

Program “dfelrtna.tsp” for conducting root-N consistent estimations of dynamic fixed effects logit models including strictly exogenous explanatory variables with or without time dummies

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1 Introduction

The program “dfelrtna.tsp” is one part of the set “DFEL-RTN (version 0.0.0)” that conducts root-N consistent estimations of dynamic fixed effects logit models. The program “dfelrtna.tsp” deals with the models including strictly exogenous explanatory variables with or without time dummies. This document illustrates the usage of “dfelrtna.tsp”. See the document “dfelrtn000.pdf” for how to install “DFEL-RTN (version 0.0.0)”.

2 Model and estimation

This section illustrates one of the models that the program deals with and the root-N consistent estimators that the program implements. The illustrative dynamic fixed effects logit model is of the ‘AR(1)’ form with both strictly exogenous continuous explanatory variables and time dummies. Through this document, i ($i = 1, \dots, N$) and t ($t = 1, \dots, T$) denote the individual and time period, respectively, where it is assumed that the number of individuals $N \rightarrow \infty$, while the number of time periods T is fixed.

2.1 Model

The ‘AR(1)’ fixed effects logit model including time dummies and K strictly exogenous continuous explanatory variables is written as follows:

$$y_{it} = \frac{\exp\left(\eta_i + TD_t + \gamma y_{i,t-1} + \sum_{k=1}^K \beta_{(k)} x_{(k)it}\right)}{1 + \exp\left(\eta_i + TD_t + \gamma y_{i,t-1} + \sum_{k=1}^K \beta_{(k)} x_{(k)it}\right)} + v_{it}, \quad \text{for } t = 2, \dots, T,$$

Where y_{it} is the binary dependent variable for i at t , η_i is the fixed effect for i , TD_t is the time dummy at t , $x_{(k)it}$ is the k th strictly exogenous continuous explanatory variable for i at t , γ and $\beta_{(k)}$ (for $k = 1, \dots, K$) are the parameters of

interest, and the disturbances v_{it} for i at t satisfy

$$E[v_{it} | \eta_i, y_{i1}, v_i^{t-1}, x_{(1)i}^T, \dots, x_{(K)i}^T] = 0$$

with v_{i1} being empty, $v_i^{t-1} = (v_{i1}, \dots, v_{i,t-1})$ and $x_{(k)i}^T = (x_{(k)i1}, \dots, x_{(k)iT})$. The binary dependent variable y_{it} takes one with the probability given by the first term on the right-hand side in the equation above.

The specification above is called the implicit form for the dynamic fixed effects logit model in this document.

2.2 Conditional moment conditions

Kitazawa (2013, 2016) proposed two types of conditional moment conditions for the ‘AR(1)’ fixed effects logit model described in the previous subsection, depending on whether the model is rewritten in “g-form” or “h-form”. These imply that root-N consistent estimation for dynamic fixed effects logit model with strictly exogenous continuous explanatory variables and time dummies is feasible.

Conditional moment conditions based on g-form

$$E[\hbar U_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1}) | \eta_i, y_{i1}, v_i^{t-2}, x_{(1)i}^T, \dots, x_{(K)i}^T] = 0, \quad \text{for } t = 3, \dots, T-1,$$

where

$$\begin{aligned} \hbar U_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1}) = & U_{it}^+ - y_{i,t-1} \\ & - \tanh \left(\frac{-\gamma y_{i,t-2} + \Delta TD_t + \Delta TD_{t+1} + \sum_{k=1}^K \beta_{(k)} (\Delta x_{(k)it} + \Delta x_{(k)i,t+1})}{2} \right) (U_{it}^+ + y_{i,t-1} - 2U_{it}^+ y_{i,t-1}) \end{aligned}$$

with

$$\begin{aligned} U_{it}^+ = & y_{it} + (1 - y_{it}) y_{i,t+1} - (1 - y_{it}) y_{i,t+1} \exp \left(-\Delta TD_{t+1} - \sum_{k=1}^K \beta_{(k)} \Delta x_{(k)i,t+1} \right) \\ & - (\exp(\gamma) - 1) y_{i,t-1} (1 - y_{it}) y_{i,t+1} \exp \left(-\Delta TD_{t+1} - \sum_{k=1}^K \beta_{(k)} \Delta x_{(k)i,t+1} \right). \end{aligned}$$

Conditional moment conditions based on h-form

$$E[\hbar Y_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1}) | \eta_i, y_{i1}, v_i^{t-2}, x_{(1)i}^T, \dots, x_{(K)i}^T] = 0, \quad \text{for } t = 3, \dots, T-1,$$

where

$$\begin{aligned} \hbar Y_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1}) &= Y_{it}^+ - y_{i,t-1} \\ &- \tanh \left(\frac{\gamma(1 - y_{i,t-2}) + \Delta TD_t + \Delta TD_{t+1} + \sum_{k=1}^K \beta_{(k)} (\Delta x_{(k)it} + \Delta x_{(k)i,t+1})}{2} \right) (Y_{it}^+ + y_{i,t-1} - 2Y_{it}^+ y_{i,t-1}) \end{aligned}$$

with

$$\begin{aligned} Y_{it}^+ &= y_{it} y_{i,t+1} + y_{it} (1 - y_{i,t+1}) \exp \left(\Delta TD_{t+1} + \sum_{k=1}^K \beta_{(k)} \Delta x_{(k)i,t+1} \right) \\ &+ (\exp(\gamma) - 1) (1 - y_{i,t-1}) y_{it} (1 - y_{i,t+1}) \exp \left(\Delta TD_{t+1} + \sum_{k=1}^K \beta_{(k)} \Delta x_{(k)i,t+1} \right). \end{aligned}$$

The operator Δ is the first-difference operator: for example, $\Delta TD_t = TD_t - TD_{t-1}$ and $\Delta x_{it} = x_{it} - x_{i,t-1}$. The parameters to be estimated root-N consistently by utilizing the conditional moment conditions are $\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \dots, \Delta TD_T$.

2.3 Unconditional moment conditions

Root-N consistent estimations are conducted by employing the GMM estimators using the unconditional moment conditions constructed from the conditional moment conditions based on g-form and h-form in the previous subsection. Typical constructions (selections) of the unconditional moment conditions based on g-form and h-form are as follows:

Typical unconditional moment conditions based on g-form

$$\begin{aligned} E[\hbar U_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1})] &= 0, & \text{for } t = 3, \dots, T-1, \\ E[y_{is} \hbar U_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1})] &= 0, & \text{for } s = 1, \dots, t-2; \ t = 3, \dots, T-1, \\ E[\Delta x_{(k)is} \hbar U_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1})] &= 0, & \text{for } s = t-1, t, t+1; \ t = 3, \dots, T-1; \\ & & k = 1, \dots, K. \end{aligned}$$

Typical unconditional moment conditions based on h-form

$$\begin{aligned} E[\hbar Y_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1})] &= 0, & \text{for } t = 3, \dots, T-1 \\ E[y_{is} \hbar Y_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1})] &= 0, & \text{for } s = 1, \dots, t-2; \ t = 3, \dots, T-1, \\ E[\Delta x_{(k)is} \hbar Y_{it}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_t, \Delta TD_{t+1})] &= 0, & \text{for } s = t-1, t, t+1; \ t = 3, \dots, T-1; \\ & & k = 1, \dots, K. \end{aligned}$$

