

# Decomposition of the linear feedback model for count panel data

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**Abstract:** This paper proposes a new estimator for the linear feedback model (LFM), which is one kind of dynamic count data models, in the case of large number of individuals and fixed number of time periods. The new estimator is a GMM estimator, based on two types of the moment restrictions generated after decomposing the LFM; those used for estimating the simple standard dynamic panel data model and those used for estimating the panel data model with multiplicative fixed effects. Although the new estimator requires (for the consistent estimation) the stationary and strict-exogenous explanatory variables composed of the fixed effects and the serially independent disturbances, a requisite and some assumptions necessary for the PSM estimator (which is an efficient estimator in small samples) are not required. Some Monte Carlo experiments exhibit that the new estimator performs well in the small samples.

**Keywords:** count panel data; linear feedback model; stationary and strict-exogenous explanatory variables composed of the fixed effects and the serially independent disturbances; decomposition of regression equations; generalized method of moments

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# 1. Introduction

The linear feedback model (LFM) for count panel data proposed by BGW (Blundell, Griffith, and Windmeijer 2002) is an attractive model for describing the dynamic structure. The LFM can appropriately take into consideration the dependent variables with large positive integer values and/or zero values specific to the count data, for the case of incorporating the dynamics into the count panel data model. For the LFM applied to the panel data set with large number of individuals and fixed number of time periods, some estimators are proposed and discussed in BGW.<sup>1</sup> The level estimator and the within-group estimator are not consistent estimators and lack of accuracy. The generalized method of moments (GMM) estimator using the quasi-differencing transformation is consistent but does not behave well in small samples. Obtaining the consistent pre-sample mean (PSM) estimator (with its small sample performances being satisfactory) require the prerequisite that the pre-sample of the dependent variables are available over long histories and the assumptions that the fixed effects composing the explanatory variables are proportional to the fixed effects in the LFM and that the moment generating functions of the disturbances (with zero mean) composing the dependent variables are equal cross-sectionally and inter-temporally and further finite.

The new estimator proposed in this paper is obtained after decomposing the LFM into two parts: the part analogous to the simple dynamic panel data model and the part where the model is the exponential regression. That is, the new estimator is a GMM estimator where the joint estimation is conducted using the pertinent moment conditions for each of the two parts. Although the new estimator requires for the consistent estimation the stationary and strict-exogenous explanatory variables composed of the fixed effects and the serially independent disturbances, it requires neither the pre-sample nor the assumptions as is needed by the PSM estimator. The weaker assumption necessitated for the new estimator to be consistent is that the moment generating functions of the disturbances (with zero mean) composing the dependent variables are equal inter-temporally. In addition, it is seen from some Monte Carlo experiments that the small sample performances of the new estimator are considerably better than those of the GMM estimator based only on the framework of the quasi-differencing transformation.

The rest of the paper is organized as follows. In section 2, the estimators existing for the LFM are introduced and then the new estimator is proposed. In section 3, the Monte Carlo experiments are carried out for these estimators. Section 4 concludes.

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<sup>1</sup> Empirical applications using the LFM and the estimators for the LFM are conducted mainly on the relationship between patents and R&D. For example, some recent papers are Abdelmoula and Bresson (2005), Bosch et al. (2005), and Uchida and Cook (2005).

## 2. Model and estimators

In this section, some discussions are conducted by using a simple illustration of the LFM for count panel data. Firstly, the assumptions on the LFM are stated, which is required through this paper. Next, the estimators for the LFM developed until now are examined. Finally, the new estimator for the LFM is proposed.

### 2.1. Model and assumptions

A simple LFM for panel data (with  $i=1, \dots, N$ ) is written as

$$y_{it} = \gamma y_{i,t-1} + \exp(\beta x_{it} + \eta_i) + v_{it}, \quad \text{for } t=2, \dots, T, \quad (1)$$

where  $y_{it}$  is the number of counts for individual  $i$  at time  $t$ ,  $x_{it}$  is an explanatory variable (or an input giving birth to the counts) for individual  $i$  at time  $t$ ,  $\eta_i$  is the individual specific effect for individual  $i$ ,  $v_{it}$  is the disturbance for individual  $i$  at time  $t$ , and  $\gamma$  and  $\beta$  are the parameters of interest.

