Consistent estimation for the full-fledged fixed effects zero-inflated Poisson model

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Abstract

- This paper advocates the transformations used for the consistent estimation of the full-fledged fixed effects zero-inflated Poisson model whose zero outcomes can arise from both of logit and Poisson parts and which equips both parts with the fixed effects.
- The valid moment conditions are constructed on the basis of the transformations.
- The finite sample behaviors of GMM and EL estimators employing the moment conditions are investigated by use of Monte Carlo experiments.

- ***Keywords:*** *fi*xed effects zero-inflated Poisson model; predetermined explanatory variables in Poisson part; moment conditions; GMM; EL; Monte Carlo experiments

1 Introduction

- The polished zero-inflated Poisson model (hereafter ZIP model) proposed by Lambert (1992) is one of the models dealing with count data with zero values being superabundant.
- Empirical studies using the ZIP model are often found in the literature on the econometric analysis: Gurmu and Trivedi (1996) on the relationship between the recreational boating trips and boat owners' attributes, Crépon & Duguet (1997) and Hu & Jefferson (2009) on the patents and R&D relationship, etc.

ZIP model (simple example)

- Count dependent variable: y_i
- Explanatory variables: w_i, x_i
- $i = 1, \ldots N \ (N \to \infty)$
- $y_i = 0$ with probability $1 - p_i$
- $y_i \sim Pois(q_i)$ with probability p_i
- Logit probability $p_i = \frac{\exp(\gamma+\delta w_i)}{1+\exp(\gamma+\delta w_i)}$
- Poisson mean $q_i = \exp(\alpha + \beta x_i)$
- Parameters $\gamma, \delta, \alpha, \beta$ are consistently estimated by Maximum likelihood method.

Incipient ZIP models with the fixed effects

- Majo (2010) and Majo & Van Soest (2011) considered the fixed effects ZIP model, but their model assumes the truncated-at-zero Poisson model in Poisson part, implying that the origin of the zero count outcomes is confined to the logit part.
- Gilles (2012) and Gilles & Kim (2013) also considered the fixed effects ZIP model, but their model incorporates no fixed effect in the logit part.

ZIP model with the fixed effects considered in this paper

- Different from the studies by Majo (2010) and Majo & Van Soest (2011) and by Gilles (2012) and Gilles & Kim (2013), the fixed effects ZIP model discussed in this paper has the Poisson part from which the zero count outcome is not improbable and the logit part with the fixed effects being built-in.
- This ZIP model is comparatively plenary.

Estimation methods for the ZIP model with the fixed effects considered in this paper

- The valid moment conditions for this ZIP model are constructed based on two transformations for different specifications of the explanatory variables in Poisson part.
- Then, the parameters of interest are consistently estimated by use of the GMM (Generalized Method of Moments) proposed by Hansen (1982) and EL (Empirical Likelihood) method proposed by Owen (1988, 1990, 1991, 2001) and advanced by Qin & Lawless (1994).

2 Model and moment conditions

- The fixed effects ZIP model is considered, which has two potential sources of outbreaks of zero count variables: logit probability and Poisson density and which furnishes both of logit and Poisson parts with the fixed effects.
- The fixed effects ZIP model is described in the implicit form and the mean and variance of its disturbance are specified. Then, presupposing that the disturbance and its square are uncorrelated with any transformations of the disturbances in past and the fixed effects, the moment conditions for consistently estimating the parameters of interest are constructed under both of the slightly strong assumptions and the mitigated ones.

2 Model and moment conditions

- Under the slightly strong assumptions, the explanatory variables in both of the logit probability and the Poisson mean are slightly exogenous, while under the mitigated assumptions, the explanatory variables in the logit probability are slightly exogenous and those in the Poisson mean are predetermined.
- The overtone of the slight exogeneity introduced in this paper is that the count dependent variable at a given period wield **no influence** over the explanatory variable at the period just behind the occurrence of its count variable, whereas it could make **some sorts of influences** on the subsequent explanatory variables.

2.1 Fixed effects ZIP model

- The fixed effects ZIP model has the following two potential sources of outbreaks of zero count dependent variables:
- $y_{it} = 0$ with probability $1 - p_{it}$
- $y_{it} \sim Pois(q_{it})$ with probability p_{it}
- Subscripts i ($i = 1, \ldots, N$) and t ($t = 1, \ldots, T$) denote the individual and the time period.
- It is assumed that $N \rightarrow \infty$, whereas T is fixed.

