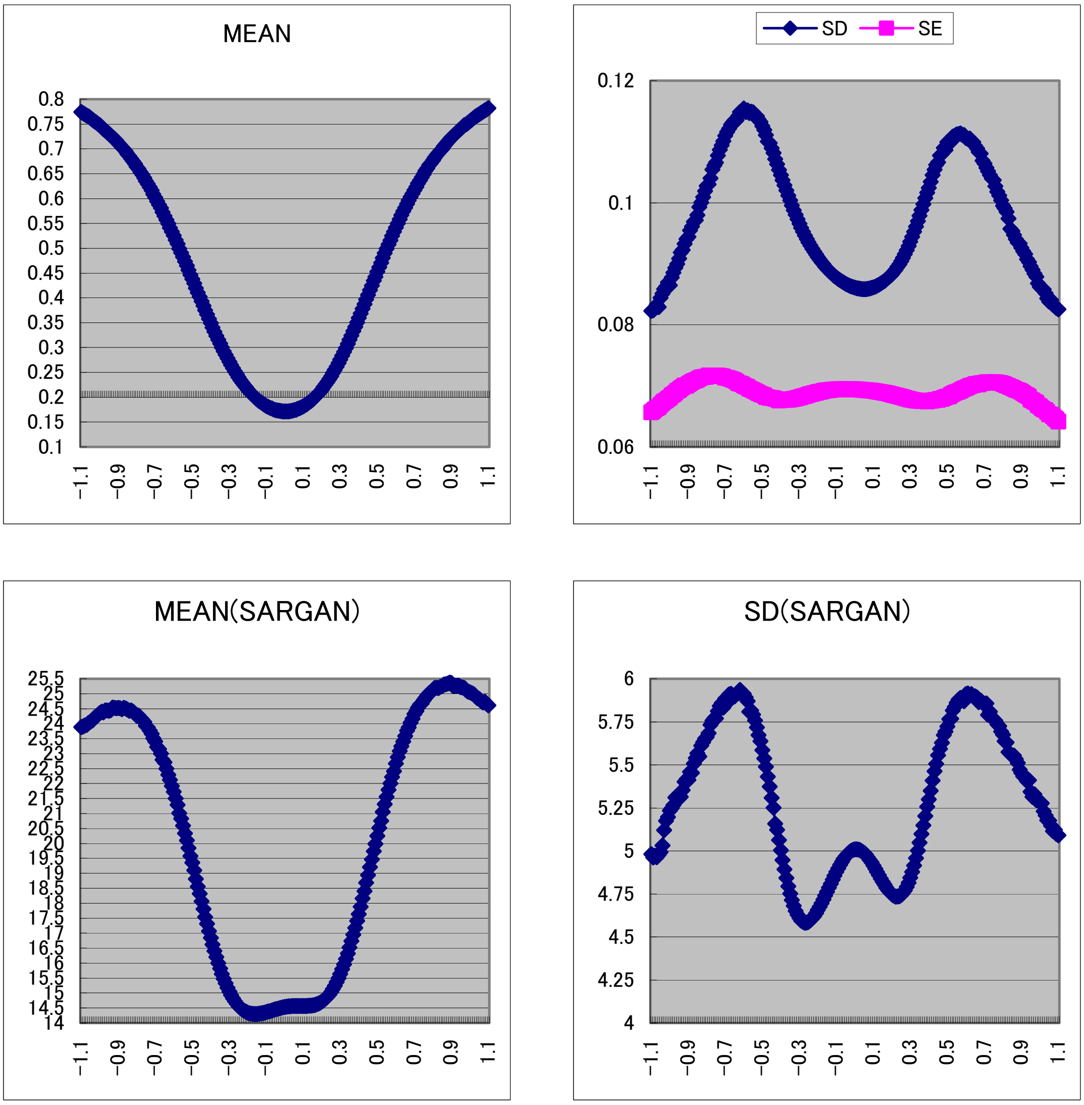


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.16	-0.16
MEAN	0.174	0.171	0.200	0.200
[SD, SE]	[0.078, 0.069]	[0.086, 0.069]	[0.090, 0.069]	[0.090, 0.069]
MEAN(SARGAN)	14.776	14.534	14.298	14.298
SD(SARGAN)	5.092	5.009	4.740	4.740

[Exhibit 0-4-0.5]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n = 0$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (4, 0.5)$ when the value of θ changes.

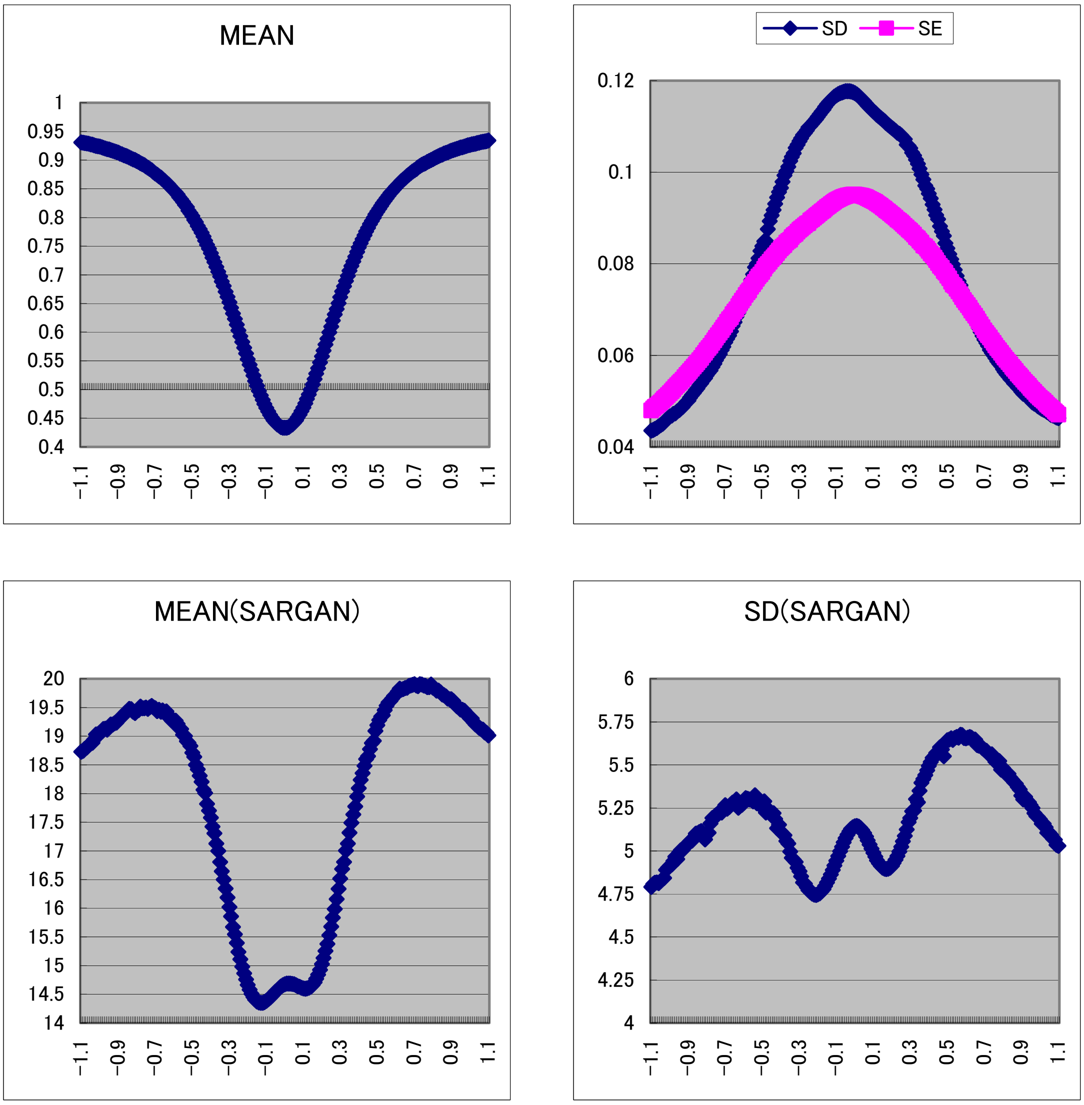


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.14	-0.13
MEAN	0.437	0.433	0.498	0.490
[SD, SE]	[0.106, 0.095]	[0.117, 0.095]	[0.115, 0.093]	[0.116, 0.093]
MEAN(SARGAN)	14.917	14.672	14.356	14.349
SD(SARGAN)	5.217	5.144	4.838	4.862

[Exhibit 0-4-0.8]

Monte Carlo Statistics using **GMM(MGF[0]-FD)** estimators ($n=0$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (4, 0.8)$ when the value of θ changes.

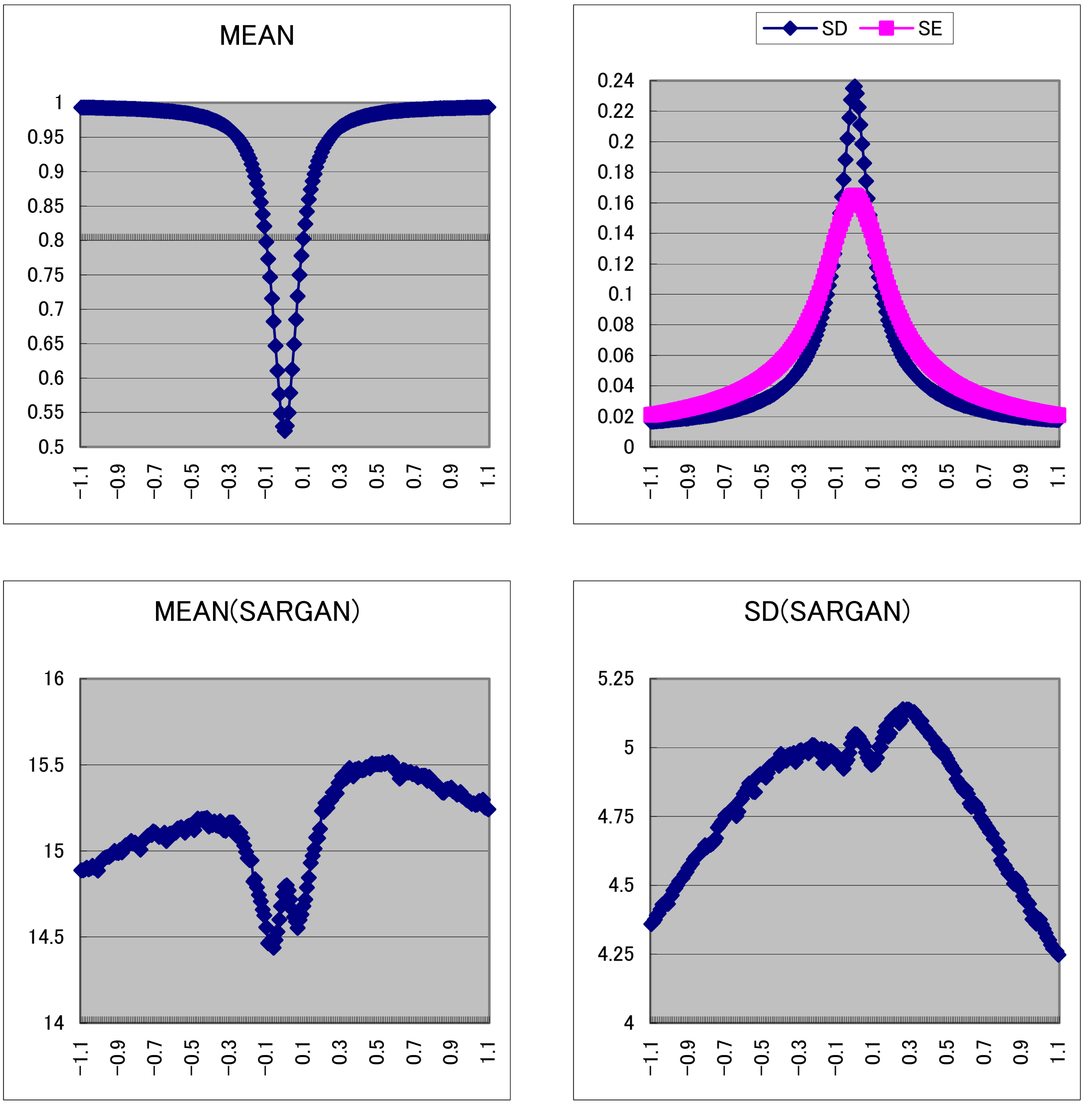


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.10	-0.06
MEAN	0.558	0.523	0.798	0.682
[SD, SE]	[0.195, 0.165]	[0.236, 0.165]	[0.134, 0.138]	[0.175, 0.153]
MEAN(SARGAN)	15.134	14.792	14.555	14.437
SD(SARGAN)	5.182	5.050	4.971	4.924