2.4 GMM estimation

The GMM estimators are illustrated using the unconditional moment conditions $E[\varphi_i(\theta)]$. With

$$\begin{aligned} & \hbar U_i^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \dots, \Delta TD_T) \\ &= [\hbar U_{i3}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \Delta TD_{T4}) \dots \hbar U_{i,T-1}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_{T-1}, \Delta TD_T)]' \end{aligned}$$

and

$$\begin{aligned} & \hbar \Upsilon_i^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \dots, \Delta TD_T) \\ &= [\hbar \Upsilon_{i3}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \Delta TD_{T4}) \dots \hbar \Upsilon_{i,T-1}^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_{T-1}, \Delta TD_T)]', \end{aligned}$$

the vector forms of the typical g-form and h-form unconditional moment conditions are

$$E[\varphi_i(\theta)] = E[\varphi_i^g(\theta)] = E[Z_i' \hbar U_i^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \dots, \Delta TD_T)]$$

and

$$E[\varphi_i(\theta)] = E[\varphi_i^h(\theta)] = E[Z_i' \hbar \Upsilon_i^+(\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \dots, \Delta TD_T)],$$

respectively. The vector form using the moment conditions based on both forms is

$$E[\varphi_i(\theta)] = E[\varphi_i^{gh}(\theta)] = E[\mathbf{vec}(\varphi_i^g(\theta) \varphi_i^h(\theta))],$$

where

$$Z_i = [Y_i \quad X_{(1)i} \quad \dots \quad X_{(K)i} \quad I_{T-3}]$$

with

$$Y_i = \begin{bmatrix} y_{i1} & & & & \\ & y_{i1} & y_{i2} & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & y_{i1} \dots y_{i,T-3} \end{bmatrix},$$

$$X_{(k)i} = \begin{bmatrix} \Delta x_{(k)i2} & \Delta x_{(k)i3} & \Delta x_{(k)i4} & & & \\ & \Delta x_{(k)i3} & \Delta x_{(k)i4} & \Delta x_{(k)i5} & & \\ & & \ddots & & \ddots & \\ & & & \Delta x_{(k)i,T-2} & \Delta x_{(k)i,T-1} & \Delta x_{(k)iT} \end{bmatrix},$$

and I_{T-3} being the identity matrix of order $T-3$.

The two-step GMM estimator for $\theta = (\gamma, \beta_{(1)}, \dots, \beta_{(K)}, \Delta TD_3, \dots, \Delta TD_T)'$ is defined as

$$\hat{\theta}_2 = \arg \min_{\theta} \bar{\varphi}(\theta)' \left(\bar{\Theta}(\hat{\theta}_1) \right)^{-1} \bar{\varphi}(\theta)$$

where $\bar{\varphi}(\theta) = (1/N) \sum_{i=1}^N \varphi_i(\theta)$ (the sample analogue of the vector form of the unconditional moment conditions $E[\varphi_i(\theta)]$ where $E[\varphi_i(\theta)]$ is selected from $E[\varphi_i(\theta)] = E[\varphi_i^g(\theta)]$, $E[\varphi_i(\theta)] = E[\varphi_i^h(\theta)]$, and $E[\varphi_i(\theta)] = E[\varphi_i^{gh}(\theta)]$) and $\bar{\Theta}(\hat{\theta}_1) = (1/N) \sum_{i=1}^N \varphi_i(\hat{\theta}_1) \varphi_i(\hat{\theta}_1)'$ (the inverse of weighting matrix for the two-step GMM estimator) with $\hat{\theta}_1$ being the one-step GMM estimator for θ . The estimator of the variance of $\hat{\theta}_2$ is defined as

$$\hat{V}(\hat{\theta}_2) = \frac{1}{N} \left(D(\hat{\theta}_2)' \left(\bar{\Theta}(\hat{\theta}_1) \right)^{-1} D(\hat{\theta}_2) \right)^{-1},$$

where

$$D(\hat{\theta}_2) = \frac{1}{N} \sum_{i=1}^N \frac{\partial \varphi_i(\theta)}{\partial \theta'} \Big|_{\theta=\hat{\theta}_2}.$$

Two types of the Sargan test statistic are defined for the two-step GMM estimator: that with the weighting matrix being estimated by the one-step GMM estimator and that with the weighting matrix being estimated by the two-step GMM estimator. The former is

$$N \bar{\varphi}(\hat{\theta}_2)' \left(\bar{\Theta}(\hat{\theta}_1) \right)^{-1} \bar{\varphi}(\hat{\theta}_2),$$

while the latter is

$$N \bar{\varphi}(\hat{\theta}_2)' \left(\bar{\Theta}(\hat{\theta}_2) \right)^{-1} \bar{\varphi}(\hat{\theta}_2),$$

both of which are asymptotically distributed as chi-square with degree of freedom being the number of moment restrictions minus the number of parameters to be estimated.

On the other hand, the one-step GMM estimator for θ is defined as

$$\hat{\theta}_1 = \arg \min_{\theta} \bar{\varphi}(\theta)' \left(\bar{\Theta}_Z \right)^{-1} \bar{\varphi}(\theta)$$

where $\bar{\Theta}_Z = (1/N) \sum_{i=1}^N Z_i' Z_i$ (the inverse of weighting matrix for the one-step GMM estimator). The estimator of the variance of $\hat{\theta}_1$ is defined as

$$\hat{V}(\hat{\theta}_1) = \hat{V}_A(\hat{\theta}_1) \left(\hat{V}_B(\hat{\theta}_1) \right)^{-1} \hat{V}_A(\hat{\theta}_1)$$

with

$$\hat{V}_A(\hat{\theta}_1) = \frac{1}{N} \left(D(\hat{\theta}_1)' \left(\bar{\Theta}_Z \right)^{-1} D(\hat{\theta}_1) \right)^{-1}$$

and

$$\hat{V}_B(\hat{\theta}_1) = \frac{1}{N} \left(D(\hat{\theta}_1)' (\bar{\Theta}_Z)^{-1} \bar{\Theta}(\hat{\theta}_1) (\bar{\Theta}_Z)^{-1} D(\hat{\theta}_1) \right)^{-1}.$$

The Wald tests for the null hypothesis that the time dummies to be estimated are jointly zero are defined for the one-step and two-step estimators. For the Wald test, see Hall and Cummins (2009).

3 Illustrative processing

This section explains the executing procedure of the program “dfelrtna.tsp” using a practical example. The dataset supplied is artificially designed. For the model with $K = 2$ in the previous section, the following variables are supplied:

Dependent variable y_{it} : fls (female labor supply, binary variable: 1=supply; 0=no supply)

Explanatory variable $x_{(1)it}$: hdbt (log of husband’s debt, continuous variable)

Explanatory variable $x_{(2)it}$: hinc (log of husband’s income, continuous variable)

The program deals with the nonlinear GMM estimation as seen from the previous section. In this case, the local minimum problem can arise presumably due to inappropriate starting values of the parameters of interest in the process of minimizing the GMM objective function. To circumvent this problem as far as possible, the program generates multiple sets of random starting values to carry out multiple minimization trials. Then, the “best” one-step and two-step GMM estimates, that is, those whose Sargan test statistic (with weighting matrix estimated using the two-step estimates) is minimal, are selected.¹ It will take a considerable amount of time to run the program to completion if a large number of trials are carried out.