2.1 Fixed effects ZIP model

- Logit probability of generating the binary process $p_{it} = \frac{\exp(\psi_i + \delta w_{it})}{1+\exp(\psi_i + \delta w_{it})}$
- Mean of generating the Poisson process $q_{it} = \exp(\phi_i + \beta x_{it})$
- ψ_i and ϕ_i: fixed effects
- w_{it} and x_{it}: (continuous) explanatory variables
- Implicit form of the fixed effects ZIP model
  $y_{it} = p_{it} \ q_{it} + v_{it}$
- v_{it}: disturbance (for the slightly strong and the mitigated assumptions)

2.1 Slightly strong assumptions and moment conditions

- Slightly Strong Assumptions:
- $E[v_{it}|\psi_i, w_i^{t+1}, \eta_i, x_i^{t+1}, v_i^{t-1}] = 0$
- $E\left[v_{it}^2\middle|\psi_i, w_i^{t+1}, \eta_i, x_i^{t+1}, v_i^{t-1}\right]$
  $$= p_{it}q_{it}(1 + (1 - p_{it})q_{it})$$
- where $w_i^{t+1} = (w_{i1}, \dots, w_{i,t+1})$, $x_i^{t+1} = (x_{i1}, \dots, x_{i,t+1})$, and $v_i^{t-1} = (v_{i0}, \dots, v_{i,t-1})$ with v_{i0} being empty.

2.1 Slightly strong assumptions and moment conditions

- Conditional moment conditions under the Slightly Strong Assumptions
- $E[\Phi_{it}(\delta, \beta) | \psi_i, w_i^t, \eta_i, x_i^t, v_i^{t-2}] = 0$
- $\Phi_{it}(\delta, \beta) = (\tanh(\delta\, \Delta w_{it}/2) - 1)\exp(-\beta\, \Delta x_{it})\left(y_{it}^2 - y_{it}\right)$
  $+(\tanh(\delta\, \Delta w_{it}/2) + 1)\exp(\beta\, \Delta x_{it})\left(y_{i,t-1}^2 - y_{i,t-1}\right)$
  $-2\tanh(\delta\, \Delta w_{it}/2)\, y_{it}\, y_{i,t-1}$
- The transformation above is referred to as the "PHI transformation" in this paper.

2.2 Mitigated assumptions and moment conditions

- Mitigated Assumptions:
- $E[v_{it}|\psi_i, w_i^{t+1}, \eta_i, x_i^{t}, v_i^{t-1}] = 0$
- $E\left[v_{it}^2|\psi_i, w_i^{t+1}, \eta_i, x_i^{t}, v_i^{t-1}\right]$
  $$= p_{it}q_{it}(1 + (1 - p_{it})q_{it})$$

2.2 Mitigated assumptions and moment conditions

- Conditional moment conditions under the Mitigated Assumptions
- $E[\Psi_{it}(\delta,\beta)|\psi_i, w_i^t, \eta_i, x_i^{t-1}, v_i^{t-2}] = 0,$
- $\Psi_{it}(\delta,\beta) = (\tanh(\delta\,\Delta w_{it}/2) - 1)\exp(-2\beta\,\Delta x_{it})\left(y_{it}^2 - y_{it}\right)$
  $+(\tanh(\delta\,\Delta w_{it}/2) + 1)\left(y_{i,t-1}^2 - y_{i,t-1}\right)$
  $-\,2\tanh(\delta\,\Delta w_{it}/2)\exp(-\beta\,\Delta x_{it})\,y_{it}\,y_{i,t-1}$
- The transformation above is referred to as the "PSI transformation" in this paper.

3 Estimation methods

- The two estimators using the unconditional moment conditions based on the PHI or PSI transformations: GMM and EL estimators
- The GMM estimator is obtained by minimizing the quadratic form composed of the sample version vector of moment conditions and a weighting matrix.
- The EL estimator, as an alternative to the GMM estimator, is obtained by maximizing the log likelihood constructed by using the implied probability under the constraint of the sample version vector weighted by the implied probability.
- Many studies reveal that the EL estimator behaves better than the GMM estimator (e.g. Newey & Smith, 2004; Anatolyev, 2005;
- Ramalho, 2005).

