Exploration of dynamic fixed effects logit models from a traditional angle

Author: Yoshitsugu Kitazawa (Kyushu Sangyo University)

21st International Panel Data Conference

June 29, 2015(Mon) ~ June 30(Tue)

Central European University, Budapest, Hungary

Paper Download: http://www.ip.kyusan-u.ac.jp/keizai-kiyo/dp60.pdf

Revised: http://paneldataconference2015.ceu.hu/Program/Yoshitsugu-Kitazawa.pdf

(Contents)

- This paper proposes the transformations for root-N-consistently estimating the dynamic fixed effects logit models.
- Transformations for the dynamic fixed effects logit model without explanatory variable

  → root-N consistent GMM estimators

  → Construction of the conditional maximum likelihood estimator (CMLE) proposed by Chamberlain (1985)
- Transformations for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables

  → root-N consistent GMM estimators（Until now, it was the dogma that no root-N consistent estimator is feasible for this model.）
- Monte Carlo experiments

（Intro.）　Motivation of this paper（significance of developing the estimators for the dynamic panel logit）

- Incorporating dynamics into the binary choice models is one of the issues which attract the interest of econometricians（allowing for the persistence of an event in past）.
- In this case, the logit specification is often used ー because of tractability
- In this case, the micro datasets are often dealt with ー which are much more accessible than before.（example: household brand choice and female labor force participation）
- In many cases, the micro datasets available have the panel structure where the number of individuals is large but the number of time periods is small.
- Accordingly, it is meaningful that we develop the estimators for the dynamic panel logit with time length being short.

(Intro.)　Objective of this paper (Proposal of the root-N consistent estimators for the dynamic fixed effects logit model)

- We deal with the fixed effects model, because the fixed effects model is more flexible than the random effects model.
- However, the fixed effects model is haunted by the incidental parameters problem.
- Solving the incidental parameters problem using the fairly traditional approach gives rise to the (asymptotically normal) root-N consistent estimators (in which the convergence rate equals the inverse of the square root of the cross-sectional sample size) for the dynamic fixed effects logit models.
- We deal with both dynamic fixed effects logit models without explanatory variables and with strictly exogenous continuous explanatory variables.

（Intro.） Conventional estimator for the dynamic fixed effects logit model without explanatory variables

- Conditional Maximum Likelihood Estimator (CMLE) proposed by Chamberlain (1985)
    - Root-N consistent estimator
    - necesiates 4 or more time periods (unless the initial conditions are specified)

（Intro.） Conventional (so-called nontraditional) estimation methods for the dynamic fixed effects logit models with strictly exogenous continuous explanatory variables

- No root-N consistent estimator is proposed until now.
- **Estimator proposed by Honoré and Kyriazidou (2000)** — uses Kernel weight.（The convergence rate is slower than root-N, depending on cross-sectional size (N) and band width）.
- **Bias-correction estimators** (Carro (2007), Bester and Hansen (2009),Fernández-Val (2009), Hahn and Kuersteiner (2011), and Yu et al. (2012)) — Unbiased estimator if number of time periods is moderately large, but not root-N consistent.
- **Pseudo CMLE**(Bartolucci and Nigro (2012)) — one of the approximation estimators and accordingly not root-N consistent for the true model.

（Intro.） Estimation methods in this paper for dynamic fixed effects logit model without explanatory variable

- Step 1： The model is transformed into the simple linear panel data models with additive fixed effects.
- Step 2： The error-components structures holding between the logit model and the transformed linear panel data models give the valid moment conditions (including the stationarity moment conditions).
- Step 3： Using these moment conditions, we root-N-consistently estimate the parameter of interest by GMM.
- Step 4： It is shown that the first-order condition of the CMLE proposed by Chamberlain (1985) can be rewritten as the combinations of some of these moment conditions.

(Intro.)　Estimation methods in this paper for dynamic fixed effects logit model with strictly exogenous continuous explanatory variables

- Step 1：　The model is transformed in order that the logit probabilities composed of the fixed effects and the explanatory variables are separated out as the additive terms.
- Step 2：　Next, the valid moment conditions, which need four or more time periods, are obtained by applying a variety of the hyperbolic tangent differencing (HTD) transformation proposed by Kitazawa (2012) to the transformed forms of the model.
- Step 3：　Using these moment conditions, we root-N-consistently estimate the parameters of interest by GMM.
- Accordingly, the root-N consistent estimators are feasible for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables, as long as the time periods are four or more.

(Intro.) Dynamic fixed effects logit model with strictly exogenous continuous explanatory variables

- Now, a window is opened into the sense of stagnation in which the recent researches on the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables seem to be packed in the atmosphere of relinquishing the pursuit of the root-N consistent estimators.
- The window is opened by dint of the extremely traditional reaction.
- Hahn's (2001) suggestion is no longer applicable to the case of four or more time periods. (The suggestion states that the root-N consistent estimation is infeasible in general specifications including strictly exogenous continuous explanatory variables in the dynamic fixed effects logit.)
- To the best of author's knowledge, this paper for the first time proposes the root-N consistent estimators for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables.

(Intro.) Structure of the paper

- Section 2： (1) Presentation of the dynamic fixed effects logit model without explanatory variable and with strictly exogenous continuous explanatory variable. (2) Derivation of valid moment conditions for both models. (3) Construction of root-N consistent GMM estimator.
- Section 3： Monte Carlo experiments for the GMM estimators constructed in section 2
- Section 4: Conclusion

（Model and estimation） Dynamic fixed effects logit model without explanatory variable (Simple dynamic fixed effects model) and its estimation （Implicit form）

- Simple dynamic fixed effects logit model

$$y_{it} = p(\eta_i, y_{i,t-1}) + v_{it} \qquad \text{for } t = 2,\dots,T$$

$$\mathrm{E}[v_{it} \mid \eta_i, y_{i1}, v_i^{t-1}] = 0$$

$$p(\eta_i, y_{i,t-1}) = \exp(\eta_i + \gamma y_{i,t-1}) / (1 + \exp(\eta_i + \gamma y_{i,t-1}))$$

Probability with which y_{it} takes one

η_i: fixed effect y_{it}: dep. var.（1 or 0） γ: Parameters of interest

v_{it}: disturbance i: individual t: time $i = 1,\dots,N$ $t = 1,\dots,T$

$N \rightarrow \infty$ T being fixed

（Model and estimation） Dynamic fixed effects logit model without explanatory variable (Simple dynamic fixed effects model) and its estimation（Shape-shifting）

- We rewrite the probability in the simple dynamic fixed effects logit model:

$$p(\eta_i, y_{i,t-1}) = f(\eta_i) y_{i,t-1} + g(\eta_i)$$

$$f(\eta_i) = h(\eta_i) - g(\eta_i)$$

$$g(\eta_i) = \exp(\eta_i) / (1 + \exp(\eta_i))$$

$$h(\eta_i) = \exp(\eta_i + \gamma) / (1 + \exp(\eta_i + \gamma))$$

- The above is the logit specification of the linear AR(1) (autoregressive model of order 1) regression form considered by Al-Sadoon et al. (2012) for the dynamic binary choice panel data model with fixed effects

（Model and estimation） Dynamic fixed effects logit model without explanatory variable (Simple dynamic fixed effects model) and its estimation（Transformation）