3.1 Data arrangement

The Microsoft EXCEL 97-2003 worksheet files for the dependent and explanatory variables must be prepared separately. The names of the files are composed of the variable names and the postfix “_dt” with the extension “.xls”. For example, if the names of the dependent and two explanatory variables are “fls”, “hdbt”, and “hinc”, respectively, the corresponding file names must be “fls_dt.xls”, “hdbt_dt.xls”, and “hinc_dt.xls”, respectively.

In the EXCEL files, the variable “icode”, which represents the individual code, must

¹ In this case, non-convergent results in either one-step or two-step estimation are excluded. In addition, we rule out the results where the rank deficiency occurs in calculating the Sargan test statistic (with weighting matrix estimated using the two-step estimates).

be in the first column. From the next column, the data variables for respective time periods are furnished, whose names are the amalgamation of [variable name] and [time period]. The [time period]s must be in ascending order from left to right, without omission.² The datasets used must be balanced panel datasets and no omission of the individuals is permitted among the files for the dependent and explanatory variables.

For the illustrative example, the files are as follows:

File: fls_dt.xls

icode	fls1986	fls1987	fls1988	fls1989	fls1990	fls1991	fls1992	fls1993
1	1	0	1	1	1	1	1	1
2	1	1	0	0	0	1	1	0
3	1	1	1	0	1	0	1	1
4	1	1	0	1	1	0	1	1
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:

File: hdbt_dt.xls

icode	hdbt1986	hdbt1987	hdbt1988	hdbt1989	hdbt1990	hdbt1991	hdbt1992	hdbt1993
1	0.008384089	0.612204731	1.243270993	1.847956657	-0.122421026	0.191675633	1.053792596	0.646528363
2	0.066455834	0.371744335	0.079718068	-0.089538522	-0.463966757	1.139615059	0.046116028	-0.356241554
3	-0.18348746	0.770572007	0.305097818	-1.286834717	-0.961165428	-1.469172478	-0.502285123	0.595689833
4	0.647827208	0.316122264	-0.021330625	-1.058970451	-0.67933619	-1.006081939	0.082888335	-0.99553293
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:

File: hinc_dt.xls

icode	hinc1986	hinc1987	hinc1988	hinc1989	hinc1990	hinc1991	hinc1992	hinc1993
1	0.291868627	0.072085671	0.177277103	-0.250184596	0.266811132	1.061818719	0.13831982	-1.254481673
2	0.248155102	0.048316468	-0.452102125	-1.146645904	-1.040278316	-0.872839212	-0.659799874	0.398704469
3	0.384251148	0.576001644	0.636672735	-1.36213088	-0.752219856	-0.261004239	-0.313866496	0.431640714
4	-0.252126038	-0.406211466	-1.192754149	-1.269641042	-0.360407323	-0.331675142	-0.263569057	0.420563102
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:

3.5 Command setup

The setup of the commands (parameters) needed for running the program is conducted in the file “prma.tsp”, which is edited by the user. The TSP command “list” gives a single name to a list of TSP variables, while “set” stores a new scalar value. The start of the comment line must be a question mark “?”. The semicolon “;” is the separator between commands. An illustration of the setup in the file “prma.tsp” (with highlighted explanations that are not part of the file) is as follows:

² In this data arrangement, the program cannot deal with the EXCEL files with more than 65536 rows of data. For the information for loading the files with more rows, see the manual “docum_ex2.pdf” in the folder “estim_ex2”. In this folder, the estimation example is completed using the files with more rows. See also section 4 in this document.

? Parameter Setting File for dfelrtna.tsp

```
list dpv fls ;
```

The **d**ependent **v**ariable used in the estimations (which is “fls” in this case) is defined as the dependent variable list “dpv”.

```
list exv hdbt hinc ;
```

The **e**xplanatory **v**ariables used in the estimations (which are “hdbt” and “hinc” in this case) are defined as the explanatory variables list “exv”.

```
list lead_hdbt 0 ; list ctmp_hdbt 1 ; list lag_hdbt 0 ;
```

The **l**eads and **l**ags of the explanatory variable “hdbt” used in the model are specified in the lists “lead_hdbt” and “lag_hdbt”, respectively. A value of 0 means none are used. To use the third and first leads write “list lead_hdbt 3 1”, to use the first and second lags write “list lag_hdbt 1 2”, and so on. The use of the **c**ontemporary value is indicated by the value of “ctmp_hdbt”: if the contemporary value is not used, its value is 0; if it is, the value is 1. In this case, no leads and no lags of “hdbt” are used, but the contemporary value is used.

```
list lead_hinc 0 ; list ctmp_hinc 1 ; list lag_hinc 0 ;
```

These are the corresponding lists for the other explanatory variable “hinc” used in the model. In this case, no leads and no lags of “hinc” are used, but the contemporary value is used

```
set cnob = 15000 ;
```

The cross-sectional sample size (number of individuals) is set to the variable “cnob”, which is 15000 in this case.

```
set drm = 1 ;
```

The variable “drm” specifies the method of reading the dataset. If “drm = 1”, the dataset is loaded directly from the EXCEL worksheet files exhibited in the previous subsection. However, this specification allows us to load up to 65536 rows of data. When loading more rows, “drm” should be set to 0 such that “drm = 0”, the appropriate statements should be written in the file “detra.tsp”, and the EXCEL worksheet files should be converted into the CSV files after eliminating the headers (see the manual “docum_ex2.pdf” in the folder “estim_ex2” for more information).

```
set spy = 1986 ; set epy = 1991 ;
```

These parameters set the time span for the dependent variable used in the estimation equations. The parameter “spy” is the start year (in this case 1986), while “epy” is the end year (in this case 1991). The number of equations plus two must equal to $epy - spy$.

```
set spu_fls = 1986 ; set epu_fls = 1991 ;
```

These parameters set the start and end years for the variable “fls” used in the estimation equations and/or the instruments. The parameter “spu_fls” is the start year (in this case 1986) for the variable “fls”, while “epu_fls” is the end year (in this case 1991).

```
set spu_hdbt = 1986 ; set epu_hdbt = 1991 ;
```

These parameters set the start and end years for the variable “hdbt” used in the estimation equations and/or the instruments. The parameter “spu_hdbt” is the start year (in this case 1986) for the variable “hdbt”, while “epu_hdbt” is the end year (in this case 1991).

```
set spu_hinc = 1986 ; set epu_hinc = 1991 ;
```

These parameters set the start and end years for the variable “hinc” used in the estimation equations or the instruments. The parameter “spu_hinc” is the start year (in this case 1986) for the variable “hinc”, while “epu_hinc” is the end year (in this case 1991).

```
set fls_sg = -2 ; set fls_eg = -99 ;
```

These parameters set the lead/lag range for the variable “fls” used as instruments for the g-form and h-form. Positive, zero, and negative values represent leads, contemporary values, and lags, respectively, for the g-form and h-form at time t . The value of “fls_sg” must be greater than or equal to that of “fls_eg”. In this case, the variables “fls” dated $t - 2$ and before (until $t - 99$) are used as instruments for the g-form and h-form at time t .

```
set d_hdbt_sg = 1 ; set d_hdbt_eg = -1 ;
```