[Exhibit 1-0.25-0.2]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*)=(0.25, 0.2)$ when the value of θ changes.

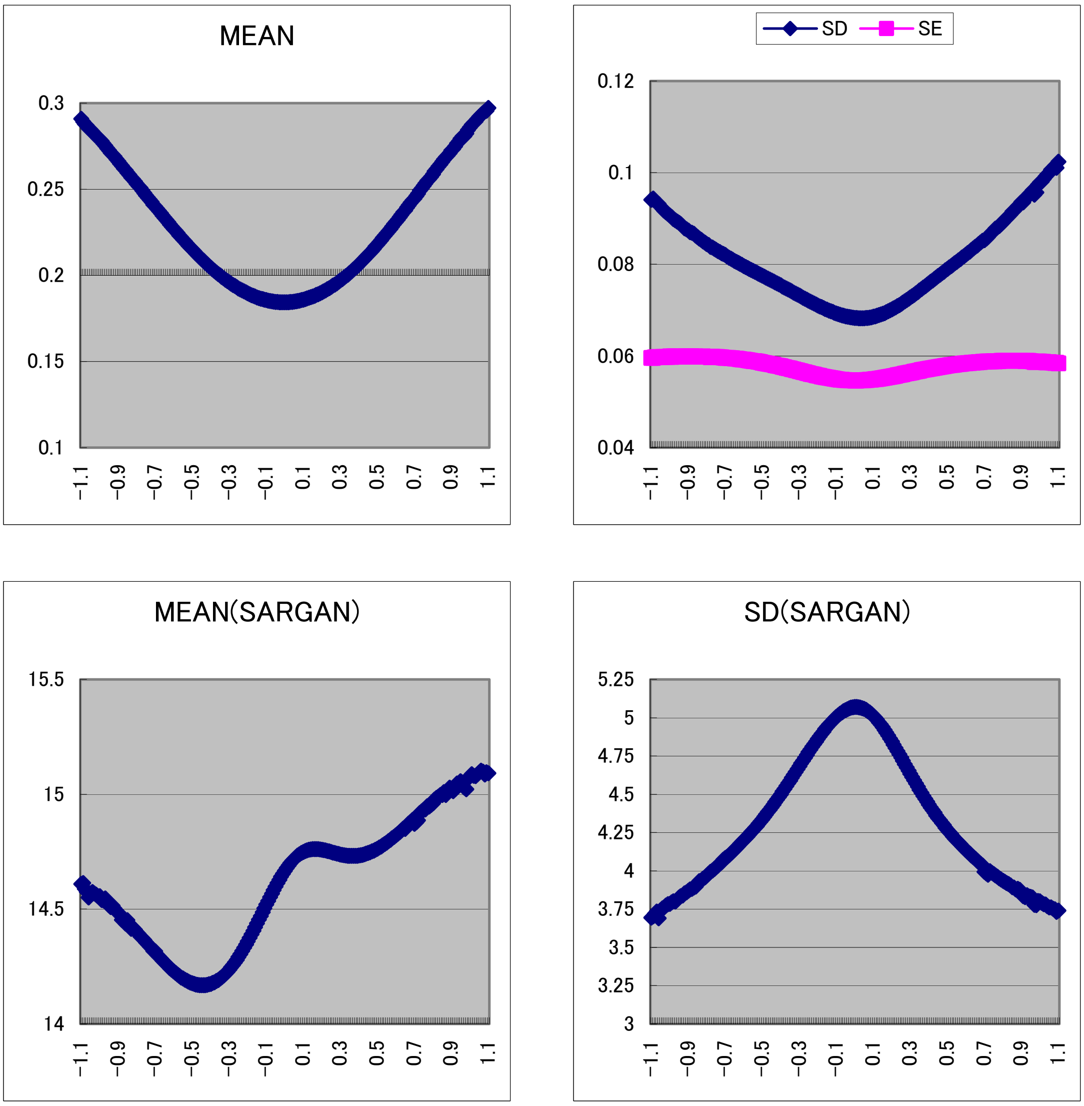


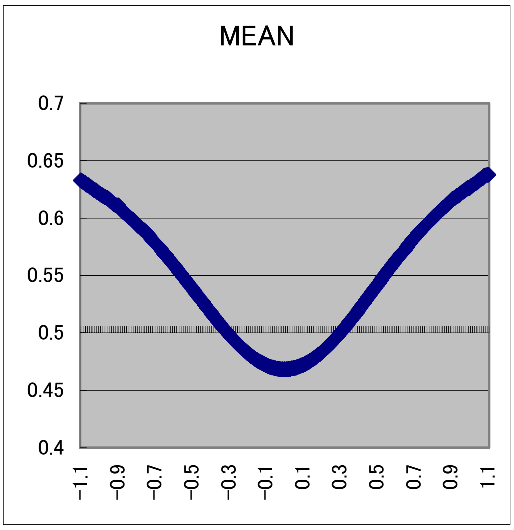
Table. Representative Monte Carlo results (DF=14)

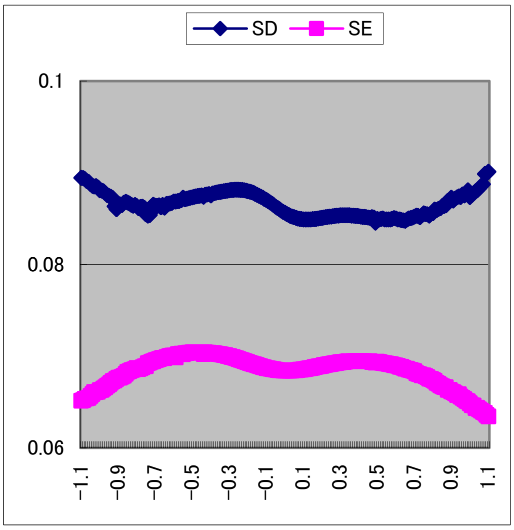
	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.35	-0.45
MEAN	0.184	0.184	0.200	0.210
[SD, SE]	[0.064, 0.055]	[0.068, 0.055]	[0.074, 0.057]	[0.076, 0.058]
MEAN(SARGAN)	14.904	14.667	14.197	14.169
SD(SARGAN)	5.147	5.069	4.590	4.416

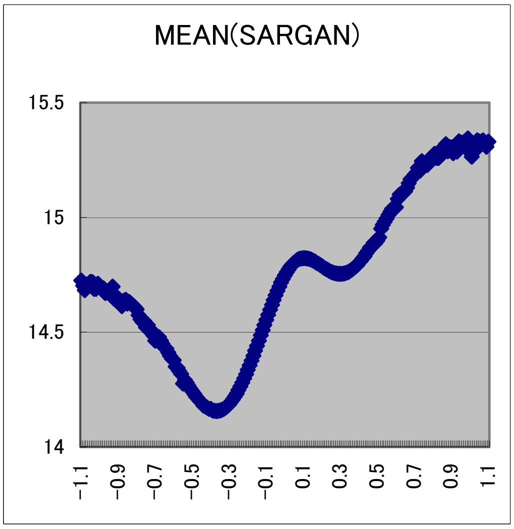
[Exhibit 1-0.25-0.5]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (0.25, 0.5)$ when the value of θ changes.







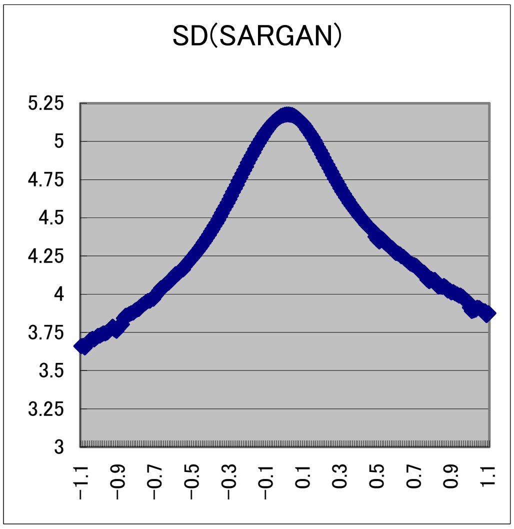


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.31	-0.37
MEAN	0.467	0.468	0.500	0.512
[SD, SE]	[0.080, 0.068]	[0.086, 0.068]	[0.088, 0.070]	[0.088, 0.070]
MEAN(SARGAN)	14.982	14.748	14.179	14.157
SD(SARGAN)	5.252	5.175	4.599	4.467

[Exhibit 1-0.25-0.8]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (0.25, 0.8)$ when the value of θ changes.