The supposition in this paper is that the explanatory variable  $x_{it}$  is written as

$$x_{it} = \lambda_i + w_{it}, \quad \text{for } t=1, \dots, T, \quad (2)$$

where  $E[w_{it}] = 0$ ,  $\lambda_i$  and  $w_{it}$  are mutually independent,  $w_{it}$  and  $w_{is}$  are mutually independent with  $t \neq s$ ,  $w_{it}$  and  $\eta_i$  are mutually independent, and  $w_{it}$  and  $y_{it}$  are mutually independent. Further, it is assumed that the relationship holds for (1) that

$E[v_{it} | y_{it}, \lambda_i, w_i^T, \eta_i, v_i^{t-1}] = 0$  with  $w_i^T = (w_{i1}, \dots, w_{iT})$  and  $v_i^{t-1} = (v_{i1}, \dots, v_{i,t-1})$ . These assumptions imply that the explanatory variable  $x_{it}$  is the stationary and strict exogenous variable composed of the fixed effect and the serially independent disturbance.

### 2.2. Level estimator

The level estimator for  $\gamma$  and  $\beta$  using the auxiliary parameter  $\beta_0$  solves the moment conditions

$$\sum_{i=1}^N \sum_{t=2}^T z_{it} [y_{it} - \gamma y_{i,t-1} - \exp(\beta_0 + \beta x_{it})] = 0, \quad (3)$$

where the vector  $z_{it} = [1 \ y_{i,t-1} \ x_{it}]'$ . This estimator is inconsistent even when  $N \rightarrow \infty$  and  $T \rightarrow \infty$ , due to the ignorance of the individual effect  $\eta_i$ .

### 2.3. Within group estimator

The within group (WG) estimator for  $\gamma$  and  $\beta$  solves the moment conditions

$$\sum_{i=1}^N \sum_{t=2}^T z_{it} [y_{it} - \gamma y_{i,t-1} - (\bar{y}_i - \gamma \bar{y}_{i,-1})(f_{it}/\bar{f}_i)] = 0 \quad , \quad (4)$$

where  $z_{it} = [y_{i,t-1} \quad x_{it}]'$  ,  $\bar{y}_i = (1/(T-1)) \sum_{t=2}^T y_{it}$  ,  $\bar{y}_{i,-1} = (1/(T-1)) \sum_{t=2}^T y_{i,t-1}$  ,

$f_{it} = \exp(\beta x_{it})$  , and  $\bar{f}_i = (1/(T-1)) \sum_{t=2}^T f_{it}$  . This estimator for the LFM is proposed by BGW (2002), which may be said to be a count panel data version of the least square dummy variable (LSDV) estimator. For the reason similar to the case of the LSDV estimator, the WG estimator for the LFM is inconsistent when  $N \rightarrow \infty$  and  $T$  is fixed, although it is consistent when  $N \rightarrow \infty$  and  $T \rightarrow \infty$  .<sup>2</sup>

#### 2.4. Pre-sample mean estimator

When the fixed effect in the explanatory variable is proportional to the fixed effect in the regression (i.e.  $\lambda_i = \kappa \eta_i$  with  $\kappa$  being constant) and  $w_{it}$  (satisfying the conditions mentioned in subsection 2.1) has a finite moment generating function for all  $i$  and  $t$  , the fixed effect  $\eta_i$  is a linear function of the pre-sample mean of the dependent variable  $y_{it}$  (i.e.

$\eta_i = \beta_0^* + \phi \log(\bar{y}_{ip})$  , where  $\beta_0^*$  and  $\phi$  are constant, and  $\bar{y}_{ip} = (1/TP) \sum_{r=0}^{TP-1} y_{i,0-r}$  with the pre-sample period  $TP \rightarrow \infty$  ). When the pre-sample variables of the dependent variable are available for long-run period, the pre-sample mean (PSM) estimator for  $\gamma$  and  $\beta$  using the auxiliary parameters  $\beta_0^*$  and  $\phi$  solves the moment conditions introducing

$$\eta_i = \beta_0^* + \phi \log(\bar{y}_{ip}) \quad \text{into (1)}$$

$$\sum_{i=1}^N \sum_{t=2}^T z_{it} [y_{it} - \gamma y_{i,t-1} - \exp(\beta_0^* + \beta x_{it} + \phi \log(\bar{y}_{ip}))] = 0 \quad , \quad (5)$$

where the vector  $z_{it} = [1 \quad y_{i,t-1} \quad x_{it} \quad \log(\bar{y}_{ip})]'$  . The PSM estimator is consistent when  $N \rightarrow \infty$  and  $TP \rightarrow \infty$  . The PSM estimator for the LFM is proposed by BGW (2002).<sup>3</sup>

#### 2.5. Quasi-type transformed GMM estimator

Chamberlain (1992) and Wooldridge (1997) contrive the GMM estimator based on the quasi-differenced transformation for the panel data model with the multiplicative fixed effect. By using the GMM estimator based on the quasi-differenced transformation, the parameters  $\gamma$  and  $\beta$  in the LFM (1) with (2) can be estimated consistently, as is shown in BGW (2002).