These parameters set the lead/lag range for the first-difference of the variable “hdbt” (i.e., d_hdbt) used as instruments for the g-form and h-form. Positive, zero, and negative values represent leads, contemporary values, and lags, respectively, for the g-form and

h-form at time t . The value of “d_hdbt_sg” must be greater than or equal to that of “d_hdbt_eg”. In this case, the variables “d_hdbt” at $t + 1$, t , and $t - 1$ are used as instruments for the g-form and h-form dated t .

```
set d_hinc_sg = 1 ; set d_hinc_eg = -1 ;
```

These parameters set the lead/lag range for the first-difference of the variable “hinc” (i.e., d_hinc) used as instruments for the g-form and h-form. Positive, zero, and negative values represent leads, contemporary values, and lags, respectively, for the g-form and h-form at time t . The value of “d_hinc_sg” must be greater than or equal to that of “d_hinc_eg”. In this case, the variables “d_hinc” at $t + 1$, t , and $t - 1$ are used as instruments for the g-form and h-form at t .

```
set cinst = 1 ;
```

The variable “cinst” specifies whether the constant (i.e. one) is used as the instrument for the g-form and h-form at time t or not. If the constant is used, “cinst = 1”; if not, “cinst = 0”..

```
set g_fls_lag1_min = -2 ; set g_fls_lag1_max = 2 ;
```

The range of the randomly generated starting values of the coefficient (“g_fls_lag1”) on the 1 lagged value of the dependent variable “fls” is set in order to carry out the nonlinear optimization in the GMM estimations. In this case, the minimal value (“g_fls_lag1_min”) is set to -2 , while the maximal value (“g_fls_lag1_max”) is set to 2 . The initial letter “g” stands for “gamma”.

```
set b_hdbt_min = -2 ; set b_hdbt_max = 2 ;
```

The minimum and maximum of the randomly generated starting values of the coefficient (“b_hdbt”) on the explanatory variable “hdbt” are set for use in the nonlinear optimization in the GMM estimations. The initial letter “b” stands for “beta”.

```
set b_hinc_min = -2 ; set b_hinc_max = 2 ;
```

The minimum and maximum of the randomly generated starting values of the coefficient (“b_hinc”) on the explanatory variable “hinc” are set for use in the nonlinear optimization in the GMM estimations.

```
set tds = 1 ;
```

The variable “tds” specifies whether the time dummies to be estimated are included in

the estimation equations for the g-form and h-form or not. If the time dummies are included, “tds = 1”; if not, “tds = 0”.

```
set dtd1988_min = -2 ; set dtd1988_max = 2 ;  
set dtd1989_min = -2 ; set dtd1989_max = 2 ;  
set dtd1990_min = -2 ; set dtd1990_max = 2 ;  
set dtd1991_min = -2 ; set dtd1991_max = 2 ;
```

The minima and maxima of the randomly generated starting values of the first-differenced time dummy coefficients for 1988 to 1991 are set for use in the nonlinear optimization in the GMM estimations. These are only used if “set tds = 1”.

```
set ufm = 3 ;
```

The variable “ufm” specifies the transformations used in the estimation. If ufm = 1, only the g-form is used; if ufm = 2, only the h-form is used; and if ufm = 3, both forms are jointly used for the estimation.

```
set ig2 = 1 ;
```

The variable “ig2” specifies whether to use the one-step GMM estimates or randomly generated starting values as the starting values of the parameters of interest in the optimization for obtaining the two-step GMM estimates. If “ig2 = 0”, randomly generated starting values are used; if “ig2 = 1”, the one-step GMM estimates are used.

```
set ntrial = 100 ;
```

The variable “ntrial” specifies the number of the optimization trials for obtaining the GMM estimates that minimizes the two-step GMM criterion function where the weighting matrix is estimated by the two-step estimates. If “ig2 = 0”, the number of sets of the starting values randomly generated in the optimization equals to “ntrial” times two, since the generation of the starting values occurs in both the one-step and two-step estimations.

```
set sdgp = 256 ;
```

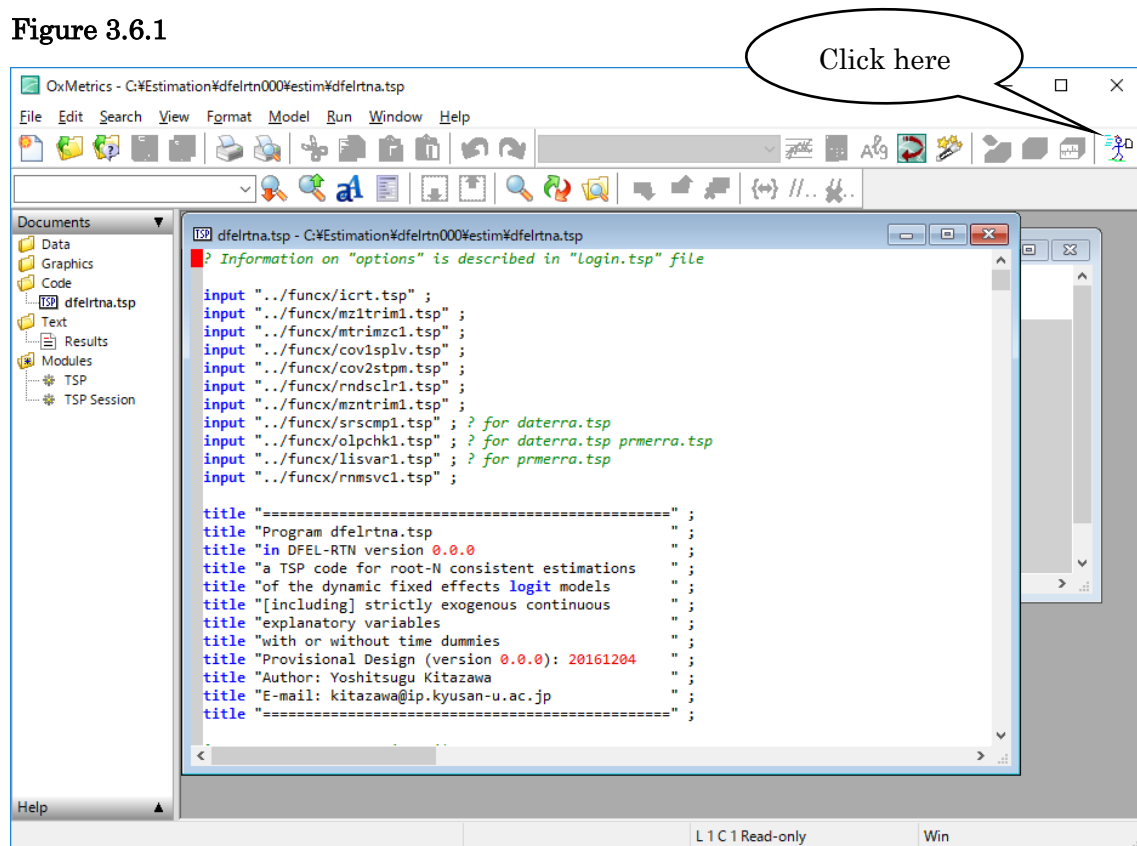
The variable “sdgp” sets the seed for generating random starting values for the parameters of interest in the optimization trials. If “sdgp = -99999”, the seed is automatically created in TSP. To obtain reproducible results, any seed value except “-99999” can be assigned to sdgp. In this case, the seed is 256.