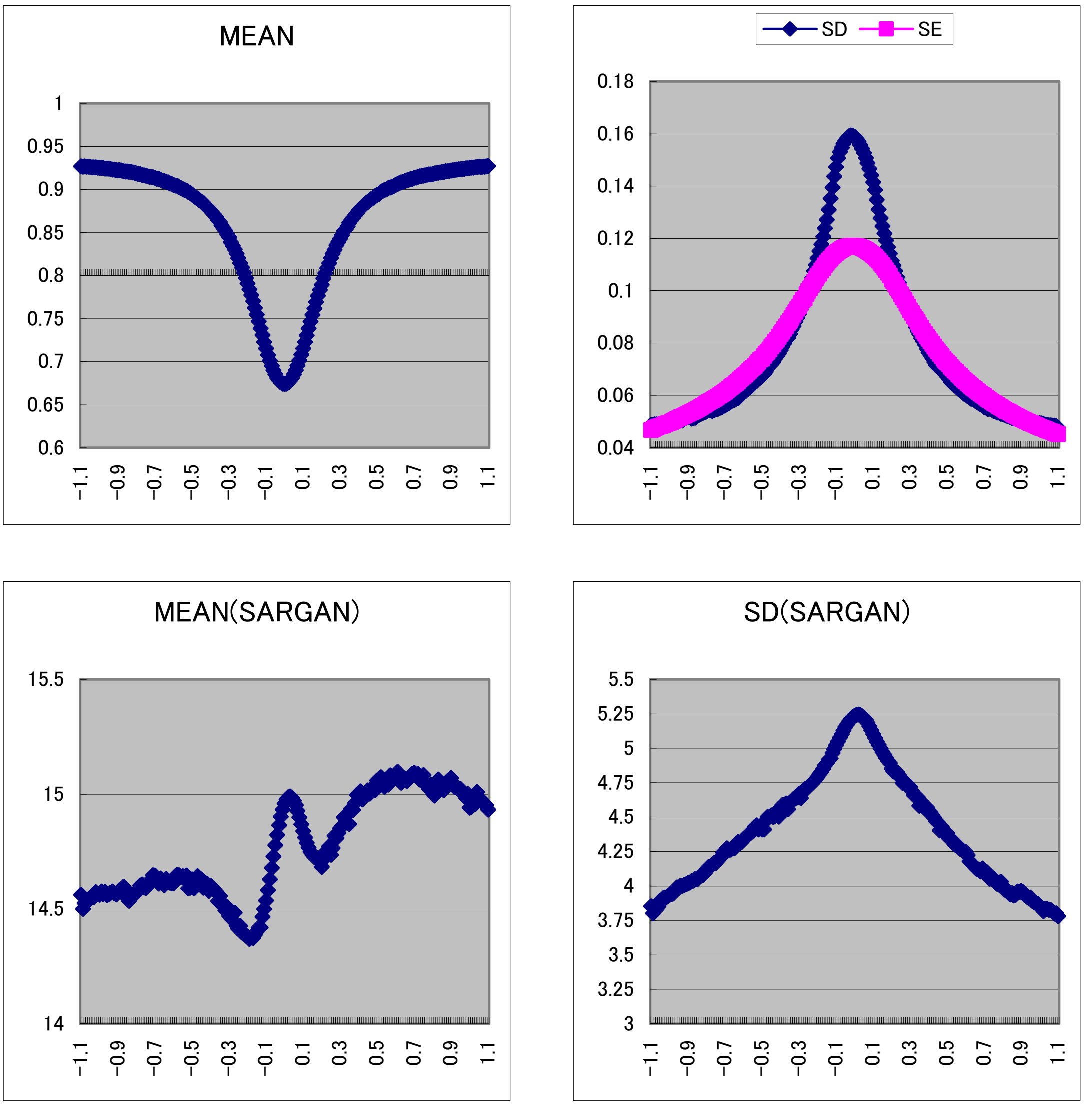


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.21	-0.19
MEAN	0.681	0.674	0.797	0.784
[SD, SE]	[0.140, 0.117]	[0.159, 0.117]	[0.110, 0.105]	[0.115, 0.107]
MEAN(SARGAN)	15.215	14.958	14.385	14.370
SD(SARGAN)	5.305	5.236	4.774	4.812

[Exhibit 1-1-0.2]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.2)$ when the value of θ changes.

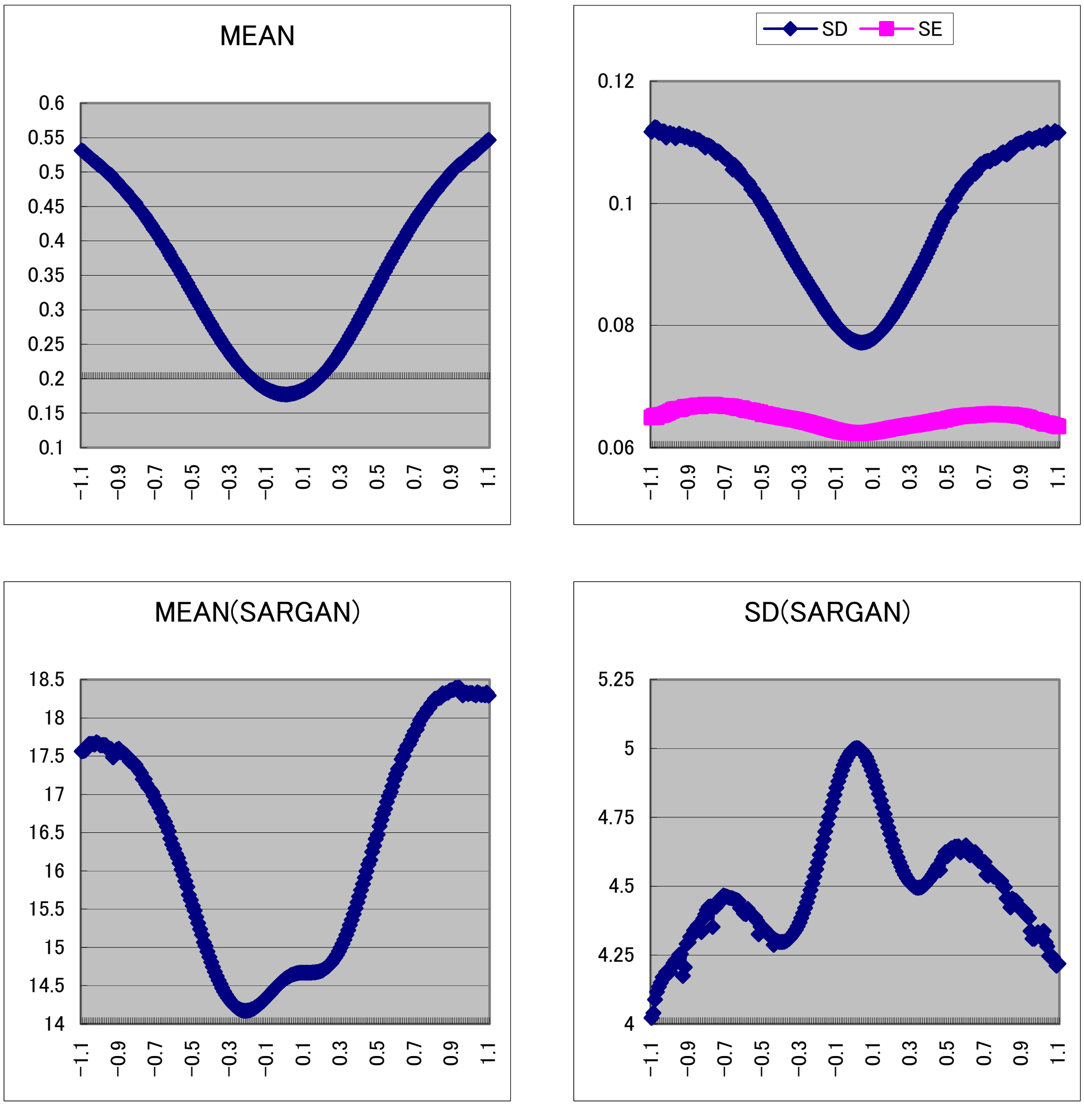


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.18	-0.21
MEAN	0.179	0.177	0.200	0.208
[SD, SE]	[0.071, 0.062]	[0.077, 0.062]	[0.083, 0.063]	[0.085, 0.064]
MEAN(SARGAN)	14.825	14.588	14.191	14.172
SD(SARGAN)	5.077	5.000	4.642	4.561

[Exhibit 1-1-0.5]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.5)$ when the value of θ changes.

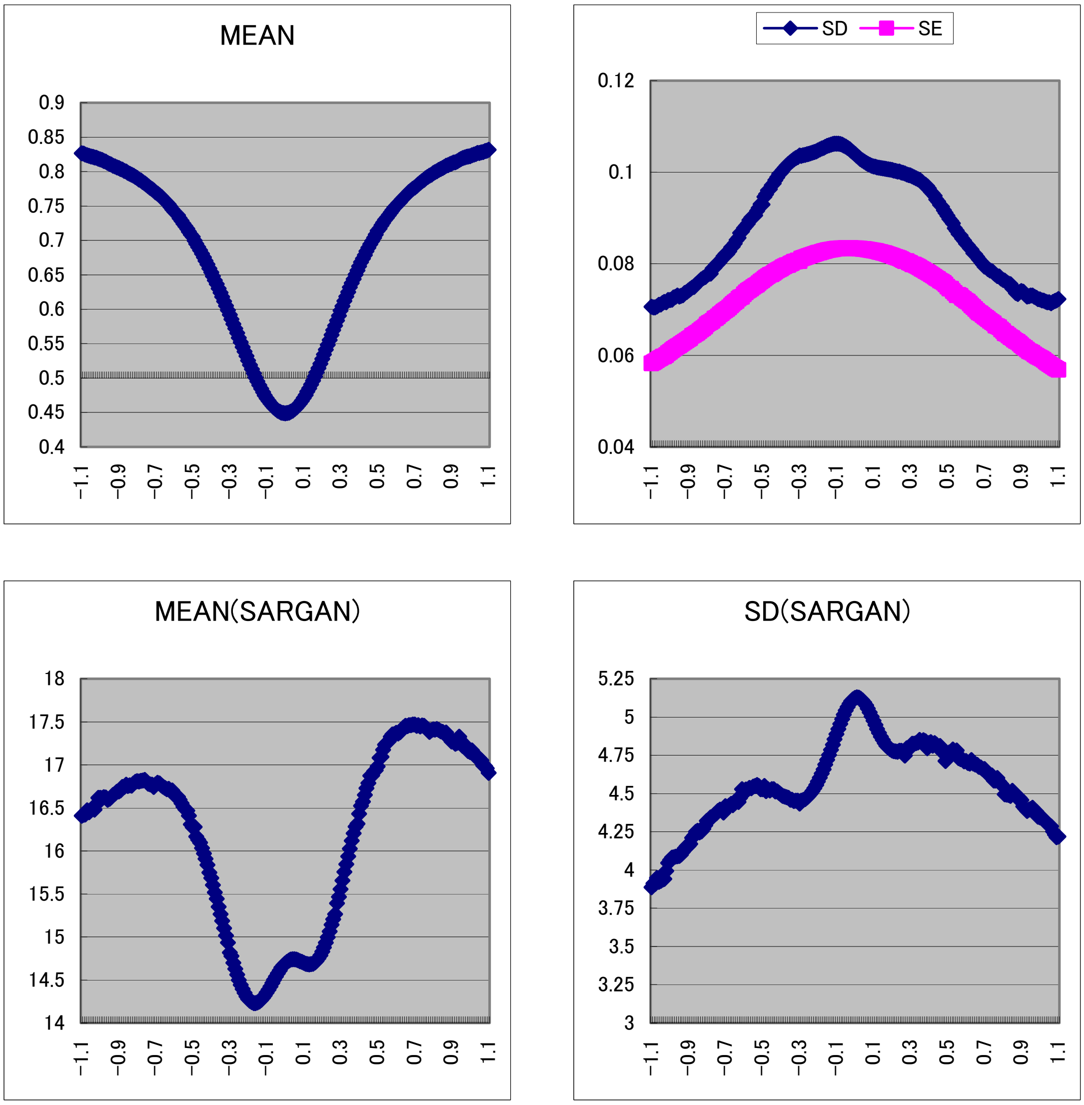


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.16	-0.16
MEAN	0.450	0.449	0.501	0.501
[SD, SE]	[0.095, 0.083]	[0.104, 0.083]	[0.106, 0.083]	[0.106, 0.083]
MEAN(SARGAN)	14.930	14.693	14.231	14.231
SD(SARGAN)	5.192	5.125	4.689	4.689

[Exhibit 1-1-0.8]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.8)$ when the value of θ changes.