2 The within group estimator coincides with the Poisson CMLE (conditional maximum likelihood estimator) used in Hausman et al. (1984) when  $\gamma = 0$  .

3 The idea that the usage of the pre-sample histories can approximate the fixed effect originates from Blundell et al. (1995). In addition, the PSM estimator is discussed in Blundell et al. (1999) for the count panel data model without dynamics.

The quasi-differenced transformation of (1) is written as

$$\Delta q_{it} = u_{it}(f_{i,t-1}/f_{it}) - u_{i,t-1}, \quad \text{for } t=3, \dots, T, \quad (6)$$

where  $u_{it} = y_{it} - \gamma y_{i,t-1}$  and  $f_{it} = \exp(\beta x_{it})$ . For the quasi-differenced transformation (6), the conditional moment conditions hold under the assumptions in subsection 2.1:

$$E[\Delta q_{it} | y_i^{t-2}, x_i^T] = 0, \quad \text{for } t=3, \dots, T, \quad (7)$$

where  $y_i^{t-2} = (y_{i1}, y_{i2}, \dots, y_{i,t-2})$  and  $x_i^T = (x_{i1}, x_{i2}, \dots, x_{iT})$ .<sup>4</sup> Then, from the conditional moment conditions (7), the following unconditional moment conditions can be obtained:

$$E[y_{is} \Delta q_{it}] = 0, \quad \text{for } t=3, \dots, T \text{ and } s=1, \dots, t-2, \quad (8)$$

and

$$E[x_{i,t-1} \Delta q_{it}] = 0 \text{ and } E[x_{it} \Delta q_{it}] = 0, \text{ for } t=3, \dots, T. \quad (9)$$

Further, in this paper, the so-called quasi-level transformation is used from the viewpoint that the explanatory variables are mean-stationary. The quasi-level transformation of the LFM (1) is written as

$$q_{it} = u_{it}/f_{it}, \quad \text{for } t=2, \dots, T, \quad (10)$$

where  $u_{it}$  and  $f_{it}$  are the same as those in the quasi-differenced transformation (6). For the quasi-level transformation (10), the conditional moment conditions hold under the assumptions in subsection 2.1:

$$E[(q_{it} - g_i) | \Delta x_i^T] = 0, \quad \text{for } t=2, \dots, T, \quad (11)$$

where  $g_i = \exp(\eta_i)$  and  $\Delta x_i^T = (\Delta x_{i2}, \Delta x_{i3}, \dots, \Delta x_{iT})$ .<sup>5</sup> Then, from the conditional moment conditions (11), the following unconditional moment conditions can be obtained:

$$E[q_{it} \Delta x_{it}] = 0, \quad \text{for } t=2, \dots, T, \quad (12)$$

where the assumptions on the LFM (1) with (2) in subsection 2.1 and the relationship  $E[g_i \Delta x_{it}] = 0$  derived from the assumptions are used.<sup>6</sup>

Finally, the quasi-type transformed GMM (QGMM) estimator for the parameter vector  $\theta = [\gamma \ \beta]'$  constructed by using the moment conditions (8), (9), and (12) is obtained by minimizing the following criterion with respect to  $\theta$ :

$$\left( (1/N) \sum_{i=1}^N \epsilon_i^Q{}' Z_i^Q \right) W_N^Q \left( (1/N) \sum_{i=1}^N Z_i^Q{}' \epsilon_i^Q \right), \quad (13)$$

where the column vector  $\epsilon_i^Q = [\Delta q_{i3} \ \Delta q_{i4} \ \dots \ \Delta q_{iT}]'$  being a  $(T-2)$  element column vector and  $q_i = [q_{i2} \ q_{i3} \ \dots \ q_{iT}]'$  being a  $(T-1)$  element

<sup>4</sup> Note that  $\Delta q_{it} = u_{it}(f_{i,t-1}/f_{it}) - u_{i,t-1} = v_{it}(f_{i,t-1}/f_{it}) - v_{i,t-1}$ .

<sup>5</sup> Note that  $E[q_{it} | \Delta x_i^T] = E[u_{it}/f_{it} | \Delta x_i^T] = E[(g_i + v_{it}/f_{it}) | \Delta x_i^T]$ .