3.6 Running the program

After completing the data arrangement and command setup, double-click the file “dfelrtna.tsp” to launch OxMetrics. Then, click the “Run” button to run the program (see Figure 3.6.1).

It might take a considerable time (e.g., several days, as the case may be) to obtain conclusive estimation results, if the number of optimization trials “ntrial” is large.

Figure 3.6.1



3.7 Run termination

If the run terminates successfully, the output is displayed in the OxMetrix screen (see Figure 3.7.1). At the end of the output, the termination sentence “END OF OUTPUT” is printed. If the optimizations do not work out in some trials, a number of “ERROR MESSAGES”, “NUMERIC WARNINGS”, “WARNING MESSAGES”, and so on are printed.

At this stage, the output file is not saved. To do this, first click the frame of the inner window “dfeirtna.out” and then click the “File” button in the OxMetrix screen (see Figure 3.7.1). Next, select the menu “Save As...” in the emerging menu bar (see Figure 3.7.2). Third, click the “Save (S)” button in the “Save as” dialog to save the output file “dfeirtna.out” in the subfolder “estim” (see Figure 3.7.3).

Figure 3.7.1

Click here.

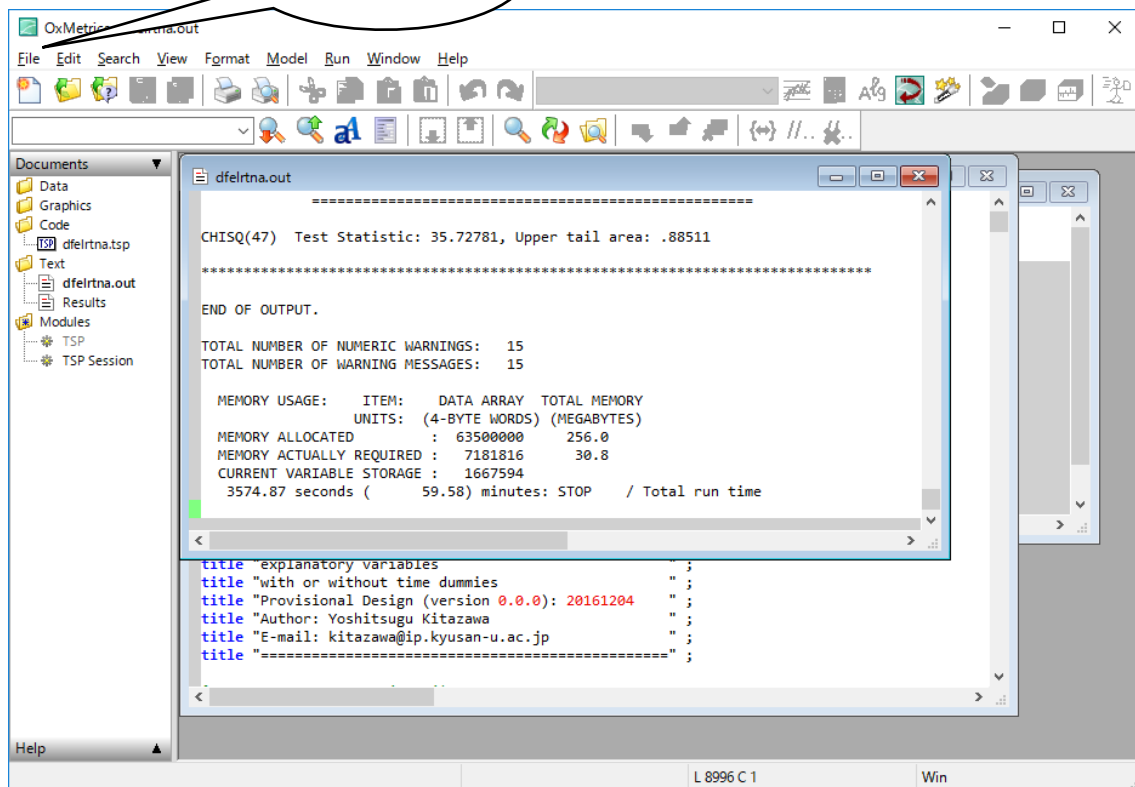


Figure 3.7.2

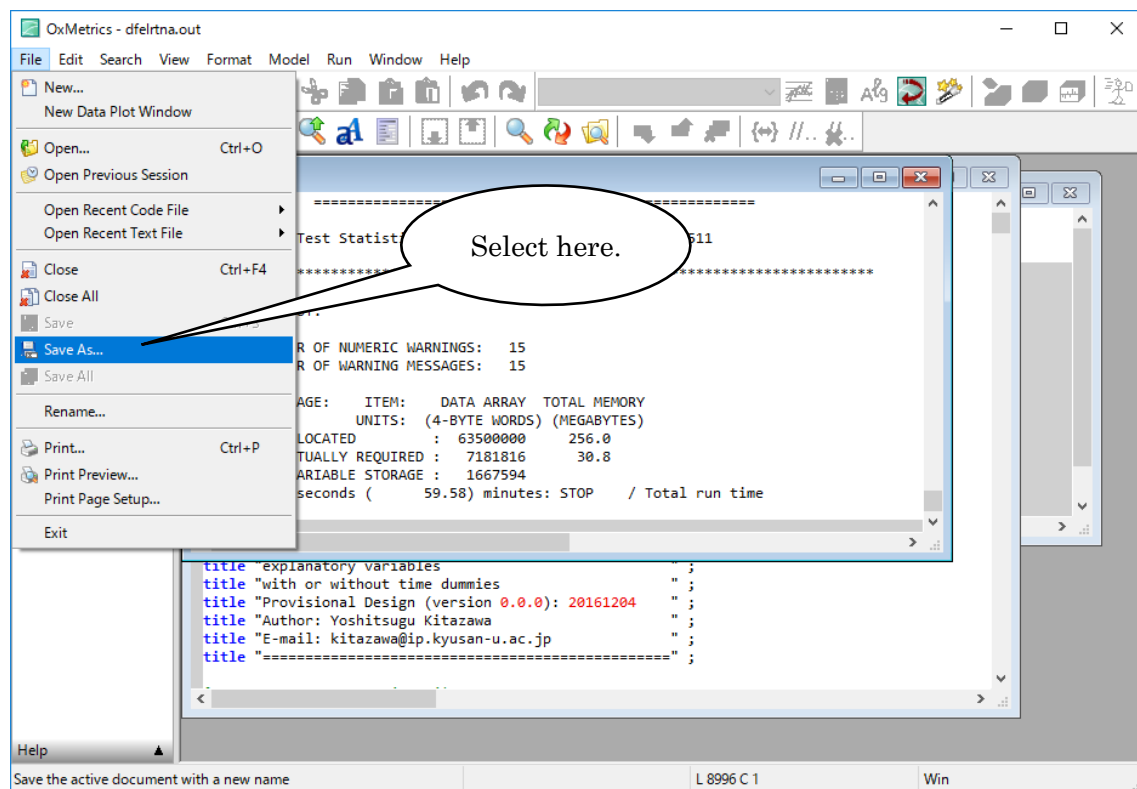
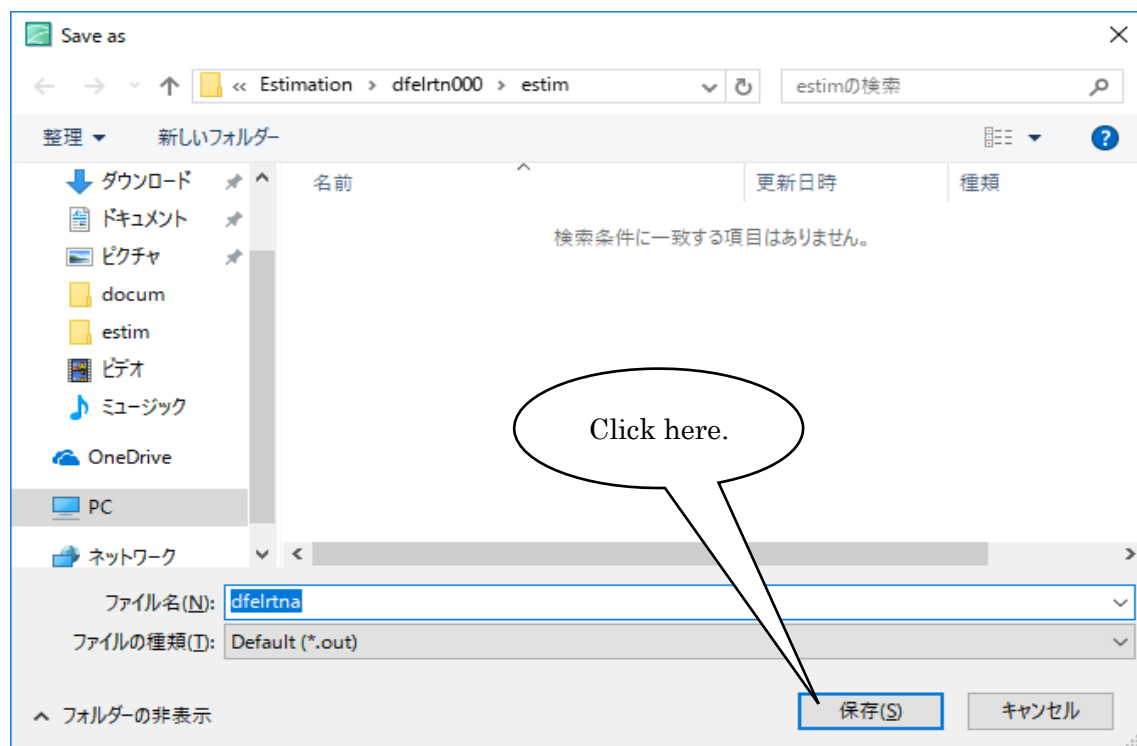