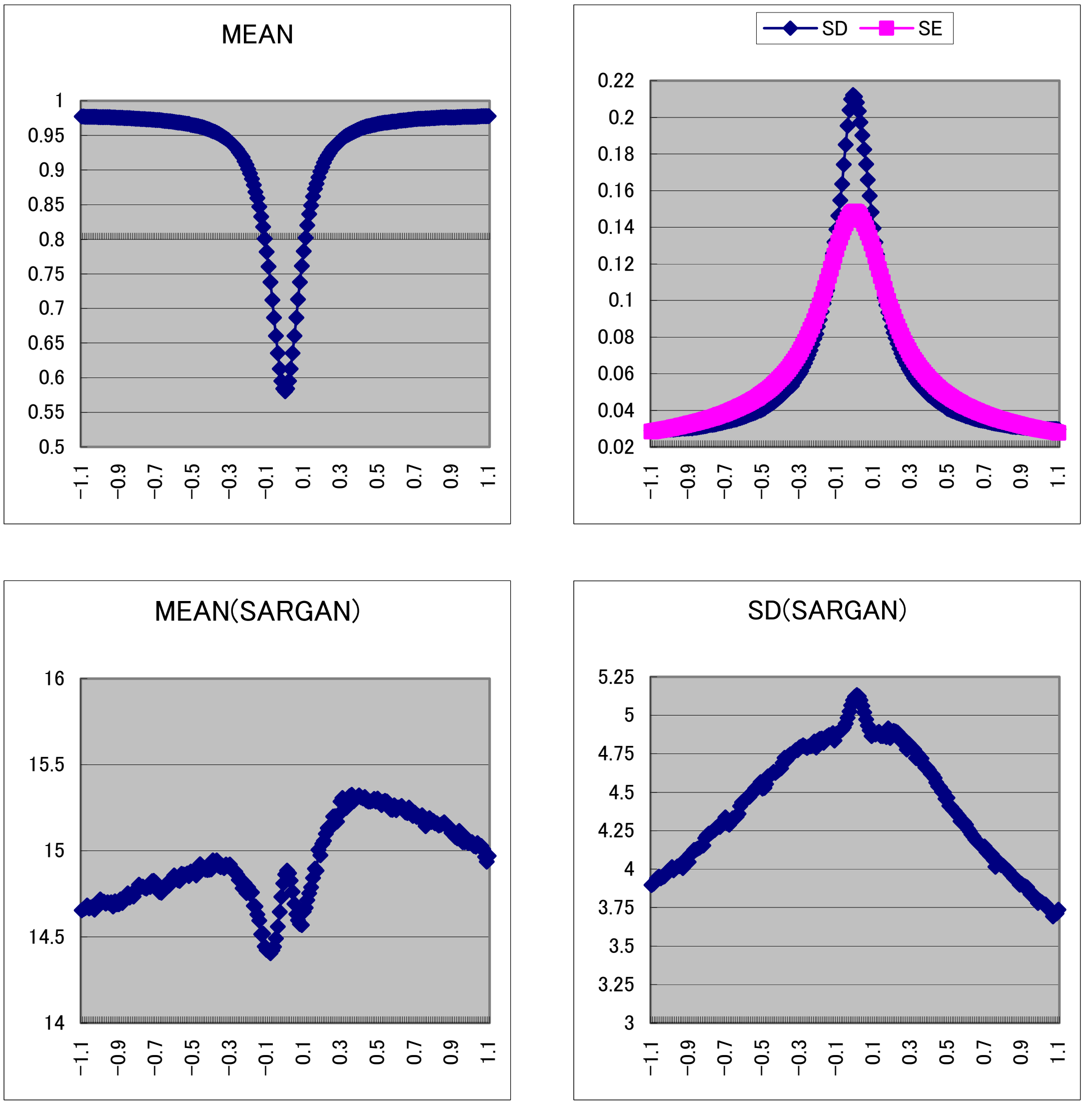


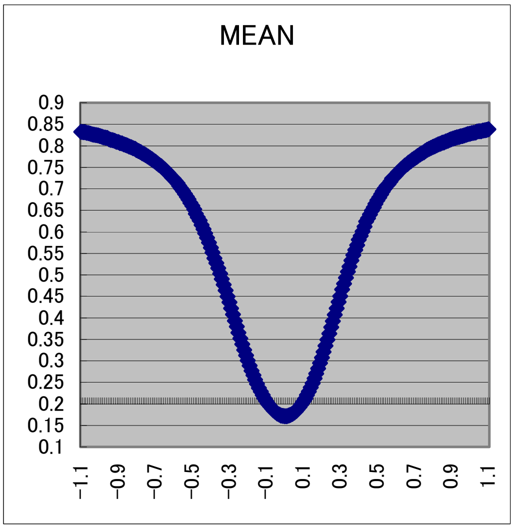
Table. Representative Monte Carlo results (DF=14)

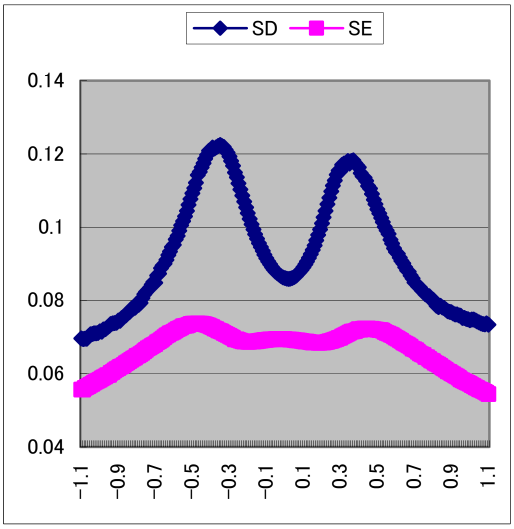
	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.11	-0.08
MEAN	0.604	0.580	0.801	0.738
[SD, SE]	[0.177, 0.149]	[0.211, 0.149]	[0.132, 0.125]	[0.155, 0.134]
MEAN(SARGAN)	15.174	14.863	14.444	14.405
SD(SARGAN)	5.228	5.122	4.836	4.899

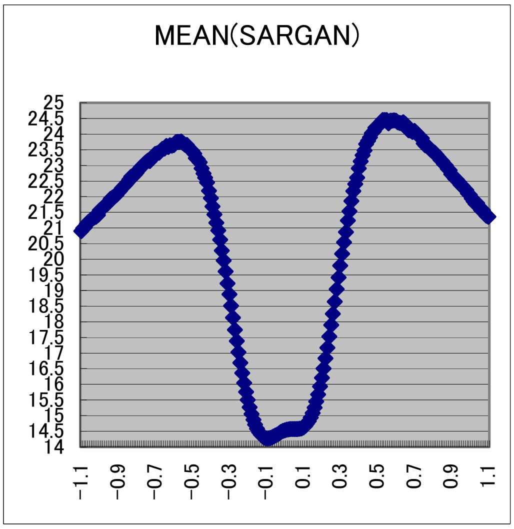
[Exhibit 1-4-0.2]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_{\eta}^2/\sigma_{v}^2,\alpha^*)=(4,\,0.2)$ when the value of θ changes.







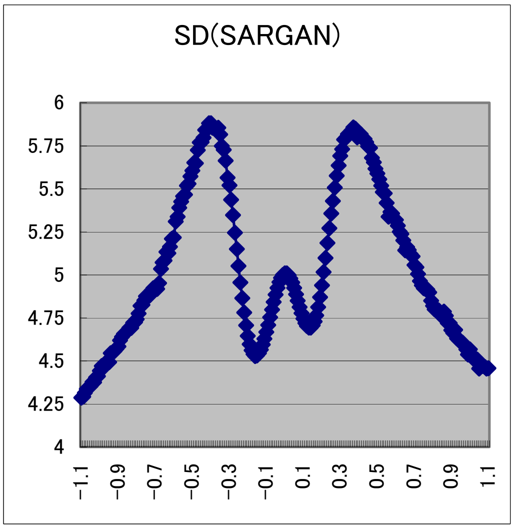


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.09	-0.09
MEAN	0.174	0.171	0.200	0.200
[SD, SE]	[0.078, 0.069]	[0.086, 0.069]	[0.091, 0.069]	[0.091, 0.069]
MEAN(SARGAN)	14.776	14.534	14.274	14.274
SD(SARGAN)	5.092	5.009	4.708	4.708

[Exhibit 1-4-0.5]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (4, 0.5)$ when the value of θ changes.