3.1 GMM estimator

Objective function

$$\hat{\theta}_{\text{GMM}} = \arg\min_{\theta} \bar{g}(\theta)'(\bar{\Omega}(\hat{\theta}_1))^{-1}\bar{g}(\theta)\,.$$

Empirical counterpart of unconditional moment conditions (m by 1)

$$\bar{g}(\theta) = (1/N)\sum_{i=1}^{N} g_i(\theta) = 0,$$

Inverse of weighting matrix (m by m)

$$\bar{\Omega}(\hat{\theta}_1) = (1/N)\sum_{i=1}^{N} g_i(\hat{\theta}_1)g_i(\hat{\theta}_1)',$$

- Vector of parameters of interest: $\theta = [\delta, \;\; \beta]$
- One-step estimator of the vector: $\hat{\theta}_1$

Unconditional moment conditions $E[g_i(\theta)] = 0$, m by 1 , constructed based on the unconditional moment conditions

3.2 EL estimator

Objective function

$$\min_{\theta,\pi_1,\ldots,\pi_N} -(1/N)\sum_{i=1}^{N}((-\ln(1/N)) - (-\ln \pi_i)),$$

Subject to

$$\sum_{i=1}^{N} \pi_i = 1. \qquad \sum_{i=1}^{N} \pi_i g_i(\theta) = 0,$$

$$\hat{\theta}_{\mathrm{EL}} = \arg\min_{\theta}(\max_{\lambda}\,(1/N)\sum_{i=1}^{N}\ln(1-\lambda' g_i(\theta))),$$

By dint of the transformation to the dual problem, the number of parameters to be estimated decreases from 2+N to 2+m, if N>m.

Probability of realization of the variables composing $g_i(\theta)$: π_i

Lagrange Multiplier (m by 1): λ

Asymptotic distribution of GMM and EL estimators

- Qin & Lawless (1994) show that the EL estimator $\hat{\theta}_{EL}$ has the same limit distribution as the GMM estimator $\hat{\theta}_{GMM}$, which is represented by

$$N^{1/2}(\hat{\theta}_{\mathrm{GMM}} - \theta_0) \xrightarrow{\mathrm{d}} \mathrm{N}(0, (D(\theta_0)'(\Omega(\theta_0))^{-1}D(\theta_0))^{-1}),$$

- where

$$D(\theta_0) = (\partial \mathrm{E}[g_i(\theta)]/\partial \theta') \mid_{\theta=\theta_0} \qquad \Omega(\theta_0) = \mathrm{E}[g_i(\theta_0)g_i(\theta_0)'].$$

- θ_0 : true value of θ

4 Monte Carlo

- The finite sample behaviors of the GMM and EL estimators based on the PHI and PSI transformations are investigated with some Monte Carlo experiments.
- The experiments are carried out by using the programming language “R" (version 3.0.2) developed by R Core Team (2013). [GMM and EL estimations: package “gmm" developed by Chaussé (2010), ML estimation: package “pscl" developed by Zeileis et al. (2008).]

4.1 Data generating process

- DGP (fixed effects ZIP model)

$$
\begin{aligned}
y_{it} &= y^p_{it} y^q_{it}, \\
y^p_{it} &\sim \text{Bin}(1, p_{it}), \\
p_{it} &= \exp(\psi_i + \delta w_{it})/(1 + \exp(\psi_i + \delta w_{it})), \\
w_{it} &= \alpha w_{i,t-1} + \iota \psi_i + \zeta_{it}, \\
w_{i1} &= (1/(1-\alpha))\iota\psi_i + (1/(1-\alpha^2)^{(1/2)})\zeta_{i1}, \\
& \quad \psi_i \sim \text{N}(0, \sigma^2_\psi);\ \zeta_{it} \sim \text{N}(0, \sigma^2_\zeta), \\
y^q_{it} &\sim \text{Pois}(q_{it}), \\
q_{it} &= \exp(\eta_i + \beta x_{it}), \\
x_{it} &= \rho x_{i,t-1} + \tau \eta_i + \varepsilon_{it}, \\
x_{i1} &= (1/(1-\rho))\tau\eta_i + (1/(1-\rho^2)^{(1/2)})\varepsilon_{i1}, \\
& \quad \eta_i \sim \text{N}(0, \sigma^2_\eta);\ \varepsilon_{it} \sim \text{N}(0, \sigma^2_\varepsilon).
\end{aligned}
$$

Cross-sectional sizes:
N=1000, 5000, 10000

Number of time periods:
T= 4, 8

Number of replications:
10000.

Values are set to the parameters δ, α, ι, σ^2_ψ, σ^2_ζ, β, ρ, τ, , σ^2_η, σ^2_ε.