<sup>6</sup> Note that  $E[g_i \Delta x_{it}] = E[g_i \Delta w_{it}] = 0$  from the assumptions on the LFM (1) with (2) in subsection 2.1.

column vector, the matrix

$$Z_i^O = \begin{bmatrix} A_i & B_i & O \\ O & O & C_i \end{bmatrix}$$

with

$$A_i = \begin{bmatrix} y_{i1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & y_{i1} & y_{i2} & \cdots & y_{i,T-2} \end{bmatrix}$$

being a  $(T-2)$  by  $(T-2)(T-1)/2$  matrix,

$$B_i = \begin{bmatrix} x_{i2} & x_{i3} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & x_{i3} & x_{i4} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & x_{i,T-1} & x_{iT} \end{bmatrix}$$

being a  $(T-2)$  by  $2(T-2)$  matrix,

$$C_i = \begin{bmatrix} \Delta x_{i2} & 0 & \cdots & 0 \\ 0 & \Delta x_{i3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta x_{iT} \end{bmatrix}$$

being a  $(T-1)$  by  $(T-1)$  diagonal matrix, and  $O$  being the zero matrix, and the weighting matrix

$$W_N^O = \left( (1/N) \sum_{i=1}^N Z_i^O{}' \epsilon_i^O(\tilde{\theta}_1) \epsilon_i^O(\tilde{\theta}_1)' Z_i^O \right)^{-1}$$

with  $\epsilon_i^O(\tilde{\theta}_1)$  being the vector  $\epsilon_i^O$  realized by incorporating an initial consistent estimate  $\tilde{\theta}_1$  for  $\theta$ .<sup>7</sup> The QGMM estimator is a consistent estimator for  $N \rightarrow \infty$ , and is constructed in the framework of the GMM estimator based on the quasi-differenced transformation.

## 2.6. Decomposed GMM estimator

Equation (1) is rewritten as the following set of two equations, by using equation (2):

$$y_{it} = \gamma y_{i,t-1} + u_{it}, \quad \text{for } t=2, \dots, T, \quad (14)$$

and

$$u_{it} = \mu_i \omega_{it} + v_{it}, \quad \text{for } t=2, \dots, T, \quad (15)$$

where  $\mu_i = \exp(\beta \lambda_i + \eta_i)$  and  $\omega_i = \exp(\beta w_i)$ . It can be recognized that the form (14) with (15)

<sup>7</sup> In this paper, the initial consistent estimate  $\tilde{\theta}_1$  is obtained by minimizing (13) with respect to  $\theta$ , after incorporating  $W_N^O = \left( (1/N) \sum_{i=1}^N Z_i^O{}' Z_i^O \right)^{-1}$  into (13).

is analogous to the form for a simple standard dynamic panel data model presented in Arellano and Bond (1991). Under the assumptions on equations (1) and (2) in subsection 2.1 and the assumption that the moment generating function of  $w_{it}$  (i.e.  $\omega_i = \exp(\beta w_i)$ ) is equal for all  $t$ , the following relationships among  $y_{it}$  and  $u_{it}$  (for  $t=2, \dots, T$ ) hold for any  $i$ :

$$E[y_{it}u_{it}] = \phi_i, \quad \text{for } t=2, \dots, T, \quad (16)$$

and

$$E[u_{it}u_{is}] = \varphi_i, \quad \text{for } t \neq s \text{ and } s, t=2, \dots, T, \quad (17)$$

where  $\phi_i = \xi_i \varpi_i$  and  $\varphi_i = \mu_i^2 \varpi_i^2$  with  $\xi_i = E[y_{it}\mu_i]$  and  $\varpi_i = E[\omega_{it}]$ . Accordingly, from the relationships (16) and (17), the following moment conditions based on the covariances of  $y_{it}$  and  $u_{it}$  (for  $t=2, \dots, T$ ) are obtained for estimating  $\gamma$  consistently:

$$E[y_{it}\Delta u_{is}] = 0, \quad \text{for } t=3, \dots, T \text{ and } s=1, \dots, t-2, \quad (18)$$

and

$$E[u_{it}\Delta u_{i,t-1}] = 0, \quad \text{for } t=4, \dots, T. \quad (19)$$

The derivation process of the moment conditions (18) and (19) is the same as that described in Ahn (1990) and Ahn and Schmidt (1995), and therefore the moment conditions (18) and (19) are the same moment conditions as the standard moment conditions proposed by Holtz-Eakin et al. (1988) and Arellano and Bond (1991) and the additional non-linear moment conditions proposed by Ahn and Schmidt (1995) respectively.<sup>8</sup>

In addition, it is seen from subsection 2.5 that since equation (15) is nothing but  $u_{it} = \exp(\beta x_{it} + \eta_i) + v_{it}$ , the moment conditions (9) and (12) are also valid especially with respect to estimating  $\beta$  consistently.