- Two types of transformations of the simple dynamic fixed effects logit model into the linear panel data model with additive fixed effects
- g-form

$$y_{it} = \delta y_{i,t-1}(1 - y_{it}) y_{i,t+1} + g(\eta_i) + w_{it} \qquad \text{for } 2 \le t \le T-1$$

$$\mathrm{E}[w_{it} \mid \eta_i, y_{i1}, v_i^{t-1}] = 0$$

Fixed effect

- h-form

$$y_{it} = -\delta(1 - y_{i,t-1}) y_{it}(1 - y_{i,t+1}) + h(\eta_i) + \omega_{it} \qquad \text{for } 2 \le t \le T-1$$

$$\mathrm{E}[\omega_{it} \mid \eta_i, y_{i1}, v_i^{t-1}] = 0$$

Fixed effect

- w_{it} , ω_{it} : new disturbances. $\delta = \exp(\gamma) - 1$: parameter of interest

（Model and estimation） Dynamic fixed effects logit model without explanatory variable (Simple dynamic fixed effects model) and its estimation（Standard moment conditions）

- Standard linear moment conditions are obtains for consistently estimating δ, by eliminating the fixed effects g(η_i) と h(η_i) (using the first-differencing transformation).
- Standard moment conditions based on g-form

$$E[\Delta u_{it}] = 0 \quad \text{for} \quad 3 \le t \le T-1$$

$$E[y_{is}\Delta u_{it}] = 0 \quad \text{for} \quad 1 \le s \le t-2; \quad 3 \le t \le T-1$$

$$u_{it} = y_{it} - \delta y_{i,t-1}(1-y_{it})y_{i,t+1}$$

- Standard moment conditions based on h-form

$$E[\Delta \upsilon_{it}] = 0 \quad \text{for} \quad 3 \le t \le T-1$$

$$E[y_{is}\Delta \upsilon_{it}] = 0 \quad \text{for} \quad 1 \le s \le t-2; \quad 3 \le t \le T-1$$

$$\upsilon_{it} = y_{it} + \delta(1-y_{i,t-1})y_{it}(1-y_{i,t+1})$$

（Model and estimation） Dynamic fixed effects logit model without explanatory variable (Simple dynamic fixed effects model) and its estimation （Stationarity）

- Stationarity for simple dynamic fixed effects logit model
- If the initial conditions for the binary dependent variables are written as follows:

$$y_{i1} = g(\eta_i)/(1 - f(\eta_i)) + v_{i1}$$

$$\mathrm{E}[v_{i1} \mid \eta_i] = 0$$

- binary dependent variables y_{it} are stationary:

$$\mathrm{E}[y_{it} \mid \eta_i] = g(\eta_i)/(1 - f(\eta_i))$$

（Model and estimation） Dynamic fixed effects logit model without explanatory variable (Simple dynamic fixed effects model) and its estimation (Stationarity moment conditions)

- (Linear) stationarity moment conditions root-N-consistently estimating δ for the simple dynamic fixed effects logit model
- Stationarity moment conditions based on g-form

$$\mathrm{E}[\Delta y_{i,t-1}\, u_{it}] = 0 \qquad \text{for} \quad 3 \le t \le T-1$$

- Stationarity moment conditions based on h-form

$$\mathrm{E}[\Delta y_{i,t-1}\, \upsilon_{it}] = 0 \qquad \text{for} \quad 3 \le t \le T-1$$

- These correspond the stationarity moment conditions for the ordinary linear dynamic panel data model（Arellano and Bover (1995); Ahn and Schmidt (1995); Blundell and Bond (1998)）.

（Model and estimation） Relationships with CMLE proposed by Chamberlain (1985)

- The CMLE proposed by Chamberlain (1985) for the dynamic fixed effects logit model without explanatory variable maximize the following likelihood with respect to γ :
  $\sum_{i=1}^{N} \ell_{it}$
  with $\ell_{it} = (\Delta y_{it})^2 (\gamma y_{i,t-1}(y_{i,t-2} - y_{i,t+1}) - \ln(1+\exp(\gamma(y_{i,t-2} - y_{i,t+1}))))$
- For *T*=4, this CMLE is effeicient under
  $(\Delta y_{it})^2 = 1$ and/or $(y_{i,t-2} - y_{i,t+1})^2 = 1$

(Model and estimation)　Relationships with CMLE proposed by Chamberlain (1985)

- The first-order condition for the CMLE proposed by Chamberlain (1985) for the dynamic fixed effects logit model without explanatory variable is written as the sum of the standard moment conditions based on g-form and h-form:

$$\mathrm{E}[(1-y_{i,t-2})\Delta u_{it} - y_{i,t-2}\Delta v_{it}] = 0$$

- If the binary dependent variable y_{it} is stationary, the above first-order condition is written as the sum of the stationarity moment conditions based on g-form and h-form:

$$\mathrm{E}[\Delta y_{i,t-1}(u_{it} + v_{it})] = 0$$

（Model and estimation） Dynamic fixed effects logit model with strictly exogenous continuous explanatory variables and its estimation（Implicit form）

- Dynamic fixed effects logit model with the strictly exogenous continuous explanatory variable

$$y_{it} = p(\eta_i, y_{i,t-1}, x_{it}) + v_{it} \quad \text{for } t = 2,\dots,T$$

$$\mathrm{E}[v_{it} \mid \eta_i, y_{i1}, v_i^{t-1}, x_i^T] = 0$$

Probability with which y_{it} takes one

$$p(\eta_i, y_{i,t-1}, x_{it}) = \exp(\eta_i + \gamma y_{i,t-1} + \beta x_{it}) / (1 + \exp(\eta_i + \gamma y_{i,t-1} + \beta x_{it}))$$

η_i: fixed effect y_{it}: dep. var.（1 or 0）

x_{it}: strictly exogenous continuous explanatory variable

γ, β: parameters of interest v_{it}: disturbance

i: individual t: time $i = 1,\dots,N$ $t = 1,\dots,T$ $N \rightarrow \infty$ T being fixed

（ Model and estimation） Dynamic fixed effects logit model with strictly exogenous continuous explanatory variables and its estimation （Shape-shifting）

- Rewriting the probability of Dynamic fixed effects logit model with the strictly exogenous continuous explanatory variable,

$$p(\eta_i, y_{i,t-1}, x_{it}) = f(\eta_i, x_{it})\, y_{i,t-1} + g(\eta_i, x_{it})$$

$$f(\eta_i, x_{it}) = h(\eta_i, x_{it}) - g(\eta_i, x_{it})$$

$$g(\eta_i, x_{it}) = \exp(\eta_i + \beta x_{it}) / (1 + \exp(\eta_i + \beta x_{it}))$$

$$h(\eta_i, x_{it}) = \exp(\eta_i + \gamma + \beta x_{it}) / (1 + \exp(\eta_i + \gamma + \beta x_{it}))$$

- The above shape-shifting is the logit specification of the linear AR(1) (autoregressive model of order 1) regression form considered by Al-Sadoon et al. (2012) for the dynamic binary choice panel data model with fixed effects

（Model and estimation） Dynamic fixed effects logit model with strictly exogenous continuous explanatory variables and its estimation (transformation)

- Two types of transformations in which the logit probabilities composed of the fixed effects and the explanatory variables are separated out as the additive terms.
- g-form $U_{it} = g(\eta_i, x_{i,t+1}) + W_{it}$ for $2 \le t \le T-1$ $\quad \mathrm{E}[W_{it} \mid \eta_i, y_{i1}, v_i^{t-1}, x_i^T] = 0$

$$U_{it} = y_{it} + (1-y_{it})y_{i,t+1} - (1-y_{it})y_{i,t+1}\exp(-\beta\,\Delta x_{i,t+1}) - \delta y_{i,t-1}(1-y_{it})y_{i,t+1}\exp(-\beta\,\Delta x_{i,t+1}),$$