4.2 Estimators assayed

- The GMM and EL using the unconditional moment conditions based on the PHI and PSI transformations:

Those based on the PHI transformation

$$E[\Phi_{it}(\delta, \beta)\ \Delta w_{it}] = 0, \qquad \text{for } t = 2, \dots, T,$$
$$E[\Phi_{it}(\delta, \beta)\ \Delta x_{it}] = 0, \qquad \text{for } t = 2, \dots, T.$$

Those based on the PSI transformation

$$E[\Psi_{it}(\delta, \beta)\ \Delta w_{it}] = 0, \qquad \text{for } t = 2, \dots, T,$$
$$E[\Psi_{it}(\delta, \beta)\ x_{is}] = 0, \text{ for } s = 1, \dots, t-1; t = 2, \dots, T.$$

- As a control, the (inconsistent) pooled maximum likelihood estimator (hereafter, the “ML(POOL)" estimator) is used, which ignores the individual heterogeneity and accordingly has the indigenous bias.

4.3 Results

- Monte Carlo results for the estimators assayed when T = 4 and 8 are shown in Table 1 and 2, respectively.
- Figure 1 and 2 are the boxplots of the GMM and EL estimators for δ and β when T = 4, respectively, while Figure 3 and 4 are those when T = 8.

Monte Carlo results for the fixed effects ZIP model, *T=8*

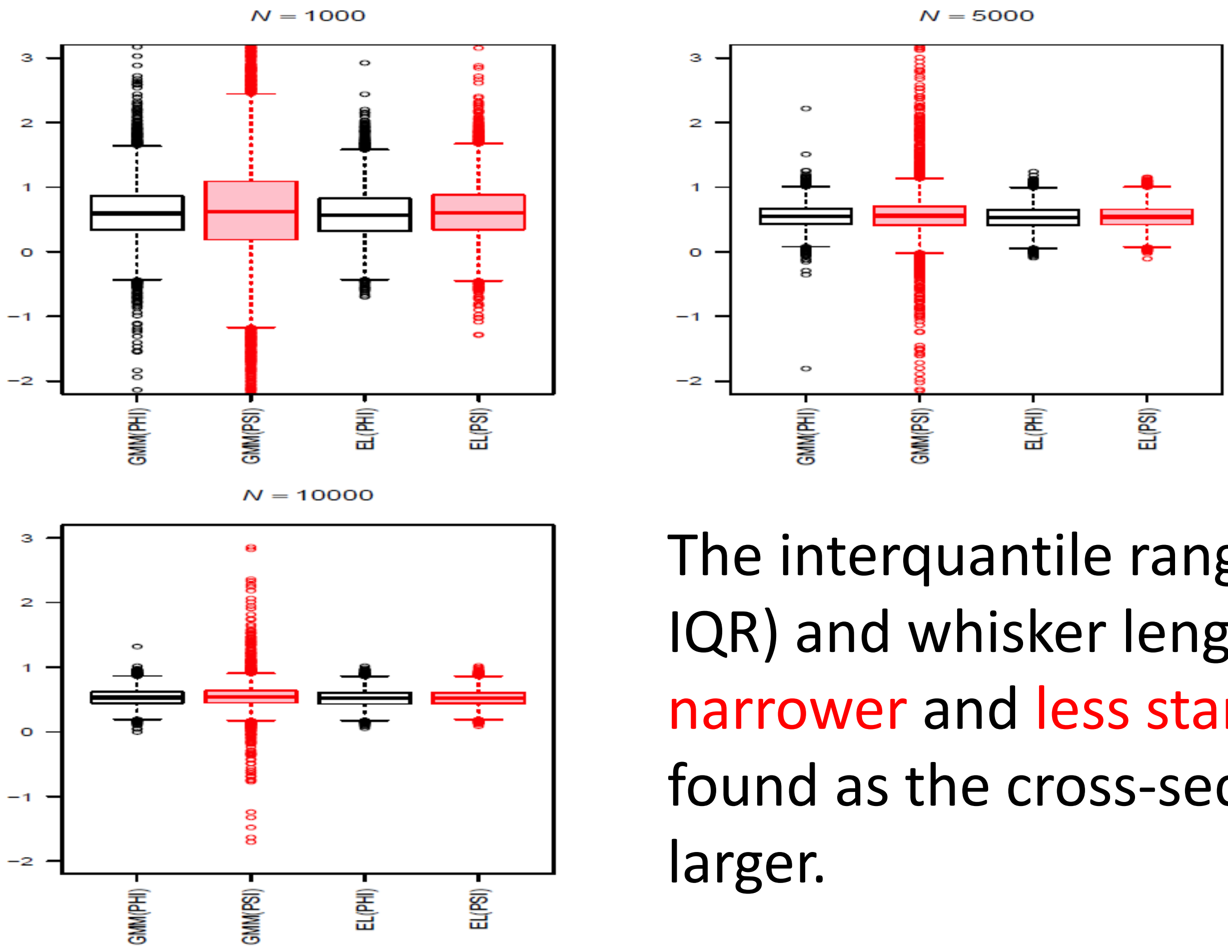
Table 2: Monte Carlo results for the fixed effects ZIP model, $T = 8$