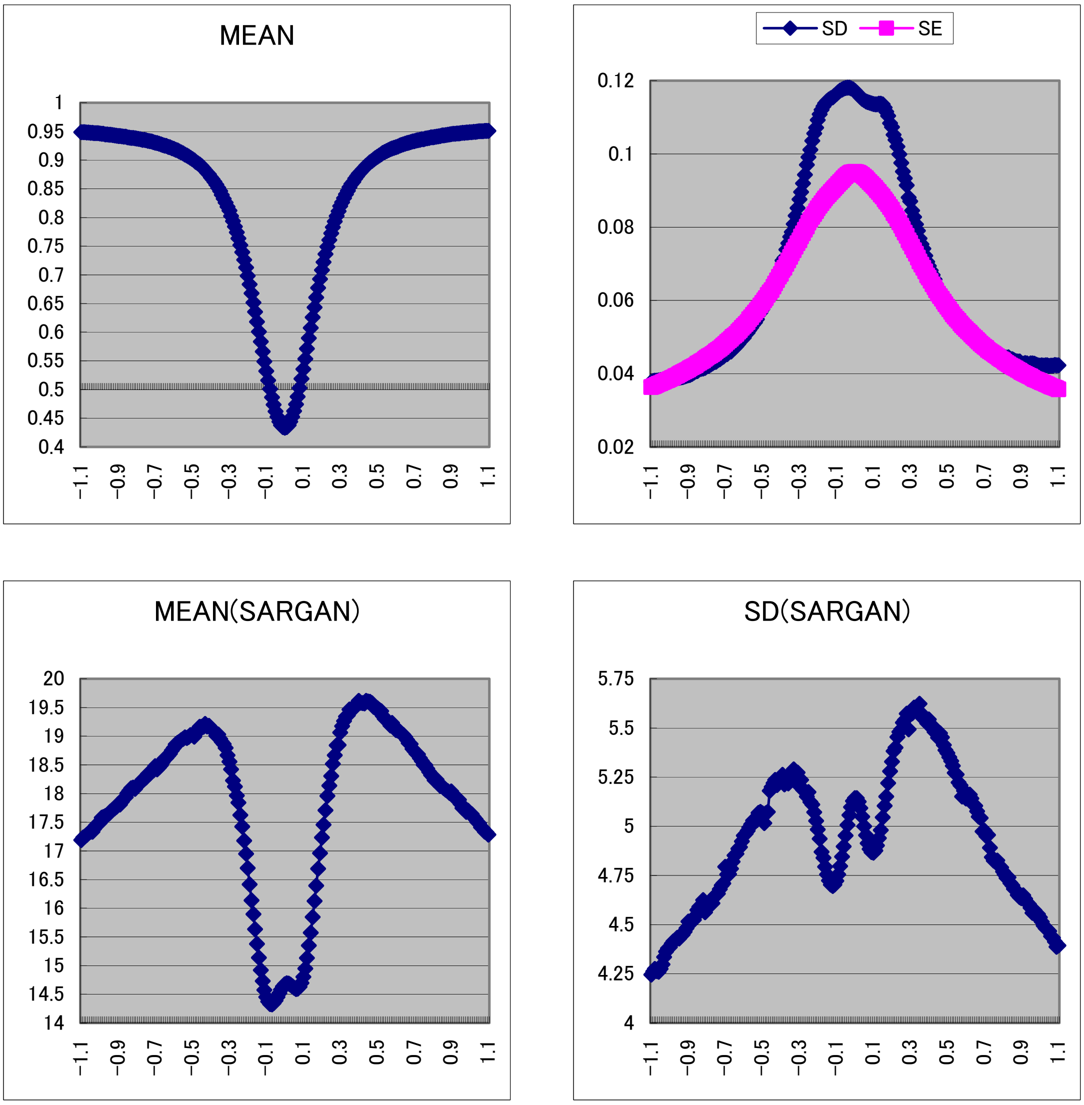


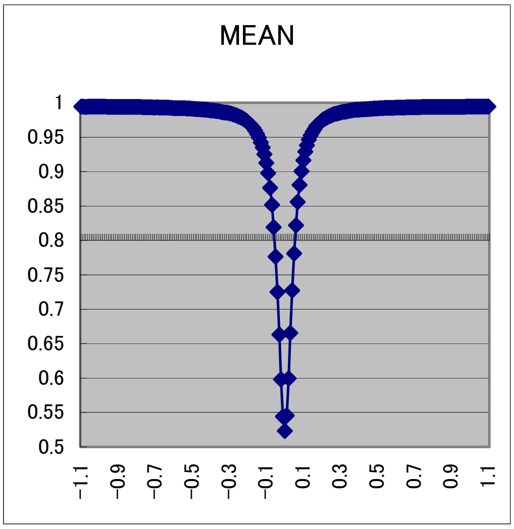
Table. Representative Monte Carlo results (DF=14)

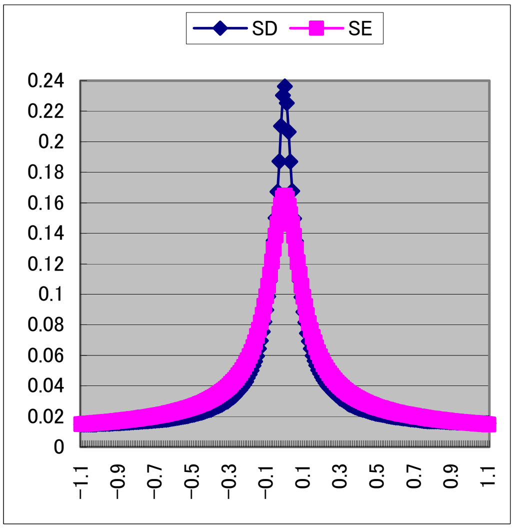
	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.08	-0.07
MEAN	0.437	0.433	0.501	0.487
[SD, SE]	[0.106, 0.095]	[0.117, 0.095]	[0.117, 0.092]	[0.118, 0.093]
MEAN(SARGAN)	14.917	14.672	14.331	14.323
SD(SARGAN)	5.217	5.144	4.796	4.845

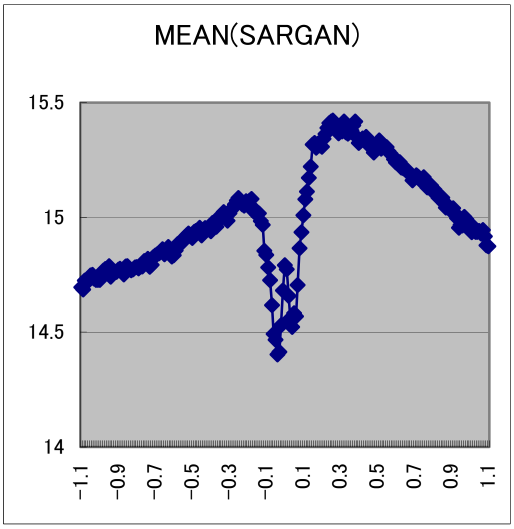
[Exhibit 1-4-0.8]

Monte Carlo Statistics using **GMM(MGF[1]-FD)** estimators ($n=1$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (4, 0.8)$ when the value of θ changes.







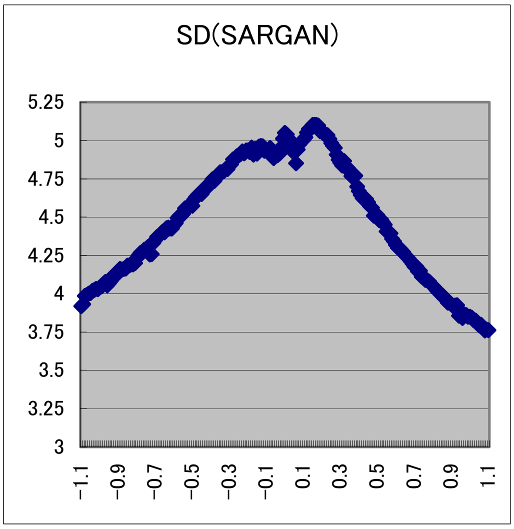


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	0	0	-0.06	-0.04
MEAN	0.558	0.523	0.819	0.725
[SD, SE]	[0.195, 0.165]	[0.236, 0.165]	[0.135, 0.130]	[0.167, 0.146]
MEAN(SARGAN)	15.134	14.792	14.493	14.403
SD(SARGAN)	5.182	5.050	4.884	4.904

[Exhibit 2-0.25-0.2]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (0.25, 0.2)$ when the value of θ changes.

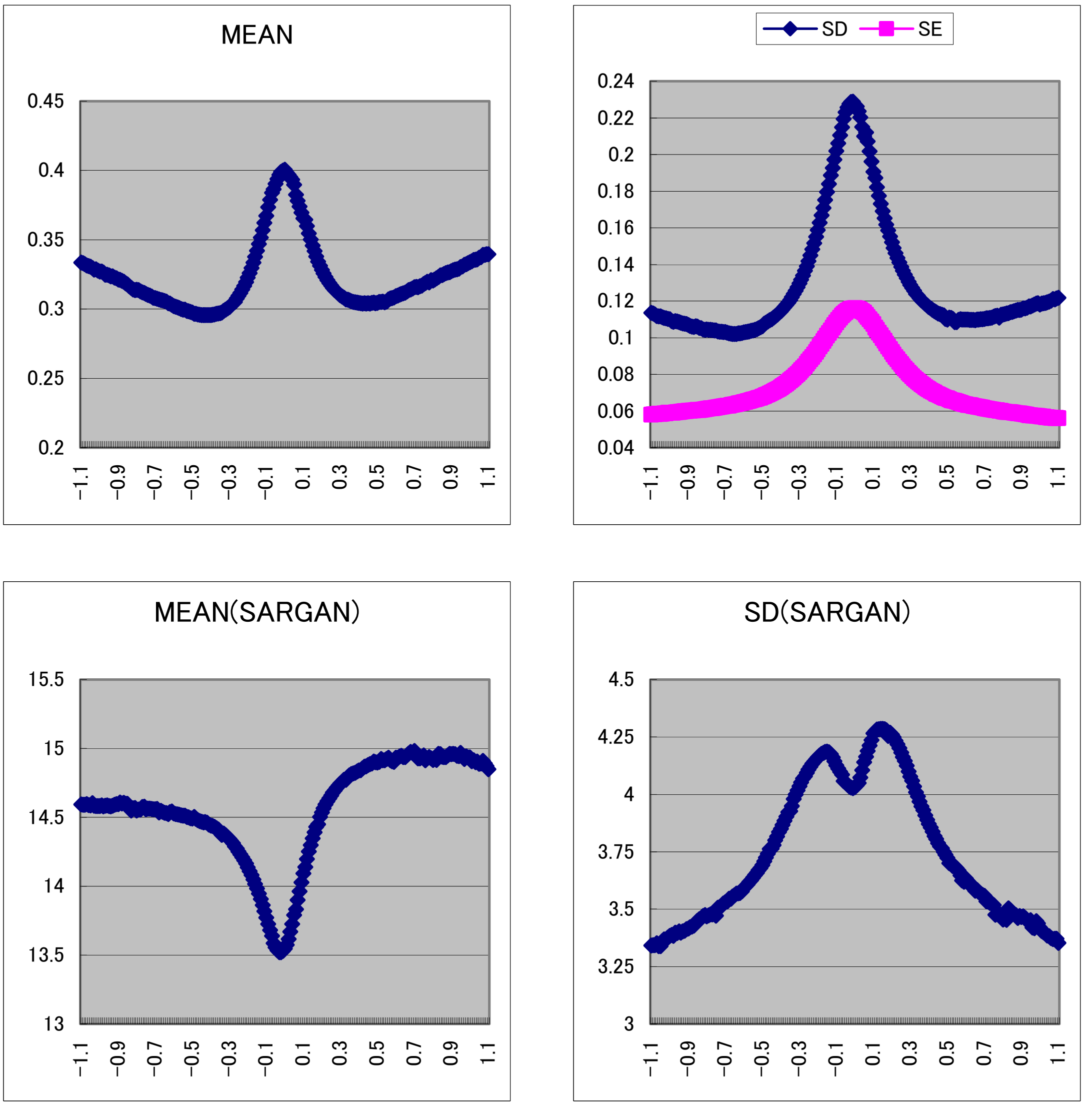


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.41	-0.03
MEAN	0.184	0.184	0.296	0.397
[SD, SE]	[0.064, 0.055]	[0.068, 0.055]	[0.114, 0.072]	[0.228, 0.115]
MEAN(SARGAN)	14.904	14.667	14.449	13.519
SD(SARGAN)	5.147	5.069	3.836	4.035

[Exhibit 2-0.25-0.5]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (0.25, 0.5)$ when the value of θ changes.