Figure 3.7.3



3.8 Output

The program outputs the files “ghresa.txt” and “ghsrta.txt” in addition to “dfelrtna.out”.

The file “dfelrtna.out” is the main output file of the estimation results, whose final part is illustrated in Exhibition 3.8.1. In the last part, the best estimation results are summarized: the transformed equations, instruments, one-step and two-step estimates, Sargan test statistics, and Wald test statistics for the null hypothesis that the time dummies are jointly zero, etc. In the middle of the file, there are a series of estimation results for the optimization trials where different randomly generated starting values are used.

The file “ghresa.txt” summarizes the estimation results. It contains the best estimation results: the one-step and two-step estimates, Sargan test statistics, and Wald test statistics for the case where the time dummies are specified. The file “ghsrta.txt” holds the randomly generated starting values used for obtaining the best estimation results. Illustrations of “ghresa.txt” and “ghsrta.txt” are shown in Exhibitions 3.8.2 and 3.8.3.

Exhibition 3.8.1

Last part of the output file “dfelrtna.out”

(Last part of the results)

```
*****
=====
***** Dynamic Fixed Effects Logit Model *****
=====
***** including Strictly Exogenous Continuous Regressors *****
=====
***** with or without time dummies *****
=====
```

Transformation Used for the root-N Consistent Estimator:

g-form + h-form (see Kitazawa, 2016)

● Equations used for the estimations (g-form dated 1988-90 and h-form dated 1988-90)

Equations Transformed (g-form + h-form)

■ Equation in g-form dated 1988

EQUATION: HTD_ELG1988

FRML HTD_ELG1988 Y_FLS1988 + (1 - Y_FLS1988)*Y_FLS1989 - (1 -
Y_FLS1988)*Y_FLS1989*EXP(-(0 + B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 +
B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988)) - DTD1989) + -(EXP(G_FLS_LAG1) -
1)*Y_FLS1987*(1 - Y_FLS1988)*Y_FLS1989*EXP(-(0 + B_HDBT*X_HDBT1989 +
B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988)) - DTD1989) -
Y_FLS1987 - (EXP(-G_FLS_LAG1*Y_FLS1986 + 0 + B_HDBT*X_HDBT1989 +
B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1987 + B_HINC*X_HINC1987) + DTD1989 +
DTD1988) - 1)/(EXP(-G_FLS_LAG1*Y_FLS1986 + 0 + B_HDBT*X_HDBT1989 +
B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1987 + B_HINC*X_HINC1987) + DTD1989 +
DTD1988) + 1)*(Y_FLS1988 + (1 - Y_FLS1988)*Y_FLS1989 - (1 -
Y_FLS1988)*Y_FLS1989*EXP(-(0 + B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 +

$$\begin{aligned}
& B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988)) - DTD1989) + -(EXP(G_FLS_LAG1) - \\
& 1)*Y_FLS1987*(1 - Y_FLS1988)*Y_FLS1989*EXP(-(0 + B_HDBT*X_HDBT1989 + \\
& B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988)) - DTD1989) + \\
& Y_FLS1987 - 2*(Y_FLS1988 + (1 - Y_FLS1988)*Y_FLS1989 - (1 - \\
& Y_FLS1988)*Y_FLS1989*EXP(-(0 + B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 + \\
& B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988)) - DTD1989) + -(EXP(G_FLS_LAG1) - \\
& 1)*Y_FLS1987*(1 - Y_FLS1988)*Y_FLS1989*EXP(-(0 + B_HDBT*X_HDBT1989 + \\
& B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988)) - \\
& DTD1989))*Y_FLS1987)
\end{aligned}$$

■ Equation in g-form dated 1989

EQUATION: HTD_ELG1989

(abbrev.)

■ Equation in g-form dated 1990

EQUATION: HTD_ELG1990

(abbrev.)