- h-form $\Upsilon_{it} = h(\eta_i, x_{i,t+1}) + \Omega_{it}$ for $2 \le t \le T-1$ $\quad \mathrm{E}[\Omega_{it} \mid \eta_i, y_{i1}, v_i^{t-1}, x_i^T] = 0$

$$\Upsilon_{it} = y_{it}y_{i,t+1} + y_{it}(1-y_{i,t+1})\exp(\beta\,\Delta x_{i,t+1}) + \delta(1-y_{i,t-1})y_{it}(1-y_{i,t+1})\exp(\beta\,\Delta x_{i,t+1}),$$

- W_{it} , Ω_{it} : new disturbances $\quad \delta = \exp(\gamma) - 1$, β : parameters of interest

（Model and estimation） Dynamic fixed effects logit model with strictly exogenous continuous explanatory variable and its estimation（moment conditions for root-N-consistently estimating γ and β）

- By ruling out the fixed effects η_i in g(η_i, $x_{i,t+1}$) and h(η_i, $x_{i,t+1}$) using the variant of the hyperbolic tangent differencing (HTD) transformation（kitazawa, 2012）, the moment conditions for root-N-consistently estimating γ and β are obtained.
- Mom. cond. based on g-form $\mathrm{E}[\hbar U_{it} \mid \eta_i, y_{i1}, v_i^{t-2}, x_i^T] = 0$ for $3 \le t \le T-1$

$$\hbar U_{it} = U_{it} - y_{i,t-1} - \tanh((1/2)(-\gamma y_{i,t-2} + \beta(\Delta x_{it} + \Delta x_{i,t+1})))(U_{it} + y_{i,t-1} - 2U_{it}\, y_{i,t-1}),$$

- Mom. cond. based on h-form $\mathrm{E}[\hbar \Upsilon_{it} \mid \eta_i, y_{i1}, v_i^{t-2}, x_i^T] = 0$ for $3 \le t \le T-1$

$$\hbar \Upsilon_{it} = \Upsilon_{it} - y_{i,t-1} - \tanh((1/2)(\gamma(1 - y_{i,t-2}) + \beta(\Delta x_{it} + \Delta x_{i,t+1})))(\Upsilon_{it} + y_{i,t-1} - 2\Upsilon_{it}\, y_{i,t-1}).$$

（Model and estimation）Dynamic fixed effects logit models and their estimations (Estimation method: GMM)

- Vector of moment conditions' set $\mathrm{E}[\varphi_i(\theta)] = 0$ (m × 1)

  m: # of moment cond., $\theta = \gamma$ （without regressor）, $\theta = (\gamma, \beta)$ （with regressor)

- Optimal GMM estimator $\hat{\theta}^{GMM}$ is obtained by minimizing the following quadratic form with respect to θ.（$\hat{\theta}_1$ is any consistent estimator of θ .)

  $\bar{\varphi}(\theta)'\left(\bar{\Theta}(\hat{\theta}_1)\right)^{-1}\bar{\varphi}(\theta)$ with $\bar{\varphi}(\theta) = (1/N)\sum_{i=1}^{N}\varphi_i(\theta)$, $\bar{\Theta}(\hat{\theta}_1) = (1/N)\sum_{i=1}^{N}\varphi_i(\hat{\theta}_1)\varphi_i(\hat{\theta}_1)'$

- Optimal GMM estimator is the (asymptotically normal) root-N consistent estimator:

  $$N^{1/2}(\hat{\theta}_{\mathrm{GMM}} - \theta_0) \xrightarrow{d} \mathrm{N}(0,\ (D(\theta_0)'(\Theta(\theta_0))^{-1} D(\theta_0))^{-1})$$

  where $D(\theta_0) = (\partial\, \mathrm{E}[\varphi_i(\theta)]/\partial\theta')|_{\theta=\theta_0}$ and $\Theta(\theta_0) = \mathrm{E}[\varphi_i(\theta_0)\varphi_i(\theta_0)']$

θ_0 : true value of θ

（Monte Carlo） Model without explanatory variables

- DGP for the model without explanatory variables

$$y_{it} = \begin{cases} 1 & \text{if } p(\eta_i, y_{i,t-1}) > \zeta_{it} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{i1} = \begin{cases} 1 & \text{if } q(\eta_i) > \zeta_{i1} \\ 0 & \text{otherwise} \end{cases}$$

$$p(\eta_i, y_{i,t-1}) = \exp(\eta_i + \gamma y_{i,t-1}) / (1 + \exp(\eta_i + \gamma y_{i,t-1}))$$

$$q(\eta_i) = 1 / (1 + (1 + \exp(\eta_i)) / (\exp(\eta_i)(1 + \exp(\eta_i + \gamma))))$$

$$\zeta_{it} \sim \mathrm{U}(0,1) \qquad \eta_i \sim \mathrm{N}(0, \sigma_\eta^2)$$

- Parameters with values being set： γ , σ_η^2
- Cross-sectional sample size N=1000, 5000, 10000 # of time periods T = 4, 8
  # of replications R_N = 10000

（Monte Carlo） Model without explanatory variables

- GMM estimator to be investigated
- GMM(g-STD): GMM estimator only using the standard moment conditions based on g-form
- GMM(g-SYS): GMM estimator using both the standard moment conditions and stationarity moment conditions based on g-form
- GMM(h-STD): GMM estimator only using the standard moment conditions based on h-form
- GMM(h-SYS): GMM estimator using both the standard moment conditions and stationarity moment conditions based on h-form
- **GMM(FOC-o)**: GMM estimator using the first-order conditions of CMLE
- **GMM(FOC-s)**: GMM estimator using the first-order conditions of CMLE assuming the stationarity of dependent variables

（Monte Carlo） Model without explanatory variables

- Table 2. Monte Carlo results for the (simple) dynamic fixed effects logit model without explanatory variable, T=8

	N = 1000		N = 5000		N = 10000	
	bias	rmse	bias	rmse	bias	rmse
Simulation (1a)						
GMM(g-STD) γ	**−0.066**	**0.107**	**−0.012**	**0.038**	**−0.006**	**0.026**
GMM(g-SYS) γ	−0.049	0.090	−0.009	0.034	−0.004	0.023
GMM(h-STD) γ	**−0.058**	**0.101**	**−0.011**	**0.038**	**−0.005**	**0.026**
GMM(h-SYS) γ	−0.050	0.089	−0.009	0.034	−0.005	0.023
GMM(FOC-o) γ	**−0.006**	**0.095**	**0.000**	**0.042**	**−0.001**	**0.030**
GMM(FOC-s) γ	−0.004	0.103	0.000	0.046	−0.001	0.033
Simulation (1b)						
GMM(g-STD) γ	**−0.090**	**0.134**	**−0.017**	**0.045**	**−0.008**	**0.031**
GMM(g-SYS) γ	−0.064	0.107	−0.012	0.038	−0.005	0.026
GMM(h-STD) γ	**−0.084**	**0.130**	**−0.016**	**0.045**	**−0.007**	**0.031**
GMM(h-SYS) γ	−0.068	0.109	−0.013	0.038	−0.006	0.026
GMM(FOC-o) γ	**−0.007**	**0.107**	**−0.001**	**0.048**	**0.000**	**0.034**
GMM(FOC-s) γ	−0.005	0.116	−0.001	0.052	0.000	0.037
Simulation (1c)						
GMM(g-STD) γ	**−1.118**	**1.243**	**−0.102**	**0.139**	**−0.037**	**0.072**
GMM(g-SYS) γ	−0.938	1.027	−0.090	0.128	−0.035	0.070
GMM(h-STD) γ	**−0.695**	**0.780**	**−0.085**	**0.125**	**−0.038**	**0.074**
GMM(h-SYS) γ	−0.566	0.639	−0.062	0.107	−0.030	0.069
GMM(FOC-o) γ	**−0.020**	**0.253**	**0.000**	**0.109**	**−0.002**	**0.078**
GMM(FOC-s) γ	−0.017	0.258	0.000	0.113	−0.002	0.081
Simulation (1d)						
GMM(g-STD) γ	**−1.176**	**1.310**	**−0.118**	**0.156**	**−0.042**	**0.077**
GMM(g-SYS) γ	−0.920	1.002	−0.096	0.135	−0.038	0.074
GMM(h-STD) γ	**−0.918**	**1.023**	**−0.108**	**0.146**	**−0.044**	**0.080**
GMM(h-SYS) γ	−0.662	0.745	−0.071	0.114	−0.032	0.071
GMM(FOC-o) γ	**−0.024**	**0.256**	**−0.002**	**0.111**	**−0.001**	**0.079**
GMM(FOC-s) γ	−0.022	0.260	−0.002	0.115	−0.001	0.082