		$N = 1000$		$N = 5000$		$N = 10000$	
		bias	rmse	bias	rmse	bias	rmse
GMM(PHI)	δ	0.085	2.633	0.046	0.212	0.032	0.131
	β	0.001	0.073	0.001	0.035	0.000	0.026
GMM(PSI)	δ	1.982	106.559	0.043	3.052	0.061	1.761
	β	-0.114	0.222	-0.032	0.080	-0.017	0.051
EL(PHI)	δ	0.082	0.395	0.027	0.177	0.018	0.130
	β	0.003	0.072	0.001	0.036	0.000	0.027
EL(PSI)	δ	0.119	0.440	0.040	0.178	0.025	0.129
	β	0.015	0.116	0.005	0.055	0.003	0.040
ML(POOL)	δ	0.342	0.346	0.341	0.342	0.341	0.342
	β	0.476	0.479	0.477	0.478	0.477	0.477

The bias and rmse for the GMM and EL estimators dwindle in size as the cross-sectional size *N* increases, reflecting the consistency, while the *considerable upward bias* of the inconsistent *ML(POOL) estimator* remains unchanged.

Monte Carlo boxplots of the GMM and EL estimators for δ, *T=8*

Figure 3: Monte Carlo boxplots of the GMM and EL estimates for δ, $T = 8$



The interquantile range (hereafter IQR) and whisker length become narrower and less standoff outliers are found as the cross-sectional size *N* is larger.

EL is superior to GMM

- When using the PSI transformations based on the mitigated assumptions, the EL estimator overwhelmingly outperforms the GMM estimator whose small sample performance is poor in the extreme, as is seen from the comparison of the performance of the EL(PSI) estimator with that of the GMM(PSI) estimator.
- The smaller sizes of bias and rmse, narrower IQR and whisker range, and less standoff outliers are recognizable for the EL estimator.

Reason for the fact that EL is superior to GMM (1)

- The GMM(PSI) estimator might suffer from the weak instruments problem pointed out by Bound et al. (1995) and Staiger & Stock (1997).
- That is, it could be that the lagged levels of the explanatory variables x_{it} in the moment conditions based on the PSI transformation are the weak instruments for the PSI transformations.
- The EL could solve the problem above.

Reason for the fact that EL is superior to GMM (2)

- The GMM(PSI) estimator (which is the two-step estimator) might be afflicted with the higher-order bias characteristic of the GMM estimator shown by Newey & Smith (2004), leading to its poor small sample performance, judging from the fact that it uses many growing instruments for the PSI transformations as the number of time periods T increases.
- The EL could solve the problem above.

Discarded sample in consistently estimating the fixed effects ZIP model

- The observations for which $(y_{it}, y_{i,t-1}) = (0,0), (0,1), (1,0)$ make no contribution to the identification using the GMM and EL estimators, as is seen from the PHI and PSI transformations.
- In the DGP, rate of the above combinations of the dependent variables attains to about 70 percent for each replication, which is discarded in the estimations.
- Accordingly, a considerable degree of sample sizes would be needed for enhancing the accuracy and precision of the GMM and EL estimators, which is reflected in the Monte Carlo results.

5 Conclusion

- The two types of moment conditions were proposed for consistently estimating the parameters of interest in the fixed effects ZIP model in which zero count outcomes could germinate from the Poisson part as well as from the logit part:
- The moment conditions for the case of **slightly exogenous explanatory variables in logit and Poisson parts** and the moment conditions for the case of **slightly exogenous explanatory variables in logit part and predetermined ones in Poisson part**.
- Monte Carlo experiments indicated that the large number of individuals would behooves for obtaining the accurate and precise GMM and EL estimates.
- It is conceivable that this would be caused by the virtual decrease of sample sizes contributing to the estimation, which is due to mass generation of zero count outcomes.