In practice, in this paper, the joint estimation using the moment conditions (18), (19), (9), and (12) is conducted to estimate  $\gamma$  and  $\beta$  consistently. That is, the decomposed GMM (DGMM) estimator for the parameter vector  $\theta = [\gamma \ \beta]'$  is obtained by minimizing the following criterion with respect to  $\theta$

$$\left( (1/N) \sum_{i=1}^N \epsilon_i^D{}' Z_i^D \right) W_N^D \left( (1/N) \sum_{i=1}^N Z_i^D{}' \epsilon_i^D \right), \quad (20)$$

where the column vector  $\epsilon_i^D = [\Delta u_i' \ n_i' \ \Delta q_i' \ q_i']'$  with  $n_i = [n_{i4} \ n_{i5} \ \dots \ n_{iT}]'$  being a  $(T-3)$  element column vector after defining  $n_{it} = u_{it} \Delta u_{i,t-1}$  for  $t=4, \dots, T$  and the remaining column vectors being the same as those in subsection 2.5, the matrix

<sup>8</sup> See also Blundell and Bond (1998).

$$Z_i^D = \begin{bmatrix} A_i & O & O & O \\ O & I & O & O \\ O & O & B_i & O \\ O & O & O & C_i \end{bmatrix}$$

with  $I$  being the  $(T-3)$  by  $(T-3)$  identity matrix and the remaining matrices being the same as those in subsection 2.5, and the weighting matrix

$$W_N^D = \left( (1/N) \sum_{i=1}^N Z_i^D{}' \epsilon_i^D(\tilde{\theta}_1) \epsilon_i^D(\tilde{\theta}_1)' Z_i^D \right)^{-1},$$

with  $\epsilon_i^D(\tilde{\theta}_1)$  being the vector  $\epsilon_i^D$  realized by incorporating an initial consistent estimate  $\tilde{\theta}_1$  for  $\theta$ .<sup>9</sup> The DGMM estimator is a consistent estimator for  $N \rightarrow \infty$ , and is constructed using two types of the moment conditions corresponding to the simple dynamic panel data model and the quasi-type transformation. On the DGMM estimator, it is considered that the usage of the moment conditions (18) and (19) can achieve a good small sample performance of the estimator for the AR(1) parameter  $\gamma$  at least, taking into account the good performance in the small sample for the simple dynamic panel data model.<sup>10</sup> Further, as is required for the PSM estimator, the DGMM estimator require (for the consistent estimation) neither the pre-sample of the dependent variables with long histories, nor the assumption that the fixed effect  $\lambda_i$  in the explanatory variable is proportional to the fixed effect  $\eta_i$  in the regression, nor the assumption that the disturbance  $w_{it}$  in the explanatory variable has a finite moment generating function for all  $i$  and  $t$ . The assumption required in order to obtain the consistent DGMM estimator is that the moment generating function of  $w_{it}$  is equal for all  $t$ . Accordingly, under the weaker restrictions than in the PSM estimator, the DGMM estimator is consistent, given the assumptions in subsection 2.1.

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9 In this paper, the initial consistent estimate  $\tilde{\theta}_1$  is obtained by minimizing (20) with respect to  $\theta$ , after incorporating  $W_N^D = \left( (1/N) \sum_{i=1}^N Z_i^D{}' Z_i^D \right)^{-1}$  into (20).

10 For example, see Kitazawa (2001).

### 3. Monte Carlo experiments

In order to investigate the performance of the DGMM estimator and to compare its performance with that of the other estimators, Monte Carlo experiments are conducted for some settings of the LFM. Through the settings, the explanatory variable is set to be the stationary and strict exogenous, composed of the fixed effect and the serially independent disturbance. The experiments are carried out for both cases that the fixed effect in the explanatory variable is proportional to the fixed effect in the LFM and that the fixed effect in the explanatory variable is correlated with the fixed effect in the LFM but is not proportional to the fixed effect in the LFM.

#### 3.1. Data generating process

The data generating process (DGP) used is

$$\begin{aligned} y_{it} &\sim \text{Poisson}(\gamma y_{i,t-1} + \exp(\beta x_{it} + \eta_i)) \quad , \\ y_{i,-TG+1} &\sim \text{Poisson}(\exp(\beta x_{i,-TG+1} + \eta_i)) \quad , \\ x_{it} &= \kappa \eta_i + \iota \zeta_i + w_{it} \quad , \\ \eta_i &\sim N(0, \sigma_\eta^2) \quad ; \quad \zeta_i \sim N(0, \sigma_\zeta^2) \quad ; \quad w_{it} \sim N(0, \sigma_w^2) \quad , \end{aligned}$$

where  $t = -TG+1, \dots, 1, \dots, T$  with  $TG$  being the number of the periods of the pre-sample to be generated. Although in the DGP the distinct parameter settings of  $\iota$  (i.e.  $\iota=0$  And  $\iota=1$ ) are conducted corresponding to the both cases above, the settings of the remaining parameters (i.e.