Simulation (1a): γ Low, σ_η^2 Small
γ=0.5, σ_η^2=0.5

Simulation (1b): γ Low, σ_η^2 Large
γ=0.5, σ_η^2=1.5

Simulation (1c): γ High, σ_η^2 Small
γ=2.5, σ_η^2=0.5

Simulation (1d): γ High, σ_η^2 Large
γ=2.5, σ_η^2=1.5

（Monte Carlo） Model without explanatory variables

- The additional usage of the stationarity moment conditions improves the small sample performances of the GMM estimators, especially for the high value of the persistence parameter γ （comparing the results of GMM(g-STD) and GMM(h-STD) with those of GMM(g-SYS) and GMM(h-SYS) ）.
- However, the dramatic improvement in terms of bias and rmse for the high value of the persistence parameter is conducted by using the **GMM(FOC-o)** estimator (which uses the first-order conditions of the CMLE written as the plain sums of fractions of the moment conditions used mainly in the GMM(g-STD) and GMM(h-STD) estimators).
- It cannot be said that the GMM(FOC-s) estimator (which uses the first-order conditions of the CMLE written using the plain sums of the stationarity moment conditions) behaves well for the low value of the persistent parameter γ and the large value of the σ_η^2.

(Monte Carlo) Model with strictly exogenous continuous explanatory variables

- DGP for the model with strictly exogenous continuous regressor

$$y_{it} = \begin{cases} 1 & \text{if } p(\eta_i, y_{i,t-1}, x_{it}) > \zeta_{it} \\ 0 & \text{otherwise} \end{cases} \qquad y_{i1} = \begin{cases} 1 & \text{if } q(\eta_i, x_{i1}) > \zeta_{i1} \\ 0 & \text{otherwise} \end{cases}$$

$$p(\eta_i, y_{i,t-1}, x_{it}) = \exp(\eta_i + \gamma y_{i,t-1} + \beta x_{it}) / (1 + \exp(\eta_i + \gamma y_{i,t-1} + \beta x_{it}))$$

$$q(\eta_i, x_{i1}) = 1 / (1 + (1 + \exp(\eta_i + \beta x_{i1})) / (\exp(\eta_i + \beta x_{i1})(1 + \exp(\eta_i + \gamma + \beta x_{i1}))))$$

$$x_{it} = \rho x_{i,t-1} + \tau \eta_i + \varepsilon_{it} \qquad x_{i1} = (\tau / (1-\rho))\eta_i + (1/(1-\rho^2)^{1/2})\varepsilon_{i1}$$

$$\zeta_{it} \sim \mathrm{U}(0,1) \qquad \eta_i \sim \mathrm{N}(0,\ \sigma_\eta^2) \qquad \varepsilon_{it} \sim \mathrm{N}(0, \sigma_\varepsilon^2)$$

- Parameters with values being set: $,\ \gamma,\ \beta,\ \rho,\ \tau,\ \sigma_\eta^2\ \sigma_\varepsilon^2$
- Cross-sectional sample size *N*=1000, 5000, 10000 # of time periods *T* = 4, 8
  # of Replications R_N = 10000

（Monte Carlo）　Model with strictly exogenous continuous explanatory variables

- GMM(g-HTD) : Moment conditions used by [GMM based on g-form]
- $\mathrm{E}[\hbar U_{it}]=0$ for $3 \le t \le T-1$
- $\mathrm{E}[y_{is}\,\hbar U_{it}]=0$ for $1 \le s \le t-2$; $3 \le t \le T-1$
- $\mathrm{E}[\Delta x_{is}\,\hbar U_{it}]=0$ for $t-1 \le s \le t+1$; $3 \le t \le T-1$
- GMM(h-HTD) : Moment conditions used by [GMM based on h-form]
- $\mathrm{E}[\hbar \Upsilon_{it}]=0$ or $3 \le t \le T-1$
- $\mathrm{E}[y_{is}\,\hbar \Upsilon_{it}]=0$ for $1 \le s \le t-2$; $3 \le t \le T-1$
- $\mathrm{E}[\Delta x_{is}\,\hbar \Upsilon_{it}]=0$ for $t-1 \le s \le t+1$; $3 \le t \le T-1$

（Monte Carlo） Model with strictly exogenous continuous explanatory variables

- Table 4. . Monte Carlo results for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variable, *T*=8

		N = 1000		N = 5000		N = 10000	
		bias	rmse	bias	rmse	bias	rmse
Simulation (2a)							
GMM(g-HTD)	γ	**−0.047**	**0.110**	**−0.010**	**0.044**	**−0.004**	**0.030**
	β	0.000	0.060	0.000	0.026	0.000	0.019
GMM(h-HTD)	γ	**−0.038**	**0.105**	**−0.008**	**0.044**	**−0.003**	**0.031**
	β	−0.005	0.060	−0.001	0.027	−0.001	0.019
Simulation (2b)							
GMM(g-HTD)	γ	**−0.079**	**0.152**	**−0.016**	**0.059**	**−0.007**	**0.040**
	β	0.007	0.082	0.002	0.036	0.001	0.026
GMM(h-HTD)	γ	**−0.067**	**0.146**	**−0.013**	**0.059**	**−0.006**	**0.041**
	β	−0.007	0.082	−0.001	0.037	0.000	0.026
Simulation (2c)							
GMM(g-HTD)	γ	**−0.198**	**0.294**	**−0.035**	**0.099**	**−0.016**	**0.067**
	β	0.025	0.128	0.010	0.059	0.005	0.041
GMM(h-HTD)	γ	**−0.179**	**0.279**	**−0.034**	**0.102**	**−0.017**	**0.069**
	β	0.006	0.125	0.006	0.059	0.004	0.042

Simulation (2a): low persistence
γ=0.5, β=0.5, ρ=0.5, σ_η^2=0.5, σ_ε^2=0.5

Simulation (2b): moderate persistence
γ=0.8, β=0.8, ρ=0.7, σ_η^2=0.5, σ_ε^2=0.5

Simulation (2c): high persistence
γ=1.1, β=1.1, ρ=0.9, σ_η^2=0.5, σ_ε^2=0.5

（Monte Carlo）　Model with strictly exogenous continuous explanatory variables

- The size alleviations of bias and rmse for the GMM(g-HTD) and GMM(h-HTD) estimators back up the presence of the root-N consistent estimators for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables.
- The larger downward biases for the GMM(g-HTD) and GMM(h-HTD) estimators of the persistence parameter γ are recognizable when the data of the dependent and explanatory variables are more persistent（presumably due to the weak instruments problem）.
- The same is true of the coefficient β on the explanatory variable.
- The sizes of bias and rmse with respect to β are small, compared to those with respect to γ.