■ Equation in h-form dated 1988

EQUATION: HTD_ELH1988

$$\begin{aligned}
& FRML HTD_ELH1988 Y_FLS1988*Y_FLS1989 + Y_FLS1988*(1 - Y_FLS1989)*EXP(0 \\
& + B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + \\
& B_HINC*X_HINC1988) + DTD1989) + (EXP(G_FLS_LAG1) - 1)*(1 - \\
& Y_FLS1987)*Y_FLS1988*(1 - Y_FLS1989)*EXP(0 + B_HDBT*X_HDBT1989 + \\
& B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988) + DTD1989) - \\
& Y_FLS1987 - (EXP(G_FLS_LAG1*(1 - Y_FLS1986) + 0 + B_HDBT*X_HDBT1989 + \\
& B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1987 + B_HINC*X_HINC1987) + DTD1989 + \\
& DTD1988) - 1)/(EXP(G_FLS_LAG1*(1 - Y_FLS1986) + 0 + B_HDBT*X_HDBT1989 + \\
& B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1987 + B_HINC*X_HINC1987) + DTD1989 + \\
& DTD1988) + 1)*(Y_FLS1988*Y_FLS1989 + Y_FLS1988*(1 - Y_FLS1989)*EXP(0 + \\
& B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + \\
& B_HINC*X_HINC1988) + DTD1989) + (EXP(G_FLS_LAG1) - 1)*(1 - \\
& Y_FLS1987)*Y_FLS1988*(1 - Y_FLS1989)*EXP(0 + B_HDBT*X_HDBT1989 + \\
& B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988) + DTD1989) + \\
& Y_FLS1987 - 2*(Y_FLS1988*Y_FLS1989 + Y_FLS1988*(1 - Y_FLS1989)*EXP(0 +
\end{aligned}$$

$$B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988) + DTD1989) + (EXP(G_FLS_LAG1) - 1)*(1 - Y_FLS1987)*Y_FLS1988*(1 - Y_FLS1989)*EXP(0 + B_HDBT*X_HDBT1989 + B_HINC*X_HINC1989 - (0 + B_HDBT*X_HDBT1988 + B_HINC*X_HINC1988) + DTD1989))*Y_FLS1987)$$

■ Equation in h-form dated 1989

EQUATION: HTD_ELH1989

(abbrev.)

■ Equation in h-form dated 1990

EQUATION: HTD_ELH1990

(abbrev.)

+++++

=====

● Equations (in g-form dated 1988-90 and in h-form dated 1988-90) and Instruments for each of them

Equations and Instruments for Each Time Period

=====

■ Instruments for equation in g-form dated 1988

EQUATION = HTD_ELH1988

INSTRUMENTS = Y_FLS1986 DX_HDBT1987 DX_HDBT1988 DX_HDBT1989
DX_HINC1987 DX_HINC1988 DX_HINC1989 C

=====

■ Instruments for equation in g-form dated 1989

EQUATION = HTD_ELH1989

INSTRUMENTS = Y_FLS1986 Y_FLS1987 DX_HDBT1988 DX_HDBT1989 DX_HDBT1990
DX_HINC1988 DX_HINC1989 DX_HINC1990 C

=====

■ Instruments for equation in g-form dated 1990

EQUATION = HTD_ELH1990

INSTRUMENTS = Y_FLS1986 Y_FLS1987 Y_FLS1988 DX_HDBT1989 DX_HDBT1990

DX_HDBT1991 DX_HINC1989 DX_HINC1990 DX_HINC1991 C

=====

■ Instruments for equation in h-form dated 1988

EQUATION = HTD_ELH1988

INSTRUMENTS = Y_FLS1986 DX_HDBT1987 DX_HDBT1988 DX_HDBT1989
DX_HINC1987 DX_HINC1988 DX_HINC1989 C

=====

■ Instruments for equation in h-form dated 1989

EQUATION = HTD_ELH1989

INSTRUMENTS = Y_FLS1986 Y_FLS1987 DX_HDBT1988 DX_HDBT1989 DX_HDBT1990
DX_HINC1988 DX_HINC1989 DX_HINC1990 C

=====

■ Instruments for equation in h-form dated 1990

EQUATION = HTD_ELH1990

INSTRUMENTS = Y_FLS1986 Y_FLS1987 Y_FLS1988 DX_HDBT1989 DX_HDBT1990
DX_HDBT1991 DX_HINC1989 DX_HINC1990 DX_HINC1991 C

=====

■ Parameters of interest (to be estimated)

PARAMETERS = G_FLS_LAG1 B_HDBT B_HINC DTD1988 DTD1989 DTD1990
DTD1991

=====

◆ The number of individuals (N) is 15000. At the 25th optimization trial (choice of starting values for the parameters to be estimated) in 100 trials, the two-step GMM criterion function where the weighting matrix is estimated by the two-step estimates is minimized. Accordingly, the GMM estimates for this trial are the best.

■ Cross-sectional sample size.

Crossectional size, N

=====

CNOB = 15000.00000

■ Number of trials (ntrial) and Best trial (try_opt)

Number of trials and Choicest trial

=====

	NTRIAL	TRY_OPT
Value	100.00000	25.00000

● Best one-step estimation results

■ Starting values used in the optimization for obtaining the best one-step estimates

Selected Starting values for the best 1-step estimates

=====

G_FLS_LAG1_I_OPT1 = 1.08562
B_HDBT_I_OPT1 = -0.65642
B_HINC_I_OPT1 = -0.81741
DTD1988_I_OPT1 = -0.057223
DTD1989_I_OPT1 = -1.43212
DTD1990_I_OPT1 = 0.87231
DTD1991_I_OPT1 = -0.36591

■ Convergence flag for the best one-step estimates: 1 if the estimates have converged; 0 otherwise

Best 1-step estimates: converge (1) or not (0)

=====

IFCONV_OPT1 = 1.00000

■ Best one-step GMM estimation result in the selected 25th optimization trial

<1-step estimation, best estimates>

=====

		Standard		
Parameter	Estimate	Error	t-statistic	P-value
G_FLS_LAG1	.485539	.045718	10.6204	[.000]
B_HDBT	.480950	.025422	18.9184	[.000]
B_HINC	-.516664	.025756	-20.0596	[.000]
DTD1988	-1.50647	.034725	-43.3827	[.000]

DTD1989	. 500580	. 030069	16. 6478	[. 000]
DTD1990	-. 495048	. 029955	-16. 5262	[. 000]
DTD1991	. 999249	. 052430	19. 0589	[. 000]

■ Wald test statistic whose null hypothesis is that the time dummies (DTD1988 - DTD1991) are jointly zero, for the best one-step estimation (with degree of freedom in parentheses)

Wald test for jointly zero Time Dummies, 1-step

=====

CHISQ(4) Test Statistic: 2506.906, Upper tail area: .00000

● Best two-step estimation results

■ Starting values used in the optimization for obtaining the best two-step estimates (In this case, the one-step estimates are used for the starting values, since “ig2 = 1”).

Selected starting values for the best 2-step estimates

=====

G_FLS_LAG1_I_OPT2 = 0. 48554

B_HDBT_I_OPT2 = 0. 48095

B_HINC_I_OPT2 = -0. 51666

DTD1988_I_OPT2 = -1. 50647

DTD1989_I_OPT2 = 0. 50058

DTD1990_I_OPT2 = -0. 49505

DTD1991_I_OPT2 = 0. 99925

■ Convergence flag for the best one-step estimates: 1 if the estimates have converged; 0 otherwise

Best 2-step estimates: converge (1) or not (0)

=====

IFCONV_OPT2 = 1. 00000

■ Best two-step GMM estimation result in the selected 25th optimization trial

<2-step estimation, best estimates>

=====

Parameter	Estimate	Standard Error	t-statistic	P-value
G_FLS_LAG1	. 491837	. 033032	14. 8899	[. 000]

B_HDBT	.505240	.018027	28.0272	[.000]
B_HINC	-.512008	.017798	-28.7674	[.000]
DTD1988	-1.53304	.030268	-50.6490	[.000]
DTD1989	.505520	.027735	18.2268	[.000]
DTD1990	-.487902	.027504	-17.7396	[.000]
DTD1991	.956973	.037553	25.4830	[.000]