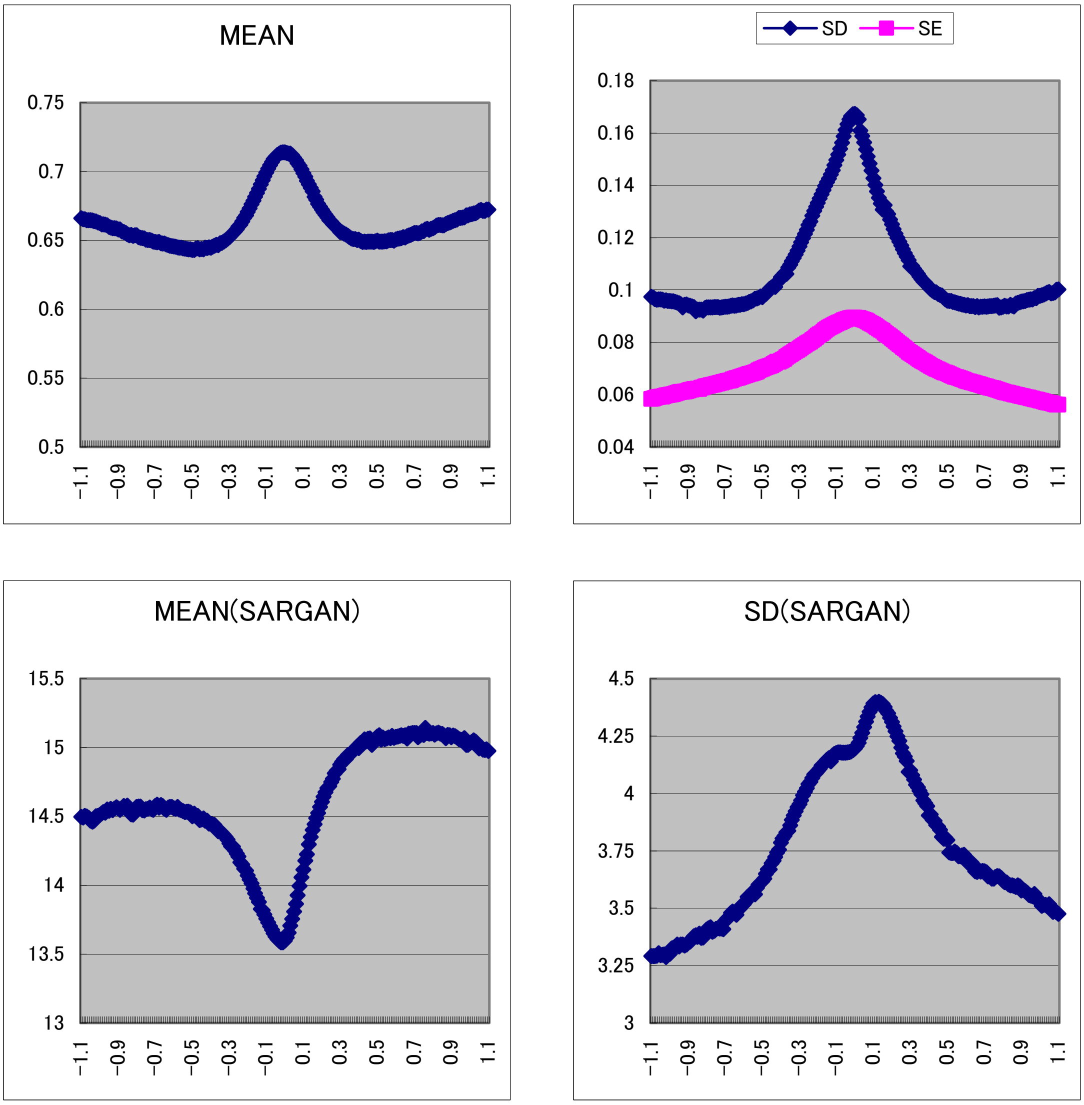


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.50	-0.02
MEAN	0.467	0.468	0.643	0.714
[SD, SE]	[0.080, 0.068]	[0.086, 0.068]	[0.097, 0.070]	[0.166, 0.089]
MEAN(SARGAN)	14.982	14.748	14.501	13.586
SD(SARGAN)	5.252	5.175	3.617	4.184

[Exhibit 2-0.25-0.8]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (0.25, 0.8)$ when the value of θ changes.

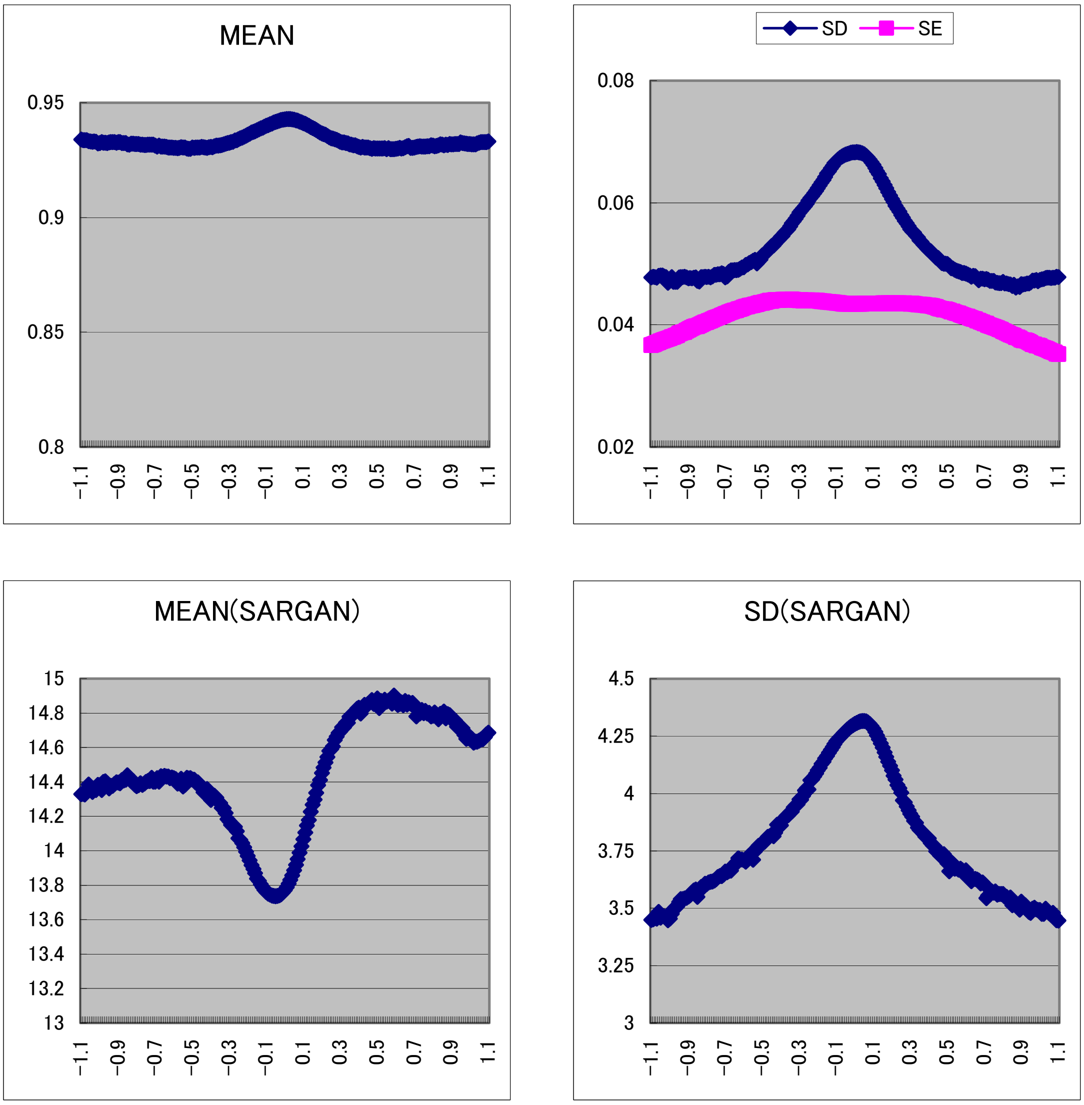


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.51	-0.05
MEAN	0.681	0.674	0.930	0.942
[SD, SE]	[0.140, 0.117]	[0.159, 0.117]	[0.051, 0.044]	[0.068, 0.043]
MEAN(SARGAN)	15.215	14.958	14.423	13.737
SD(SARGAN)	5.305	5.236	3.775	4.272

[Exhibit 2-1-0.2]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (1, 0.2)$ when the value of θ changes.

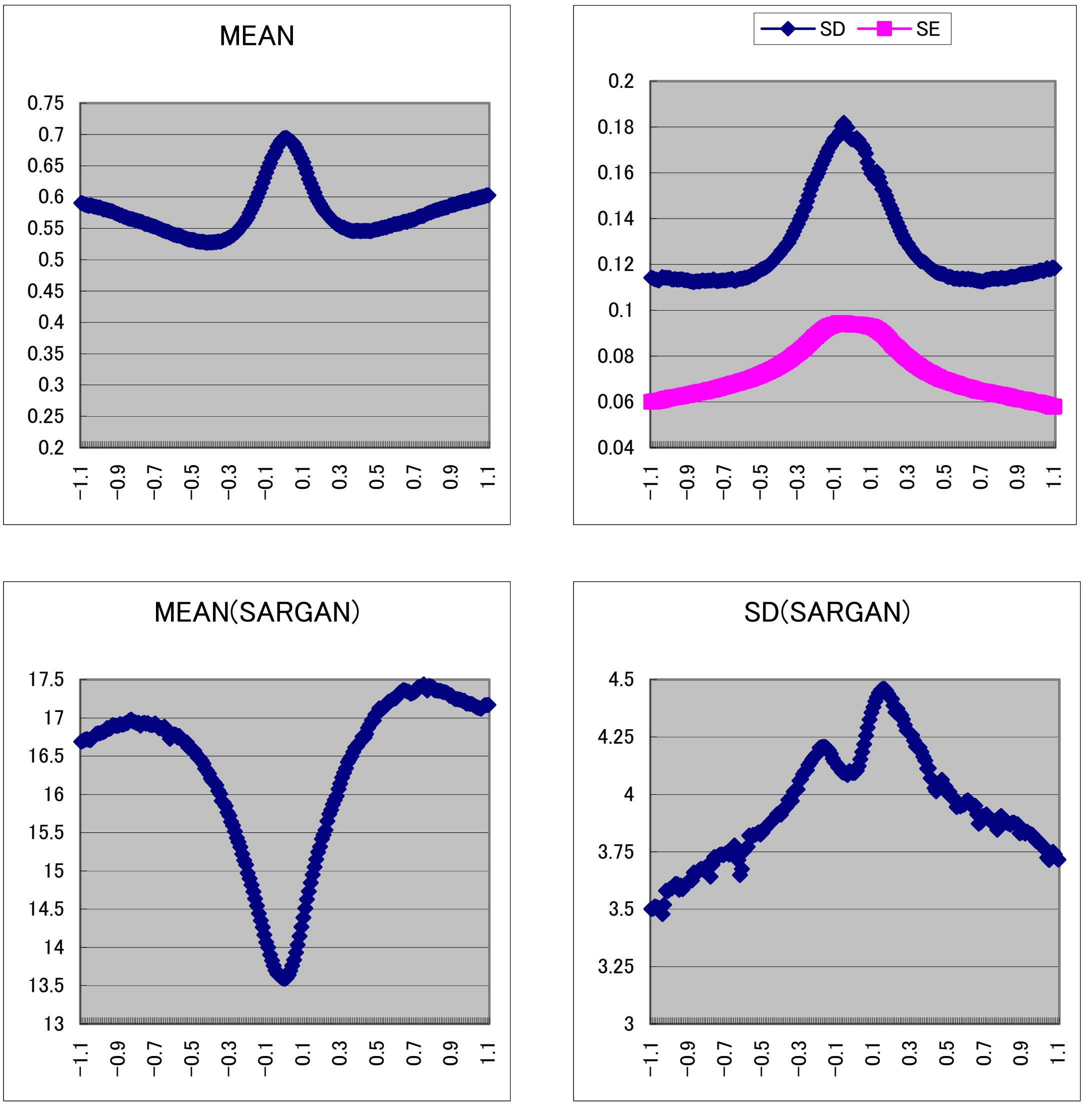


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.42	0
MEAN	0.179	0.177	0.527	0.694
[SD, SE]	[0.071, 0.062]	[0.077, 0.062]	[0.123, 0.075]	[0.175, 0.094]
MEAN(SARGAN)	14.825	14.588	16.306	13.589
SD(SARGAN)	5.077	5.000	3.913	4.094

[Exhibit 2-1-0.5]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.5)$ when the value of θ changes.