#（Conclusion）

- In this paper, the transformations and valid moment conditions were advocated for the dynamic fixed effects logit models without explanatory variable and with strictly exogenous continuous explanatory variables.
- For the model without explanatory variable, the valid moment conditions are constructed based on the error-components structures after the model is transformed into the simple linear panel data models with additive fixed effects.
- For the model with strictly exogenous continuous explanatory variables, the valid moment conditions are constructed by applying a variety of the HTD transformation (Kitazawa, 2012) after the model is transformed in order that the logit probabilities composed of the fixed effects and the explanatory variables are separated out as the additive terms.

(Conclusion)

- The climax of the paper is that if number of time periods of panel data is four or more, **the root-N consistent GMM estimators** can be constructed for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables.
- It was the traditional approach that was conducive to constructing the root-N consistent estimators for the dynamic fixed effects logit model with strictly exogenous continuous explanatory variables.

(Postscript)

- Honoré and Kyriazidou's (2000) estimator cannot estimate the dynamic fixed effects logit models with time dummies, since they use the kernel weight（see Carro, 2007, etc）.
- Hahn's (2000) suggestion states that the root-N consistent estimations of the time dummies for these models are infeasible.
- However, the estimator proposed in this paper can root-N-consistently estimate the time dummies as well as other parameters of interest in this models.

(Postscript)

- We illustrate the feasibility of the root-N-consistent estimation for the dynamic fixed effects logit model with time dummies by using some Monte Carlo experiments.
- Convergence rates are faster than or approximately $\sqrt{N}$ in almost all cases in Monte Carlo.

Memo: The convergence rate of the vector of parameters of interest to be estimated by the estimators proposed in this paper for the dynamic fixed effects logit model with explanatory variables is $\sqrt{N}$ even if number of explanatory variables increase. On the contrary, Honoré and Kyriazidou's (2000) estimator becomes much slower and slower than $\sqrt{N}$ for the larger number of explanatory variables.

(P.S. 1) Dynamic fixed effects logit model（with strictly exogenous explanatory variable + time dummies）

- Dynamic fixed effects logit model（with strictly exogenous explanatory variable + time dummies）

$$y_{it} = \frac{\exp\left(\eta_i + TD_t + \gamma\, y_{i,t-1} + \beta x_{it}\right)}{1 + \exp\left(\eta_i + TD_t + \gamma\, y_{i,t-1} + \beta x_{it}\right)} + v_{it}$$

- y_{it}: binary dependent variable for individual i at time t（1 or 0）, taking 1 with above probability
- η_i: fixed effect TD_t: time dummy x_{it}:strictly exo. continuous regressor v_{it}: disturbance
- γ, β: parameters of interest
- Assumptions on disturbances: $\mathrm{E}[v_{it} \mid \eta_i, y_{i1}, v_i^{t-1}, x_i^T] = 0$
- $v_i^{t-1} = (v_{i1}, \dots, v_{i,t-1})$ $x_i^T = (x_{i1}, \dots, x_{iT})$

i: individual t: time
i = 1,…,N t =2,…,T
N → ∞ T being fixed

(P.S. 1) Dynamic fixed effects logit model（with strictly exogenous explanatory variable + time dummies）

- Conditional moment conditions（g-form）, derived the same method as kitazawa (2013)

$$\mathrm{E}[\hbar U_{it}^{+} \mid \eta_i, y_{i1}, v_{i,t\text{-}2}, x_i^{T}] = 0$$

where
$$\hbar U_{it}^{+} = U_{it}^{+} - y_{i,t-1}$$
$$-\tanh\left(\frac{-\gamma y_{i,t-2} + \Delta TD_t + \Delta TD_{t+1} + \beta(\Delta x_{it} + \Delta x_{i,t+1})}{2}\right)(U_{it}^{+} + y_{i,t-1} - 2U_{it}^{+}\, y_{i,t-1})$$

with
$$U_{it}^{+} = y_{it} + (1 - y_{it})\, y_{i,t+1} - (1 - y_{it})\, y_{i,t+1} \exp\left(-\Delta TD_{t+1} - \beta\, \Delta x_{i,t+1}\right)$$
$$-(\exp(\gamma) - 1)\, y_{i,t-1}(1 - y_{it})\, y_{i,t+1} \exp\left(-\Delta TD_{t+1} - \beta\, \Delta x_{i,t+1}\right)$$

(P.S. 1) Dynamic fixed effects logit model（with strictly exogenous explanatory variable + time dummies）

- Conditional moment conditions（g-form），derived the same method as kitazawa (2013)

$$\mathrm{E}[\hbar\Upsilon_{it}^{+} \mid \eta_i, y_{i1}, v_{i,t\text{-}2}, x_i^T] = 0$$

where $$\hbar\Upsilon_{it}^{+} = \Upsilon_{it}^{+} - y_{i,t-1}$$

$$-\tanh\left(\frac{\gamma(1-y_{i,t-2}) + \Delta TD_t + \Delta TD_{t+1} + \beta(\Delta x_{it} + \Delta x_{i,t+1})}{2}\right)(\Upsilon_{it}^{+} + y_{i,t-1} - 2\Upsilon_{it}^{+} y_{i,t-1})$$

with $$\Upsilon_{it}^{+} = y_{it} y_{i,t+1} + y_{it}(1-y_{i,t+1})\exp\left(\Delta TD_{t+1} + \beta\Delta x_{i,t+1}\right)$$

$$+(\exp(\gamma)-1)(1-y_{i,t-1})y_{it}(1-y_{i,t+1})\exp\left(\Delta TD_{t+1} + \beta\Delta x_{i,t+1}\right)$$

(P.S. 1) Dynamic fixed effects logit model（with strictly exogenous explanatory variable + time dummies）

Unconditional moment conditions for $\sqrt{N}$-consistent estimation

- Using two types of unconditional moment conditions based on both g-form and h-form, the joint estimations of γ, β and time dummies（$\Delta TD_3, \cdots, \Delta TD_T$）are conducted.
- Moment conditions used in GMM（$3 \leq t \leq T-1; t-1 \leq s \leq t+1$）

g-form

$$\mathrm{E}[\hbar U_{it}^{+}] = 0$$
$$\mathrm{E}[y_{i,t-2}\,\hbar U_{it}^{+}] = 0$$
$$\mathrm{E}[\Delta x_{is}\,\hbar U_{it}^{+}] = 0$$

h-form

$$\mathrm{E}[\hbar \Upsilon_{it}^{+}] = 0$$
$$\mathrm{E}[y_{i,t-2}\,\hbar \Upsilon_{it}^{+}] = 0$$
$$\mathrm{E}[\Delta x_{is}\,\hbar \Upsilon_{it}^{+}] = 0$$

(P.S. 1) Dynamic fixed effects logit model（with strictly exogenous explanatory variable + time dummies）Monte Carlo

- DGP

$$y_{it} = \begin{cases} 1 & \text{if } p(\eta_i, y_{i,t-1}, x_{it}, TD_t) > \zeta_{it} \\ 0 & \text{otherwise} \end{cases} \qquad y_{i1} = \begin{cases} 1 & \text{if } q(\eta_i, x_{i1}, TD_1) > \zeta_{i1} \\ 0 & \text{otherwise} \end{cases}$$

$$p(\eta_i, y_{i,t-1}, x_{it}, TD_t) = \exp(\eta_i + \gamma y_{i,t-1} + \beta x_{it} + TD_t) / (1 + \exp(\eta_i + \gamma y_{i,t-1} + \beta x_{it} + TD_t))$$

$$q(\eta_i, x_{i1}, TD_1) = \exp(\eta_i + \beta x_{i1} + TD_1) / (1 + \exp(\eta_i + \beta x_{i1} + TD_1))$$

$$x_{it} = \rho x_{i,t-1} + \tau \eta_i + \varepsilon_{it} \qquad x_{i1} = (\tau / (1-\rho))\eta_i + (1/(1-\rho^2)^{1/2})\varepsilon_{i1}$$

$$\zeta_{it} \sim \mathrm{U}(0,1) \qquad \eta_i \sim \mathrm{N}(0, \sigma_\eta^2) \qquad \varepsilon_{it} \sim \mathrm{N}(0, \sigma_\varepsilon^2)$$

- Parameters with values being set：
- $TD_t \ (t = 1, \dots, T), \gamma, \delta, \rho, \tau, \sigma_\eta^2, \sigma_\varepsilon^2$

N=1,000, 10,000, 100,000
T=4, 8
$R_N = 2{,}500$

(P.S. 1) Monte Carlo results (case of strictly exogenous explanatory variable + time dummies,　T=4)

As N increases such as 1,000 -> 10,000 -> 100,000, precision and accuracy of the estimator increase.