■ Wald test statistic whose null hypothesis is that the time dummies (DTD1988 - DTD1991) are jointly zero, for the best two-step estimation (with degree of freedom in parentheses)

Wald test for jointly zero Time Dummies, 2-step

=====

CHISQ(4) Test Statistic: 3364.939, Upper tail area: .00000

■ Sargan test statistic where the weighting matrix is estimated by using the best one-step residuals (with degree of freedom in parentheses)

Sargan test for the overidentification, wmat(1-step)

=====

CHISQ(47) Test Statistic: 35.39479, Upper tail area: .89295

■ Sargan test statistic where the weighting matrix is estimated by using the best two-step residuals (with degree of freedom is in parentheses)

Sargan test for the overidentification, wmat(2-step)

=====

CHISQ(47) Test Statistic: 35.72781, Upper tail area: .88511

Exhibition 3.8.2

Contents of the output file “ghresa.txt”

■ Only using g-form if “UFM = 1”; only using h-form if “UFM = 2”; using both g-form and h-form if “UFM = 3”

UFM = 3.00000

■ Cross-sectional size

CNOB = 15000.00000

■ Number of trials and Best trial

		NTRIAL_TRYOPT
		1
1	100.00000	■ Number of trials
2	25.00000	■ Best trial

■ Convergence flag for the best one-step estimates: 1 if the estimates have converged; 0 otherwise

IFCONV_OPT1 = 1.00000

■ Best one-step GMM estimates

G_FLS_LAG1 = 0.48554

B_HDBT = 0.48095

B_HINC = -0.51666

DTD1988 = -1.50647

DTD1989 = 0.50058

DTD1990 = -0.49505

DTD1991 = 0.99925

■ Best one-step GMM estimates (left, Est.) and their standard errors (right, S.E.) given in the same order as the list of parameters (to be estimated) above

COEF_SES1					
	1	2			
1	0.48554	0.045718	■ G_FLS_LAG1	Est.	S. E.
2	0.48095	0.025422	■ B_HDBT	Est.	S. E.
3	-0.51666	0.025756	■ B_HINC	Est.	S. E.
4	-1.50647	0.034725	■ DTD1988	Est.	S. E.
5	0.50058	0.030069	■ DTD1989	Est.	S. E.
6	-0.49505	0.029955	■ DTD1990	Est.	S. E.
7	0.99925	0.052430	■ DTD1991	Est.	S. E.

■ Wald test statistic whose null hypothesis is that the time dummies (DTD1988 - DTD1991) are jointly zero, for the best one-step estimation

WALD0TD_DF_PV1			
	1		
1	2506.90630	■ Wald test statistic	
2	4.00000	■ Degree of freedom	
3	0.00000	■ P-value	

■ Convergence flag for the best two-step estimates: 1 if the estimates have converged; 0 otherwise

IFCONV_OPT2 = 1.00000

■ Best two-step GMM estimates

G_FLS_LAG1 = 0.49184

B_HDBT = 0.50524

B_HINC = -0.51201

DTD1988 = -1.53304

DTD1989 = 0.50552

DTD1990 = -0.48790

DTD1991 = 0.95697

■ Best two-step GMM estimates (left, Est.) and their standard errors (right, S.E.) given in the same order as the list of parameters (to be estimated) above

COEF_SES2					
	1	2			
1	0.49184	0.033032	■ G_FLS_LAG1	Est.	S. E.
2	0.50524	0.018027	■ B_HDBT	Est.	S. E.
3	-0.51201	0.017798	■ B_HINC	Est.	S. E.
4	-1.53304	0.030268	■ DTD1988	Est.	S. E.
5	0.50552	0.027735	■ DTD1989	Est.	S. E.
6	-0.48790	0.027504	■ DTD1990	Est.	S. E.
7	0.95697	0.037553	■ DTD1991	Est.	S. E.

■ Wald test statistic whose null hypothesis is that the time dummies (DTD1988 - DTD1991) are jointly zero, for the best two-step estimation

WALD0TD_DF_PV2			
	1		
1	3364.93917	■ Wald test statistic	
2	4.00000	■ Degree of freedom	
3	0.00000	■ P-value	

■ Sargan test statistic where the weighting matrix is estimated using the best one-step residuals

		SARGANW1_DF_PV2
1		
1	35.39479	■ Sargan test statistic
2	47.00000	■ Degree of freedom
3	0.89295	■ P-value

■ Sargan test statistic where the weighting matrix is estimated using the best two-step residuals

		SARGANW2_DF_PV2
1		
1	35.72781	■ Sargan test statistic
2	47.00000	■ Degree of freedom
3	0.88511	■ P-value

Exhibition 3.8.3

Contents of the output file “ghsrta.txt”

■ Only using g-form if “UFM = 1”; only using h-form if “UFM = 2”; using both g-form and h-form if “UFM = 3”

UFM = 3.00000

NTRIAL = 100.00000 ■ Number of trials

TRY_OPT = 25.00000 ■ Choicest trial

■ Randomly generated starting values used for obtaining the best one-step estimates

G_FLS_LAG1_I_OPT1 = 1.08562

B_HDBT_I_OPT1 = -0.65642

B_HINC_I_OPT1 = -0.81741

DTD1988_I_OPT1 = -0.057223

DTD1989_I_OPT1 = -1.43212

DTD1990_I_OPT1 = 0.87231

DTD1991_I_OPT1 = -0.36591

■ Starting values used for obtaining the best two-step estimates; these are the best one-step estimates, since “ig2 = 1”

G_FLS_LAG1_I_OPT2 = 0.48554

B_HDBT_I_OPT2 = 0.48095

B_HINC_I_OPT2 = -0.51666

DTD1988_I_OPT2 = -1.50647

DTD1989_I_OPT2 = 0.50058

DTD1990_I_OPT2 = -0.49505

DTD1991_I_OPT2 = 0.99925

4 Notes

The default setting of the memory size (in Mb) used by the TSP program “dfelrtna.tsp” is 256. If the estimations are conducted using standard personal computers and reasonably large datasets, there will be no trouble. However, if old computers or exceedingly large datasets are used, we should revise the special input file “login.tsp”. For the former case, we should decrease the memory size within that of the personal computer. In the latter case, we should increase the allocated memory size, convert EXCEL worksheet files into CSV files after eliminating the headers and write the adequate statements in the file “detra.tsp” (see the manual “docum_ex2.pdf” and “detra.tsp” in the folder “estim_ex2” for more information). For example, when we change the allocated memory size from 256 Mb to 128 Mb, we convert “memory = 256” into “memory = 128” in the statement starting from “options” in the file “login.tsp”.

References

Hall, B.H., Cummins, C., 2009. TSP 5.1 User's Guide. TSP International, Palo Alto, CA.

Kitazawa, Y., 2013. Exploration of dynamic fixed effects logit models from a traditional angle. Discussion Paper Series, Faculty of Economics, Kyushu Sangyo University, April 2013, No. 60.

<http://www.ip.kyusan-u.ac.jp/keizai-kiyo/dp60.pdf>

Kitazawa, Y., 2016. Root-N consistent estimations of time dummies for the dynamic fixed effects logit models: Monte Carlo illustrations. Discussion Paper Series, Faculty of Economics, Kyushu Sangyo University, March 2016, No. 72.

<http://www.ip.kyusan-u.ac.jp/keizai-kiyo/dp72.pdf>