$\gamma=0.5$  ,  $\beta=0.5$  ,  $\kappa=0.2$  ,  $\sigma_\eta^2=0.5$  ,  $\sigma_\zeta^2=0.5$  , and  $\sigma_w^2=2/3$  ) are common to the both cases. For these settings of the parameters, the experiments are carried out for  $N=100$  ,  $500$  , and  $1000$  and  $T=4$  and  $T=8$  , after generating the pre-sample with  $TG=50$  . The number of the Monte Carlo replications is  $1000$  . Results for the experiments are described as follows.<sup>11</sup>

#### 3.2. Results for the case of $\iota=0$

In this case, the fixed effect in the explanatory variable is proportional to the fixed effect in the LFM, where the PSM (using the pre-sample with long histories), QGMM, and DGMM estimators are consistent for  $N \rightarrow \infty$  . The results for the experiments are shown in Table 1 and 2. The upper and lower biases are found in the results for the inconsistent Level and WG estimators respectively. The sizes of the bias and the rmse (root mean squared error) for these estimators are considerable and do not become smaller with larger  $N$  . It is considered that these are endemic to the Level and WG estimates. The PSM estimators using the long pre-sample histories (i.e. those with

<sup>11</sup> The experiments are implemented with an econometric software TSP 4.5.

$TP=25$  and  $TP=50$  ) behave well even in the case of the smaller cross-sectional size (i.e.  $N=100$  ). For the smaller  $N$  , the QGMM estimates are biased downwards and the size of their bias and rmse are large, but these decrease with  $N$  increasing, reflecting the property of the consistent estimator. Finally, the bias sizes of the DGMM estimates are considerably small compared with those of the QGMM estimates for all  $N$  , and are a match for those of the PSM estimates using long pre-sample histories even when  $N=100$  . Although the rmse sizes of the DGMM estimates are not almost different from those of the QGMM estimates for the same  $N$  when  $T=4$  however, the former is considerably smaller than the latter when  $T=8$  . The small sample performance of the DGMM estimator is preferable especially when  $T=8$  .<sup>12</sup>

### 3.3. Results for the case of $\iota=1$

In this case, the fixed effect in the explanatory variable is correlated with the fixed effect in the LFM but is not proportional to the fixed effect in the LFM, where for  $N \rightarrow \infty$  the QGMM and DGMM estimators are consistent, while the PSM estimators are inconsistent even if using the long pre-sample histories. The results for the experiments are shown in Table 3 and 4. As is similar to the case of  $\iota=0$  , the inaccuracies endemic to using the inconsistent estimators are found in the results using the Level and WG estimators. Different from the case of  $\iota=0$  , the bias and rmse sizes of the PSM estimates are not small even when using the long pre-sample histories, for all  $N$  and  $T$  . Contrary to the results for these inconsistent estimators, the accuracies of the consistent QGMM and DGMM estimates improve with  $N$  increasing, as is similar to the case of  $\iota=0$  . Further, as is similar to the case of  $\iota=0$  , the bias sizes of the DGMM estimates are much smaller than those of the QGMM estimates, and for  $T=8$  the rmse sizes are considerably smaller than those of the QGMM estimates.

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<sup>12</sup> The results of the Monte Carlo experiments for the Level, WG, PSM, QGMM estimators are considerably similar to those in BGW (2002).

## 5. Conclusion

This paper has proposed the DGMM estimator applicable to the LFM for count panel data. The DGMM estimator requires the stationary and strict exogenous explanatory variables composed of the fixed effects and the serially independent disturbances, but (as is required for the PSM estimator) requires neither the long pre-sample histories of the dependent variables, nor the assumption that the fixed effects in the dependent variables are proportional to the fixed effects in the LFM, nor the assumption that the moment generating functions of the disturbances (with zero mean) in explanatory variables are finite and equal among individuals and over time. That is, for the LFM with the explanatory variables composed of the fixed effects and the serially independent disturbances, the DGMM estimator is consistent under the assumptions that the fixed effects in the dependent variables are not proportional to the fixed effects in the LFM and that the moment generating functions of the disturbances (with their mean zero) in explanatory variables are equal over time without the moment generating functions being equal among individuals and being finite. Some Monte Carlo experiments show that the small sample performances of the DGMM estimator are fairly favorable, superior to the QGMM estimator, and a match for those of the consistent PSM estimator.