Simulation(A-a)

$\gamma = 0.5, \quad \beta = 0.5, \quad \rho = 0.5,$
$\tau = 0.1, \quad \sigma_\eta^2 = 0.5, \quad \sigma_\varepsilon^2 = 0.5,$
$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.0,$
$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0$

Simulation(A-a)

$\gamma = 1.1, \quad \beta = 1.1, \quad \rho = 0.9,$
$\tau = 0.1, \quad \sigma_\eta^2 = 0.5, \quad \sigma_\varepsilon^2 = 0.5,$
$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.5,$
$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0$

	N=1,000		N=10,000		N=100,000	
	true	**mcm**	**true**	**mcm**	**true**	**mcm**
	mcsd	mcse	mcsd	mcse	mcsd	mcse
	bias	rmse	bias	rmse	bias	rmse
Simulation(A-a)						
GMM(gh-HTD) γ	**0.500**	**0.321**	**0.500**	**0.482**	**0.500**	**0.499**
	0.259	0.234	0.076	0.074	0.023	0.023
	−0.179	0.315	−0.018	0.078	−0.001	0.023
β	**0.500**	**0.405**	**0.500**	**0.503**	**0.500**	**0.500**
	0.191	0.117	0.044	0.041	0.014	0.013
	−0.095	0.214	0.003	0.045	0.000	0.014
ΔTD3	**−1.500**	**−1.568**	**−1.500**	**−1.505**	**−1.500**	**−1.500**
	0.143	0.133	0.040	0.039	0.012	0.012
	−0.068	0.159	−0.005	0.040	0.000	0.012
ΔTD4	**0.500**	**0.206**	**0.500**	**0.485**	**0.500**	**0.498**
	0.280	0.135	0.072	0.056	0.018	0.018
	−0.294	0.406	−0.015	0.073	−0.002	0.018
Sargan, df	**5.864**	**6**	**6.166**	**6**	**5.988**	**6**
Simulation(A-b)						
GMM(gh-HTD) γ	**1.100**	**0.592**	**1.100**	**1.052**	**1.100**	**1.097**
	0.542	0.432	0.152	0.141	0.045	0.045
	−0.508	0.743	−0.048	0.159	−0.003	0.045
β	**1.100**	**0.924**	**1.100**	**1.111**	**1.100**	**1.101**
	0.365	0.215	0.089	0.081	0.027	0.027
	−0.176	0.405	0.011	0.090	0.001	0.027
ΔTD3	**−2.000**	**−2.139**	**−2.000**	**−2.008**	**−2.000**	**−2.001**
	0.333	0.249	0.066	0.064	0.020	0.020
	−0.139	0.361	−0.008	0.066	−0.001	0.020
ΔTD4	**0.500**	**0.122**	**0.500**	**0.472**	**0.500**	**0.498**
	0.451	0.197	0.135	0.100	0.033	0.033
	−0.378	0.588	−0.028	0.138	−0.002	0.033
Sargan, df	**8.156**	**6**	**6.740**	**6**	**6.060**	**6**

(P.S. 1) Monte Carlo results (case of strictly exogenous explanatory variable + time dummies,　T=8)

As N increases such as 1,000 -> 10,000 -> 100,000, precision and accuracy of the estimator increase.

Simulation(A-a)

$\gamma = 0.5, \quad \beta = 0.5, \quad \rho = 0.5,$
$\tau = 0.1, \quad \sigma_\eta^2 = 0.5, \quad \sigma_\varepsilon^2 = 0.5,$
$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.0,$
$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0,$
$\mathrm{TD}_5 = -0.5, \quad \mathrm{TD}_6 = 0.5,$
$\mathrm{TD}_7 = 0.0, \quad \mathrm{TD}_8 = -1.0$

	N=1,000		N=10,000		N=100,000	
	true	**mcm**	**true**	**mcm**	**true**	**mcm**
	mcsd	mcse	mcsd	mcse	mcsd	mcse
	bias	rmse	bias	rmse	bias	rmse
Simulation(A-a)						
GMM(gh-HTD) γ	**0.500**	**0.391**	**0.500**	**0.490**	**0.500**	**0.499**
	0.111	0.082	0.031	0.030	0.009	0.009
	−0.109	0.156	−0.010	0.033	−0.001	0.009
β	**0.500**	**0.456**	**0.500**	**0.496**	**0.500**	**0.500**
	0.069	0.046	0.017	0.017	0.005	0.005
	−0.044	0.082	−0.004	0.018	0.000	0.005
ΔTD3	**−1.500**	**−1.516**	**−1.500**	**−1.501**	**−1.500**	**−1.501**
	0.125	0.110	0.037	0.036	0.012	0.011
	−0.016	0.126	−0.001	0.037	−0.001	0.012
ΔTD4	**0.500**	**0.427**	**0.500**	**0.492**	**0.500**	**0.499**
	0.115	0.083	0.033	0.032	0.010	0.010
	−0.073	0.136	−0.008	0.034	−0.001	0.010
ΔTD5	**−0.500**	**−0.438**	**−0.500**	**−0.492**	**−0.500**	**−0.499**
	0.113	0.083	0.033	0.032	0.010	0.010
	0.062	0.129	0.008	0.034	0.001	0.010
ΔTD6	**1.000**	**0.963**	**1.000**	**0.996**	**1.000**	**1.000**
	0.119	0.088	0.035	0.033	0.011	0.011
	−0.037	0.125	−0.004	0.035	0.000	0.011
ΔTD7	**−0.500**	**−0.479**	**−0.500**	**−0.499**	**−0.500**	**−0.500**
	0.146	0.084	0.038	0.036	0.012	0.012
	0.021	0.147	0.001	0.038	0.000	0.012
ΔTD8	**−1.000**	**−0.812**	**−1.000**	**−0.998**	**−1.000**	**−1.000**
	0.383	0.102	0.044	0.042	0.014	0.014
	0.188	0.427	0.002	0.044	0.000	0.014
Sargan, df	**51.316**	**42**	**43.213**	**42**	**42.359**	**42**

(P.S. 1) Monte Carlo results （case of strictly exogenous explanatory variable + time dummies,　T=8）

As N increases such as 1,000 -> 10,000 -> 100,000, precision and accuracy of the estimator increase.