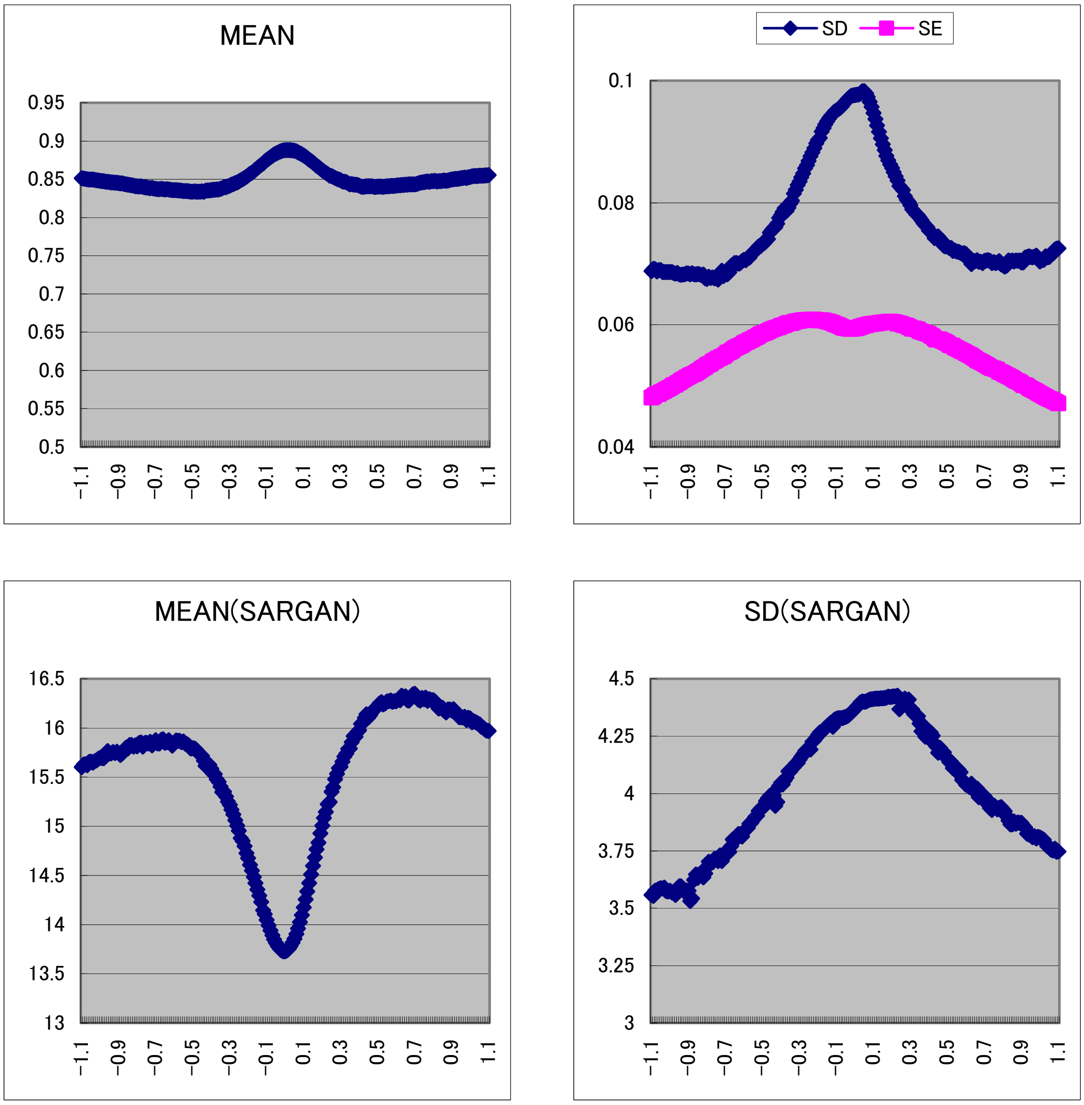


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.51	-0.01
MEAN	0.450	0.449	0.834	0.887
[SD, SE]	[0.095, 0.083]	[0.104, 0.083]	[0.073, 0.058]	[0.098, 0.059]
MEAN(SARGAN)	14.930	14.693	15.790	13.729
SD(SARGAN)	5.192	5.125	3.934	4.360

[Exhibit 2-1-0.8]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (1, 0.8)$ when the value of θ changes.

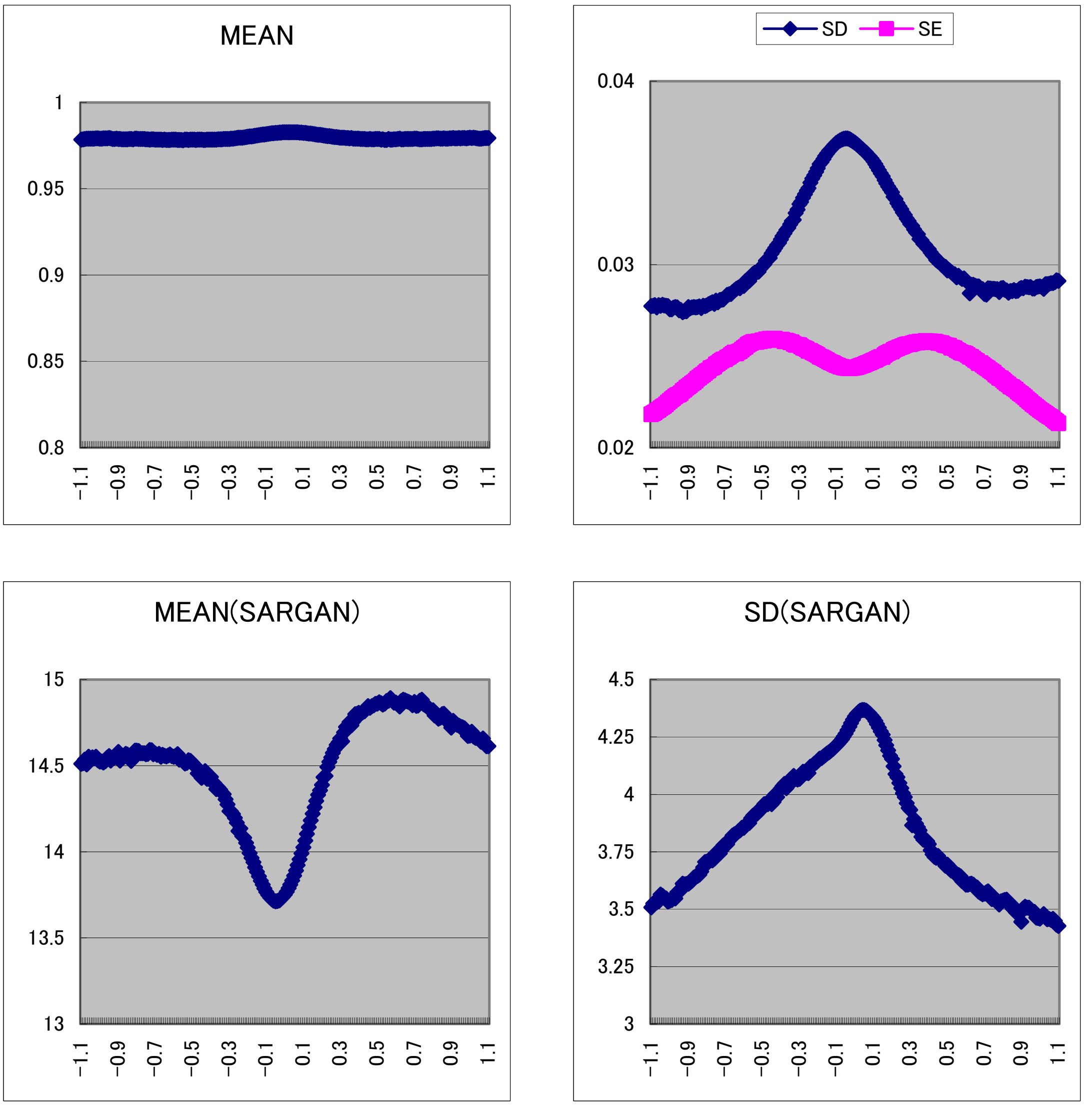


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.55	-0.05
MEAN	0.604	0.580	0.978	0.982
[SD, SE]	[0.177, 0.149]	[0.211, 0.149]	[0.029, 0.026]	[0.037, 0.024]
MEAN(SARGAN)	15.174	14.863	14.532	13.712
SD(SARGAN)	5.228	5.122	3.907	4.265

[Exhibit 2-4-0.2]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (4, 0.2)$ when the value of θ changes.