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Table 1.

Monte Carlo results for the LFM, with  $T=4$ 

		$\gamma=0.5$ ; $\beta=0.5$ ; $\kappa=0.2$ ; $\sigma_{\eta}^2=0.5$ ; $\iota=0$ ; $\sigma_{\zeta}^2=0.5$ ; $\sigma_w^2=2/3$					
		$N=100$		$N=500$		$N=1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	$\gamma$	0.255	0.262	0.269	0.271	0.272	0.273
	$\beta$	0.738	0.869	0.736	0.764	0.732	0.745
WG	$\gamma$	-0.450	0.462	-0.446	0.449	-0.444	0.445
	$\beta$	-0.285	0.293	-0.290	0.291	-0.290	0.290
PSM	$\gamma(4)$	0.131	0.156	0.149	0.154	0.153	0.156
	$\beta(4)$	0.265	0.365	0.272	0.292	0.275	0.286
	$\gamma(8)$	0.100	0.124	0.116	0.122	0.121	0.124
	$\beta(8)$	0.185	0.267	0.191	0.209	0.196	0.206
	$\gamma(25)$	0.041	0.085	0.056	0.066	0.059	0.065
	$\beta(25)$	0.076	0.158	0.079	0.101	0.082	0.094
	$\gamma(50)$	0.016	0.076	0.030	0.047	0.033	0.043
	$\beta(50)$	0.039	0.133	0.041	0.071	0.044	0.060
QGMM	$\gamma$	-0.133	0.218	-0.070	0.106	-0.047	0.079
	$\beta$	-0.124	0.194	-0.076	0.123	-0.055	0.103
DGMM	$\gamma$	-0.050	0.192	-0.032	0.092	-0.014	0.067
	$\beta$	-0.072	0.200	-0.032	0.119	-0.005	0.112

Notes: Level is the level estimator, WG is the within group estimator, PSM is the pre-sample mean estimator where the number of the last pre-sample periods used for the estimation is described in the parentheses next to  $\gamma$  and  $\beta$ , QGMM is the quasi-type GMM estimator, and DGMM is the decomposed GMM estimator. The Monte Carlo results are almost invariant to some different starting values in the optimizations implementing for these estimators.

Table 2.  
Monte Carlo results for the LFM, with  $T=8$

		$\gamma=0.5$ ; $\beta=0.5$ ; $\kappa=0.2$ ; $\sigma_{\eta}^2=0.5$ ; $\iota=0$ ; $\sigma_{\xi}^2=0.5$ ; $\sigma_w^2=2/3$					
		$N=100$		$N=500$		$N=1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	$\gamma$	0.258	0.262	0.272	0.273	0.272	0.273
	$\beta$	0.704	0.757	0.735	0.749	0.734	0.743
WG	$\gamma$	-0.190	0.198	-0.189	0.190	-0.190	0.191
	$\beta$	-0.154	0.164	-0.157	0.159	-0.158	0.159
PSM	$\gamma(4)$	0.139	0.148	0.155	0.158	0.156	0.158
	$\beta(4)$	0.254	0.292	0.276	0.285	0.277	0.285
	$\gamma(8)$	0.108	0.120	0.124	0.127	0.125	0.126
	$\beta(8)$	0.181	0.217	0.198	0.207	0.200	0.206
	$\gamma(25)$	0.050	0.071	0.062	0.067	0.063	0.066
	$\beta(25)$	0.075	0.117	0.085	0.096	0.085	0.092
	$\gamma(50)$	0.025	0.056	0.035	0.043	0.036	0.040
	$\beta(50)$	0.038	0.090	0.046	0.060	0.046	0.054
QGMM	$\gamma$	-0.196	0.215	-0.091	0.099	-0.060	0.067
	$\beta$	-0.208	0.217	-0.118	0.125	-0.084	0.091
DGMM	$\gamma$	0.025	0.128	-0.004	0.045	-0.004	0.028
	$\beta$	-0.059	0.131	-0.024	0.071	-0.017	0.046

Notes: Level is the level estimator, WG is the within group estimator, PSM is the pre-sample mean estimator where the number of the last pre-sample periods used for the estimation is described in the parentheses next to  $\gamma$  and  $\beta$ , QGMM is the quasi-type GMM estimator, and DGMM is the decomposed GMM estimator. The Monte Carlo results are almost invariant to some different starting values in the optimizations implementing for these estimators.