Simulation(A-b)

$$\gamma = 1.1, \quad \beta = 1.1, \quad \rho = 0.9,$$
$$\tau = 0.1, \quad \sigma_\eta^2 = 0.5, \quad \sigma_\varepsilon^2 = 0.5,$$
$$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.5,$$
$$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0,$$
$$\mathrm{TD}_5 = -1.5, \quad \mathrm{TD}_6 = 0.5,$$
$$\mathrm{TD}_7 = -1.0, \quad \mathrm{TD}_8 = -0.5$$

	N=1,000		N=10,000		N=100,000	
	true	**mcm**	**true**	**mcm**	**true**	**mcm**
	mcsd	mcse	mcsd	mcse	mcsd	mcse
	bias	rmse	bias	rmse	bias	rmse
Simulation(A-b)						
GMM(gh-HTD) γ	**1.100**	**0.677**	**1.100**	**1.079**	**1.100**	**1.098**
	0.284	0.126	0.052	0.050	0.016	0.016
	−0.423	0.509	−0.021	0.057	−0.002	0.016
β	**1.100**	**1.076**	**1.100**	**1.102**	**1.100**	**1.100**
	0.133	0.072	0.029	0.028	0.009	0.009
	−0.024	0.135	0.002	0.029	0.000	0.009
ΔTD3	**−2.000**	**−2.040**	**−2.000**	**−2.001**	**−2.000**	**−2.000**
	0.243	0.164	0.056	0.055	0.018	0.018
	−0.040	0.247	−0.001	0.056	0.000	0.018
ΔTD4	**0.500**	**0.274**	**0.500**	**0.488**	**0.500**	**0.498**
	0.221	0.110	0.049	0.046	0.015	0.015
	−0.226	0.316	−0.012	0.051	−0.002	0.015
ΔTD5	**−1.500**	**−1.443**	**−1.500**	**−1.494**	**−1.500**	**−1.498**
	0.210	0.124	0.050	0.047	0.015	0.015
	0.057	0.218	0.006	0.050	0.002	0.016
ΔTD6	**2.000**	**1.878**	**2.000**	**1.993**	**2.000**	**1.999**
	0.231	0.123	0.053	0.049	0.016	0.016
	−0.122	0.261	−0.007	0.053	−0.001	0.016
ΔTD7	**−1.500**	**−1.305**	**−1.500**	**−1.491**	**−1.500**	**−1.499**
	0.254	0.116	0.051	0.048	0.016	0.016
	0.195	0.320	0.009	0.052	0.001	0.016
ΔTD8	**0.500**	**0.166**	**0.500**	**0.492**	**0.500**	**0.500**
	0.438	0.141	0.103	0.079	0.027	0.027
	−0.334	0.551	−0.008	0.103	0.000	0.027
Sargan, df	**94.012**	**42**	**50.423**	**42**	**43.407**	**42**

(P.S. 2) Dynamic fixed effects logit model（with no explanatory variable but with time dummies）

- Dynamic fixed effects logit model（with no explanatory variable but with time dummies）

$$y_{it} = \frac{\exp\left(\eta_i + TD_t + \gamma\, y_{i,t-1}\right)}{1 + \exp\left(\eta_i + TD_t + \gamma\, y_{i,t-1}\right)} + v_{it}$$

- y_{it}:　binary dependent variable for individual i at time t（1 or 0）, taking 1 with above probability
- η_i: fixed effect　TD_t: time dummy　　v_{it}: disturbance
- γ : parameter of interest
- Assumptions on disturbances：　$\mathrm{E}[v_{it} \mid \eta_i, y_{i1}, v_i^{t-1}] = 0$
- $v_i^{t-1} = (\, v_{i1}, \ldots, v_{i,t-1})$

i: individual　t: time
i = 1,…,N　 t =2,…,T
N　→　∞　 T being fixed

(P.S. 2) Dynamic fixed effects logit model（with no explanatory variable but with time dummies）

- Conditional moment conditions（g-form）, derived the same method as kitazawa (2013)

$$\mathrm{E}[\hbar U_{it}^{-} \mid \eta_i, y_{i1}, v_i^{t-2}] = 0$$

where
$$\hbar U_{it\,it}^{-} = U_{it}^{-} - y_{i,t-1}$$
$$- \tanh\left(\frac{-\gamma y_{i,t-2} + \Delta TD_t + \Delta TD_{t+1}}{2} \right) (U_{it}^{-} + y_{i,t-1} - 2U_{it}^{-}\, y_{i,t-1})$$

with
$$U_{it}^{-} = y_{it} + (1 - y_{it}) y_{i,t+1} - (1 - y_{it}) y_{i,t+1} \exp\left(-\Delta TD_{t+1}\right)$$
$$- (\exp(\gamma) - 1) y_{i,t-1} (1 - y_{it}) y_{i,t+1} \exp\left(-\Delta TD_{t+1}\right)$$

(P.S. 2) Dynamic fixed effects logit model（with no explanatory variable but with time dummies）

- Conditional moment conditions（h-form）, derived the same method as kitazawa (2013)

$$\mathrm{E}[\hbar\Upsilon_{it}^{-} \mid \eta_i, y_{i1}, v_i^{t-2}] = 0$$

where

$$\hbar\Upsilon_{it}^{-} = \Upsilon_{it}^{-} - y_{i,t-1}$$
$$- \tanh\left(\frac{\gamma(1-y_{i,t-2}) + \Delta TD_t + \Delta TD_{t+1}}{2}\right)(\Upsilon_{it}^{-} + y_{i,t-1} - 2\Upsilon_{it}^{-} y_{i,t-1})$$

with

$$\Upsilon_{it}^{-} = y_{it} y_{i,t+1} + y_{it}(1-y_{i,t+1})\exp\left(\Delta TD_{t+1}\right)$$
$$+ (\exp(\gamma)-1)(1-y_{i,t-1}) y_{it}(1-y_{i,t+1})\exp\left(\Delta TD_{t+1}\right)$$

(P.S. 2) Dynamic fixed effects logit model（with no explanatory variable but with time dummies）

Unconditional moment conditions for $\sqrt{N}$-consistent estimation

- Using two types of unconditional moment conditions based on both g-form and h-form, the joint estimations of γ and time dummies（$\Delta TD_3, \cdots, \Delta TD_T$）are conducted.
- Moment conditions used in GMM（$3 \leq t \leq T-1$）

g-form

$$\mathrm{E}[\hbar U_{it}^{-}] = 0 \qquad \mathrm{E}[\hbar \Upsilon_{it}^{-}] = 0$$

$$\mathrm{E}[\, y_{i,t-2}\, \hbar U_{it}^{-}] = 0 \qquad \mathrm{E}[\, y_{i,t-2}\, \hbar \Upsilon_{it}^{-}] = 0$$

h-form

(P.S. 2) Dynamic fixed effects logit model（with no explanatory variable but with time dummies）Monte Carlo

- DGP

$$y_{it} = \begin{cases} 1 & \text{if } p(\eta_i, y_{i,t-1}, TD_t) > \zeta_{it} \\ 0 & \text{otherwise} \end{cases} \qquad y_{i1} = \begin{cases} 1 & \text{if } q(\eta_i, TD_1) > \zeta_{i1} \\ 0 & \text{otherwise} \end{cases}$$

$$p(\eta_i, y_{i,t-1}, TD_t) = \exp(\eta_i + \gamma y_{i,t-1} + TD_t) / (1 + \exp(\eta_i + \gamma y_{i,t-1} + TD_t))$$

$$q(\eta_i, TD_1) = \exp(\eta_i + TD_1) / (1 + \exp(\eta_i + TD_1))$$

$$\zeta_{it} \sim \mathrm{U}(0,1) \qquad \eta_i \sim \mathrm{N}(0, \sigma_\eta^2)$$

- Parameters with values being set：
- $TD_t \ (t = 1, \ldots, T), \gamma, \sigma_\eta^2$

N=1,000, 10,000, 100,000
T=4, 8
$R_N = 2{,}500$

(P.S. 2) Monte Carlo results （case of no explanatory variable + time dummies, T=4）

As N increases such as 1,000 -> 10,000 -> 100,000, precision and accuracy of the estimator increase.