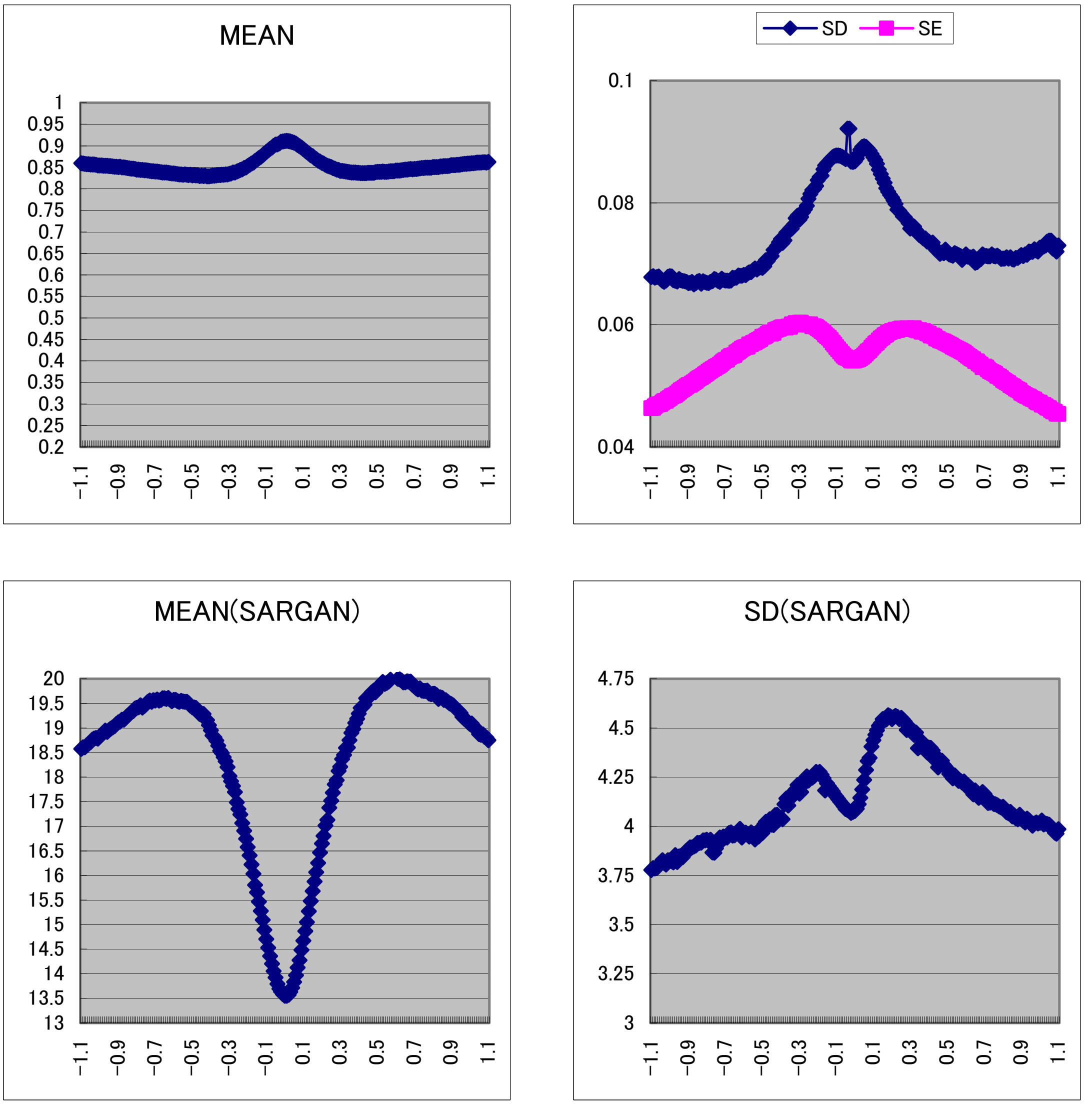


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.42	-0.01
MEAN	0.174	0.171	0.829	0.910
[SD, SE]	[0.078, 0.069]	[0.086, 0.069]	[0.073, 0.059]	[0.087, 0.054]
MEAN(SARGAN)	14.776	14.534	19.159	13.586
SD(SARGAN)	5.092	5.009	4.059	4.074

[Exhibit 2-4-0.5]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2 / \sigma_v^2, \alpha^*) = (4, 0.5)$ when the value of θ changes.

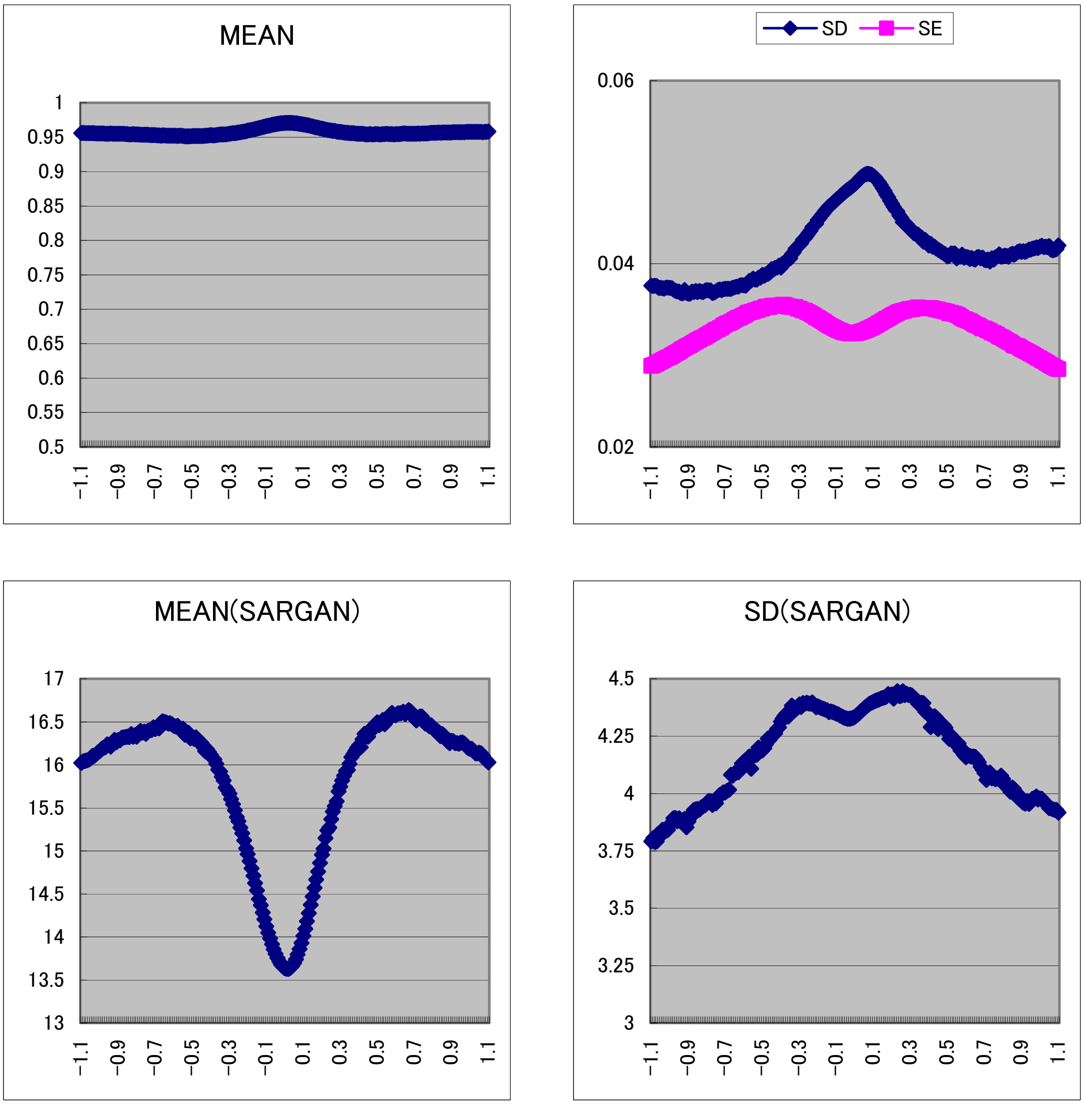


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-0.54	-0.01
MEAN	0.437	0.433	0.951	0.970
[SD, SE]	[0.106, 0.095]	[0.117, 0.095]	[0.038, 0.035]	[0.049, 0.032]
MEAN(SARGAN)	14.917	14.672	16.349	13.654
SD(SARGAN)	5.217	5.144	4.167	4.330

[Exhibit 2-4-0.8]

Monte Carlo Statistics using **GMM(MGF[2]-FD)** estimators ($n = 2$)

for $(\sigma_\eta^2/\sigma_v^2, \alpha^*) = (4, 0.8)$ when the value of θ changes.

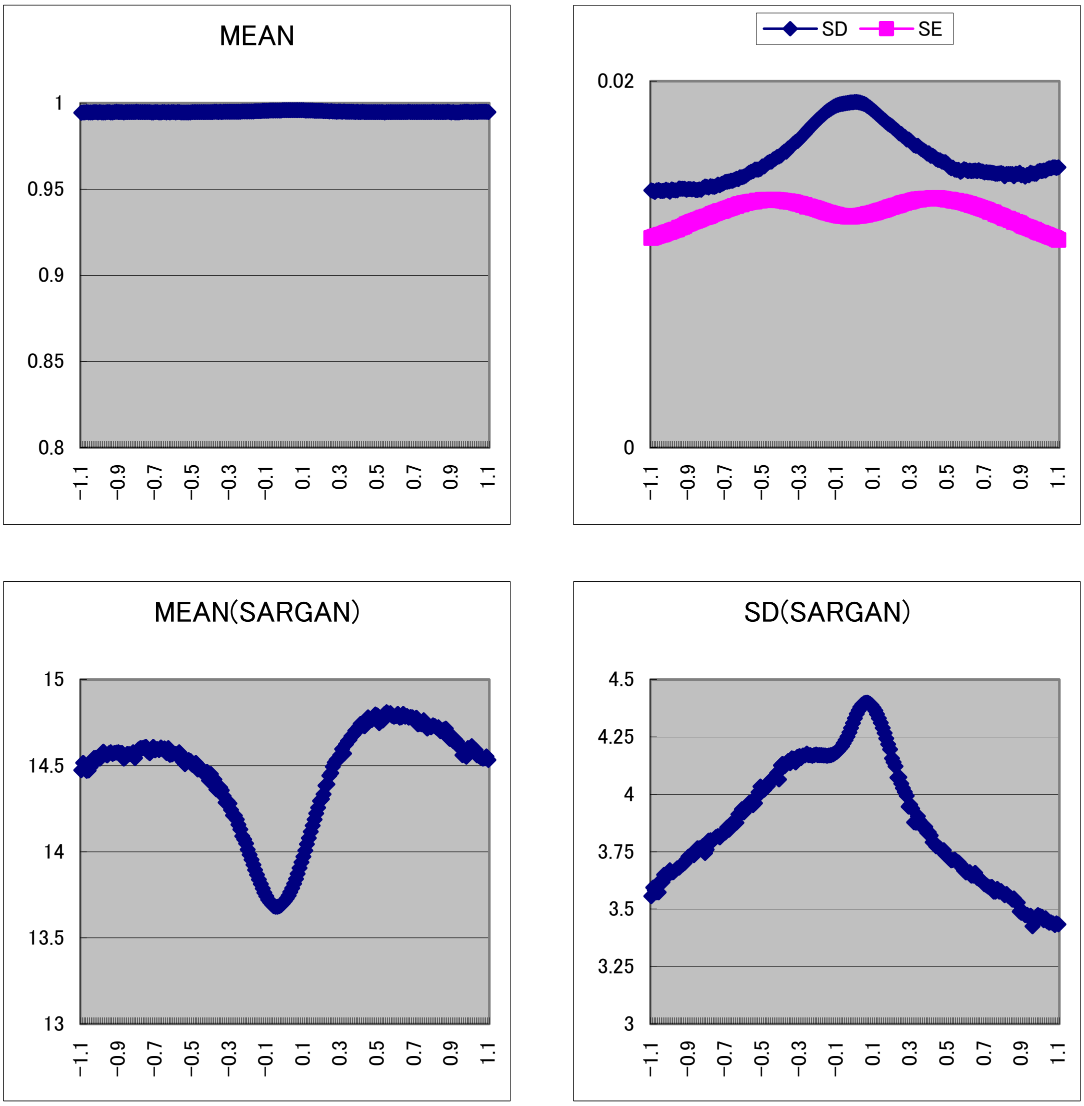


Table. Representative Monte Carlo results (DF=14)

	A-B(ONE-STEP)	A-B(TWO-STEP)	NEAREST	MINIMUM
θ	-	-	-1.1	-0.04
MEAN	0.558	0.523	0.994	0.996
[SD, SE]	[0.195, 0.165]	[0.236, 0.165]	[0.014, 0.011]	[0.019, 0.013]
MEAN(SARGAN)	15.134	14.792	14.474	13.680
SD(SARGAN)	5.182	5.050	3.556	4.252