Table 3.  
Monte Carlo results for the LFM, with  $T=4$

		$\gamma=0.5$ ; $\beta=0.5$ ; $\kappa=0.2$ ; $\sigma_{\eta}^2=0.5$ ; $\iota=1$ ; $\sigma_{\zeta}^2=0.5$ ; $\sigma_w^2=2/3$					
		$N=100$		$N=500$		$N=1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	$\gamma$	0.253	0.261	0.271	0.274	0.272	0.273
	$\beta$	0.422	0.541	0.439	0.473	0.428	0.440
WG	$\gamma$	-0.430	0.443	-0.430	0.433	-0.429	0.431
	$\beta$	-0.280	0.288	-0.283	0.285	-0.284	0.284
PSM	$\gamma(4)$	0.130	0.157	0.152	0.159	0.154	0.158
	$\beta(4)$	0.059	0.212	0.065	0.130	0.062	0.093
	$\gamma(8)$	0.101	0.134	0.120	0.129	0.121	0.127
	$\beta(8)$	-0.005	0.162	-0.004	0.084	-0.005	0.057
	$\gamma(25)$	0.044	0.103	0.059	0.075	0.060	0.070
	$\beta(25)$	-0.094	0.167	-0.098	0.114	-0.099	0.108
	$\gamma(50)$	0.020	0.095	0.032	0.059	0.033	0.050
	$\beta(50)$	-0.125	0.174	-0.129	0.144	-0.131	0.136
QGMM	$\gamma$	-0.142	0.235	-0.081	0.119	-0.053	0.081
	$\beta$	-0.138	0.198	-0.086	0.130	-0.061	0.100
DGMM	$\gamma$	-0.035	0.217	-0.032	0.124	-0.020	0.070
	$\beta$	-0.096	0.210	-0.037	0.151	-0.015	0.108

Notes: Level is the level estimator, WG is the within group estimator, PSM is the pre-sample mean estimator where the number of the last pre-sample periods used for the estimation is described in the parentheses next to  $\gamma$  and  $\beta$ , QGMM is the quasi-type GMM estimator, and DGMM is the decomposed GMM estimator. The Monte Carlo results are almost invariant to some different starting values in the optimizations implementing for these estimators.

Table 4.  
Monte Carlo results for the LFM, with  $T=8$

		$\gamma=0.5$ ; $\beta=0.5$ ; $\kappa=0.2$ ; $\sigma_{\eta}^2=0.5$ ; $\iota=1$ ; $\sigma_{\zeta}^2=0.5$ ; $\sigma_w^2=2/3$					
		$N=100$		$N=500$		$N=1000$	
		bias	rmse	bias	rmse	bias	rmse
Level	$\gamma$	0.256	0.261	0.271	0.272	0.272	0.272
	$\beta$	0.410	0.458	0.428	0.440	0.425	0.432
WG	$\gamma$	-0.185	0.194	-0.181	0.183	-0.181	0.183
	$\beta$	-0.152	0.162	-0.151	0.153	-0.152	0.154
PSM	$\gamma(4)$	0.135	0.148	0.153	0.157	0.155	0.157
	$\beta(4)$	0.044	0.129	0.056	0.084	0.054	0.072
	$\gamma(8)$	0.106	0.123	0.122	0.128	0.124	0.127
	$\beta(8)$	-0.014	0.107	-0.006	0.054	-0.006	0.040
	$\gamma(25)$	0.051	0.081	0.062	0.072	0.064	0.069
	$\beta(25)$	-0.098	0.127	-0.097	0.105	-0.097	0.102
	$\gamma(50)$	0.027	0.069	0.035	0.051	0.037	0.046
	$\beta(50)$	-0.128	0.148	-0.129	0.134	-0.129	0.132
QGMM	$\gamma$	-0.209	0.228	-0.095	0.103	-0.063	0.070
	$\beta$	-0.214	0.223	-0.120	0.127	-0.087	0.094
DGMM	$\gamma$	0.053	0.169	0.009	0.054	0.005	0.038
	$\beta$	-0.072	0.152	-0.010	0.071	-0.007	0.055

Notes: Level is the level estimator, WG is the within group estimator, PSM is the pre-sample mean estimator where the number of the last pre-sample periods used for the estimation is described in the parentheses next to  $\gamma$  and  $\beta$ , QGMM is the quasi-type GMM estimator, and DGMM is the decomposed GMM estimator. The Monte Carlo results are almost invariant to some different starting values in the optimizations implementing for these estimators.