Simulation(A-a)

$$\gamma = 0.5,$$
$$\sigma_\eta^2 = 0.5,$$
$$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.0,$$
$$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0$$

Simulation(A-a)

$$\gamma = 1.1,$$
$$\sigma_\eta^2 = 0.5,$$
$$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.5,$$
$$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0$$

	N=1,000		N=10,000		N=100,000	
	true	**mcm**	**true**	**mcm**	**true**	**mcm**
	mcsd	mcse	mcsd	mcse	mcsd	mcse
	bias	rmse	bias	rmse	bias	rmse
Simulation(B-a)						
GMM(gh-HTD) γ	**0.500**	**0.443**	**0.500**	**0.498**	**0.500**	**0.499**
	0.230	0.222	0.073	0.071	0.022	0.022
	−0.057	0.237	−0.002	0.073	−0.001	0.022
ΔTD3	**−1.500**	**−1.539**	**−1.500**	**−1.503**	**−1.500**	**−1.500**
	0.124	0.131	0.038	0.037	0.012	0.012
	−0.039	0.130	−0.003	0.038	0.000	0.012
ΔTD4	**0.500**	**0.342**	**0.500**	**0.488**	**0.500**	**0.500**
	0.255	0.173	0.074	0.051	0.015	0.015
	−0.158	0.300	−0.012	0.075	0.000	0.015
Sargan, df	**0.558**	**1**	**0.953**	**1**	**0.998**	**1**
Simulation(B-b)						
GMM(gh-HTD) γ	**1.100**	**0.959**	**1.100**	**1.090**	**1.100**	**1.100**
	0.324	0.329	0.097	0.101	0.032	0.031
	−0.141	0.353	−0.010	0.097	0.000	0.032
ΔTD3	**−2.000**	**−2.091**	**−2.000**	**−2.022**	**−2.000**	**−2.001**
	0.201	0.203	0.066	0.058	0.016	0.015
	−0.091	0.221	−0.022	0.070	−0.001	0.016
ΔTD4	**0.500**	**0.292**	**0.500**	**0.430**	**0.500**	**0.498**
	0.341	0.267	0.187	0.114	0.030	0.029
	−0.208	0.400	−0.070	0.199	−0.002	0.030
Sargan, df	**0.813**	**1**	**0.870**	**1**	**0.966**	**1**

(P.S. 2) Monte Carlo results （case of no explanatory variable + time dummies, T=8）

As N increases such as 1,000 -> 10,000 -> 100,000, precision and accuracy of the estimator increase.

Simulation(A-a)

$$\gamma = 0.5,$$
$$\sigma_\eta^2 = 0.5,$$
$$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.0,$$
$$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0,$$
$$\mathrm{TD}_5 = -0.5, \quad \mathrm{TD}_6 = 0.5,$$
$$\mathrm{TD}_7 = 0.0, \quad \mathrm{TD}_8 = -1.0$$

	N=1,000		N=10,000		N=100,000	
	true	**mcm**	**true**	**mcm**	**true**	**mcm**
	mcsd	mcse	mcsd	mcse	mcsd	mcse
	bias	rmse	bias	rmse	bias	rmse
Simulation(B-a)						
GMM(gh-HTD) γ	**0.500**	**0.450**	**0.500**	**0.495**	**0.500**	**0.499**
	0.099	0.081	0.028	0.027	0.009	0.009
	−0.050	0.111	−0.005	0.029	−0.001	0.009
ΔTD3	**−1.500**	**−1.511**	**−1.500**	**−1.501**	**−1.500**	**−1.501**
	0.115	0.108	0.035	0.035	0.011	0.011
	−0.011	0.116	−0.001	0.035	−0.001	0.011
ΔTD4	**0.500**	**0.463**	**0.500**	**0.495**	**0.500**	**0.500**
	0.108	0.086	0.031	0.031	0.010	0.010
	−0.037	0.115	−0.005	0.031	0.000	0.010
ΔTD5	**−0.500**	**−0.467**	**−0.500**	**−0.495**	**−0.500**	**−0.500**
	0.105	0.086	0.032	0.031	0.010	0.010
	0.033	0.110	0.005	0.032	0.000	0.010
ΔTD6	**1.000**	**0.976**	**1.000**	**0.997**	**1.000**	**1.000**
	0.111	0.089	0.033	0.032	0.010	0.010
	−0.024	0.114	−0.003	0.033	0.000	0.010
ΔTD7	**−0.500**	**−0.454**	**−0.500**	**−0.497**	**−0.500**	**−0.500**
	0.138	0.092	0.035	0.034	0.011	0.011
	0.046	0.146	0.003	0.035	0.000	0.011
ΔTD8	**−1.000**	**−0.986**	**−1.000**	**−1.001**	**−1.000**	**−1.000**
	0.243	0.108	0.037	0.037	0.012	0.012
	0.014	0.244	−0.001	0.037	0.000	0.012
Sargan, df	**12.929**	**13**	**13.111**	**13**	**13.067**	**13**

(P.S. 2) Monte Carlo results （case of no explanatory variable + time dummies, T=8）

As N increases such as 1,000 -> 10,000 -> 100,000, precision and accuracy of the estimator increase.

Simulation(A-b)

$$\gamma = 1.1,$$
$$\sigma_\eta^2 = 0.5,$$
$$\mathrm{TD}_1 = 0.5, \quad \mathrm{TD}_2 = 1.5,$$
$$\mathrm{TD}_3 = -0.5, \quad \mathrm{TD}_4 = 0.0,$$
$$\mathrm{TD}_5 = -1.5, \quad \mathrm{TD}_6 = 0.5,$$
$$\mathrm{TD}_7 = -1.0, \quad \mathrm{TD}_8 = -0.5$$

	N=1,000		N=10,000		N=100,000	
	true	**mcm**	**true**	**mcm**	**true**	**mcm**
	mcsd	mcse	mcsd	mcse	mcsd	mcse
	bias	rmse	bias	rmse	bias	rmse
Simulation(B-b)						
GMM(gh-HTD) γ	**1.100**	**1.010**	**1.100**	**1.096**	**1.100**	**1.100**
	0.138	0.100	0.033	0.033	0.011	0.010
	−0.090	0.165	−0.004	0.034	0.000	0.011
ΔTD3	**−2.000**	**−2.030**	**−2.000**	**−2.003**	**−2.000**	**−2.001**
	0.159	0.141	0.045	0.044	0.014	0.014
	−0.030	0.162	−0.003	0.045	−0.001	0.014
ΔTD4	**0.500**	**0.449**	**0.500**	**0.498**	**0.500**	**0.500**
	0.138	0.101	0.034	0.034	0.011	0.011
	−0.051	0.147	−0.002	0.034	0.000	0.011
ΔTD5	**−1.500**	**−1.501**	**−1.500**	**−1.499**	**−1.500**	**−1.500**
	0.127	0.104	0.036	0.034	0.011	0.011
	−0.001	0.127	0.001	0.036	0.000	0.011
ΔTD6	**2.000**	**2.006**	**2.000**	**1.998**	**2.000**	**2.000**
	0.135	0.109	0.039	0.036	0.012	0.012
	0.006	0.135	−0.002	0.039	0.000	0.012
ΔTD7	**−1.500**	**−1.500**	**−1.500**	**−1.496**	**−1.500**	**−1.500**
	0.129	0.109	0.037	0.035	0.011	0.011
	0.000	0.129	0.004	0.037	0.000	0.011
ΔTD8	**0.500**	**0.341**	**0.500**	**0.486**	**0.500**	**0.499**
	0.279	0.147	0.076	0.059	0.019	0.019
	−0.159	0.321	−0.014	0.078	−0.001	0.019
Sargan, df	**18.037**	**13**	**13.416**	**13**	**12.975**	**13**